

ENCOURAGE HOME TALENT.

LOVELL'S SERIES OF SCHOOL BOOKS.

# KEY

TO THE

## NATIONAL ARITHMETIC;

CONTAINING

Full Solutions to nearly all the Problems.

DESIGNED FOR THE

USE OF TEACHERS AND PRIVATE STUDENTS.

BY JOHN HERBERT SANGSTER, M.A., M.D.,

MATHEMATICAL MASTER AND LECTURER IN CHEMISTRY AND NATURAL  
PHILOSOPHY IN THE NORMAL SCHOOL FOR UPPER CANADA.

SECOND EDITION—CAREFULLY REVISED.

Montreal

PRINTED AND PUBLISHED BY JOHN LOVELL.

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1867.

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## P R E F A C E .

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It was the original intention of the Author to give, in the Key, merely a series of brief hints upon the Solutions of the more difficult Problems. He was led to modify this plan, and to issue the work in its present form, chiefly from the consideration that as there are in the country many young persons who, from various causes, are unable to avail themselves of the advice and assistance of a teacher, it would be a great boon to these to have access to a book to which they might refer with the certainty of having every doubt removed as to the correctness of their work and methods of solution. He offers the work to his fellow-teachers with the hope that they will accord it the same favorable reception that they have so kindly given to the National Arithmetic.

TORONTO, *May*, 1861.





# CONTENTS.

|   | PAGE |
|---|------|
| Reduction Descending.....                             | 7    |
| Reduction Ascending .....                             | 8    |
| Reduction of Canadian Currency to Dollars and Cents.. | 10   |
| Multiplication.....                                   | 11   |
| Division.....   | 16   |
| Miscellaneous Exercises, Arith. page 118.....         | 22   |
| Resolution of Numbers into their prime factors.....   | 29   |
| Divisors of Numbers. ....                             | 29   |
| Number of Divisors .....                              | 30   |
| Common measure of two or more Numbers.....            | 31   |
| Greatest Common Measure.....                          | 32   |
| Least Common Multiple.....                            | 34   |
| Reduction of Scales of Notation.....                  | 36   |
| Fundamental Rules in different scales.....            | 41   |
| Duodecimal Multiplication.....                        | 42   |
| Miscellaneous Exercises, Arith. page 149 .....        | 43   |
| Reduction of Fractions.....                           | 50   |
| Addition of Fractions .....                           | 60   |
| Subtraction of Fractions .....                        | 65   |
| Multiplication of Fractions.....                      | 67   |
| Division of Fractions .....                           | 71   |
| Miscellaneous Exercises in Vulgar Fractions.....      | 76   |
| Reduction of Decimals.....                            | 79   |
| Addition of Decimals.....                             | 85   |
| Subtraction of Decimals .....                         | 87   |
| Multiplication and Division of Decimals.....          | 88   |
| Miscellaneous Exercises in Decimals.....              | 89   |
| Miscellaneous Exercises, Arith. page 198.....         | 93   |
| Simple Proportion.....                                | 104  |
| Compound Proportion .....                             | 111  |

|  | PAGE |
|--|------|
| Conjoined Proportion.....                      | 114  |
| Miscellaneous Exercises, Arith. page 222 ..... | 116  |
| Practice .....                                 | 123  |
| Bills of Parcels.....                          | 129  |
| Miscellaneous Exercises, Arith. page 231.....  | 131  |
| Percentage .....                               | 137  |
| Commission.....                                | 138  |
| Brokerage .....                                | 139  |
| Insurance.....                                 | 141  |
| Custom House Business .....                    | 142  |
| Assessment of Taxes.....                       | 143  |
| Simple Interest .....                          | 144  |
| Compound Interest .....                        | 156  |
| Discount.....                                  | 160  |
| Bank Discount.....                             | 163  |
| Equation of Payments .....                     | 164  |
| Partnership.....                               | 165  |
| Profit and Loss.....                           | 171  |
| Barter.....                                    | 175  |
| Alligation.....                                | 177  |
| Reduction of Currencies.....                   | 182  |
| Exchange.....                                  | 184  |
| Arbitration of Exchange.....                   | 185  |
| Involution.....                                | 186  |
| Square Root.....                               | 187  |
| Cube Root.....                                 | 192  |
| Extraction of other Roots.....                 | 201  |
| *Logarithms .....                              | 202  |
| Logarithmic Arithmetic.....                    | 210  |
| Arithmetical Progression .....                 | 217  |
| Geometrical Progression.....                   | 222  |
| Single Position .....                          | 228  |
| Double Position .....                          | 231  |
| Compound Interest .....                        | 235  |
| Annuities at Simple Interest.....              | 237  |
| Annuities at Compound Interest.....            | 238  |
| Examination Problems .....                     | 241  |
| Arithmetical Recreations.....                  | 286  |

# KEY TO NATIONAL ARITHMETIC.

## EXERCISE 5—Page 50.

| (1)             | (2)            | (3)            | (4)             | (5)             |
|-----------------|----------------|----------------|-----------------|-----------------|
| d               | £              | £ s.           | £ s.            | £ s.            |
| 23328           | 348            | 38 10          | 58 13           | 58 13           |
| 4               | 20             | 20             | 20              | 20              |
| <u>93312 f.</u> | <u>6960 s.</u> | <u>770 s.</u>  | <u>1173 s.</u>  | <u>1173 s.</u>  |
|                 |                | 12             | 12              | 12              |
|                 |                | <u>9240 d.</u> | <u>14076 d.</u> | <u>14076 d.</u> |
|                 |                |                |                 | 4               |
|                 |                |                |                 | <u>56304 f.</u> |

| (6)                   | (7)             | (8)              | (9)              |
|-----------------------|-----------------|------------------|------------------|
| £ s. d.               | £ s. d.         | cwt. qrs. lbs.   | cwt. qrs. lbs.   |
| 59 13 6 $\frac{3}{4}$ | 63 0 9          | 16 2 16          | 14 3 16          |
| 20                    | 20              | 4                | 4                |
| <u>1193 s.</u>        | <u>1260 s.</u>  | <u>66 qrs.</u>   | <u>59 qrs.</u>   |
| 12                    | 12              | 25               | 25               |
| <u>14322 d.</u>       | <u>15129 d.</u> | <u>346</u>       | <u>311</u>       |
| 4                     |                 | 132              | 118              |
| <u>57291 f.</u>       |                 | <u>1666 lbs.</u> | <u>1491 lbs.</u> |

| (10)               | (11)               | (12)             | (13)            | (14)                |
|--------------------|--------------------|------------------|-----------------|---------------------|
| lbs. oz. dwt. grs. | lbs. oz. dwt. grs. | grs. yrs.        | mile.           | yrs. d. h. m.       |
| 3 5 12 16          | 7 11 15 14         | 20               | 1               | 46 21 8 56          |
| 12                 | 12                 | 365              | 8               | 365                 |
| <hr/> 41 oz.       | <hr/> 95           | <hr/> 7300 dys.  | <hr/> 8 fur.    | <hr/> 251           |
| 20                 | 20                 | 24               | 40              | 276                 |
| <hr/> 832 dwts.    | <hr/> 1915 dwts.   | <hr/> 29200      | <hr/> 320 per.  | <hr/> 138           |
| 24                 | 24                 | 14600            | 5½              | 16811 days.         |
| <hr/> 3344         | <hr/> 7674         | <hr/> 175200 hrs | <hr/> 1600      | <hr/> 24            |
| 1664               | 3830               |                  | 160             | 67252               |
| <hr/>              | <hr/>              |                  | <hr/> 1760 yds. | <hr/> 33622         |
| 19984 grs.         | 45974 grs.         |                  | 3               | 403472 hrs.         |
|                    |                    |                  | <hr/> 5280 ft.  | <hr/> 60            |
|                    |                    |                  |                 | <hr/> 24208376 min. |

| (15)                  | (16)            | (17)                | (18)             | (19)            | (20)            |
|-----------------------|-----------------|---------------------|------------------|-----------------|-----------------|
| sq. per.              | a. r. per.      | sq. miles.          | cub. ft.         | pks.            | pks.            |
| 74                    | 46 3 12         | 767                 | 767              | 767             | 797             |
| 30½                   | 4               | 640                 | 1728             | 2               | 2               |
| <hr/> 2220            | <hr/> 187 r.    | <hr/> 30680         | <hr/> 6136       | <hr/> 1534 gals | <hr/> 1594 gals |
| 18½                   | 40              | 4602                | 1534             | 4               | 4               |
| <hr/> 2238½ sq. yds.  | <hr/> 7492 per. | <hr/> 490880 sq. a. | <hr/> 5369       | <hr/>           | <hr/>           |
|                       | 30½             |                     | 767              | 6136 qts.       | 6376 qts.       |
| <hr/> 224760          | <hr/>           | <hr/>               | <hr/>            | <hr/>           | <hr/> 2         |
| 1873                  |                 |                     | 1325376 cub. in. |                 | 12752 pts.      |
| <hr/> 226633 sq. yds. | <hr/>           | <hr/>               | <hr/>            | <hr/>           | <hr/>           |

## EXERCISE 6—Page 51.

| (1)          | (2)                     | (3)                            |
|--------------|-------------------------|--------------------------------|
| f.           | grs.                    | yds.                           |
| 4)32756      | 24)23547                | 5½)397024                      |
| <hr/>        | <hr/>                   | <hr/> 2 2                      |
| 12)8189 d.   | 20)981 dwt. 3 grs.      | <hr/> 11)794048                |
| <hr/>        | <hr/>                   | <hr/>                          |
| 20)682s. 5d. | 12)49 oz. 1 dwt. 3 grs. | <hr/>                          |
| <hr/>        | <hr/>                   | <hr/> 40)72186r. 2hf-yds.=1yd. |
| £34 2s. 5d.  | 4 lbs. 1 oz. 1 dt. 3 g. | 8)1804 fur. 26 r. 1 yd.        |
|              |                         | <hr/> 225 m. 4 f. 26 r. 1 y.   |

$$\begin{array}{r} (4) \\ \text{sec.} \\ 60 \overline{) 28635} \end{array}$$

$$60 \overline{) 477} \text{ m. 15 sec.}$$

$$7 \text{ hrs. } 57 \text{ m. 15 sec. } 16 \text{ cwt. } 2 \text{ q. } 16 \text{ lbs. } 14 \text{ cwt. } 3 \text{ q. } 16 \text{ lbs.}$$

$$\begin{array}{r} (5) \\ \text{lbs.} \\ 25 \overline{) 1666} \end{array}$$

$$4 \overline{) 66} \text{ qrs. } 16 \text{ lbs.}$$

$$\begin{array}{r} (6) \\ \text{lbs.} \\ 25 \overline{) 1491} \end{array}$$

$$4 \overline{) 59} \text{ qrs. } 16 \text{ lbs.}$$

$$\begin{array}{r} (7) \\ \text{grs.} \\ 24 \overline{) 115200} \end{array}$$

$$20 \overline{) 4800} \text{ dwt.}$$

$$12 \overline{) 240} \text{ oz.}$$

$$20 \text{ lbs.}$$

$$\begin{array}{r} (8) \\ \text{oz.} \\ 16 \overline{) 107520} \end{array}$$

$$6720 \text{ lbs.}$$

$$\begin{array}{r} (9) \\ \text{cub. in.} \\ 1728 \overline{) 1674674} \end{array}$$

$$969 \text{ ft } 242 \text{ in.}$$

$$\begin{array}{r} (10) \\ \text{Fl. e.} \\ 767 \\ 3 \end{array}$$

$$4 \overline{) 2301} \text{ qrs.}$$

$$575 \text{ yds. } 1 \text{ qr.}$$

$$\begin{array}{r} (11) \\ \text{ft.} \\ 3 \overline{) 183810} \end{array}$$

$$5 \frac{1}{2} \overline{) 61270} \text{ yds.}$$

$$11 \overline{) 122540}$$

$$40 \overline{) 11140} \text{ per.}$$

$$8 \overline{) 278} \text{ fur. } 20 \text{ per.}$$

$$3 \overline{) 34} \text{ m. } 6 \text{ fur. } 20 \text{ per.}$$

$$11 \text{ lea. } 1 \text{ m. } 6 \text{ fur. } 20 \text{ per.}$$

$$\begin{array}{r} (12) \\ \text{cub. in.} \\ 1728 \overline{) 138297} \end{array}$$

$$27 \overline{) 80} \text{ ft. } 57 \text{ in.}$$

$$2 \text{ c. yds. } 26 \text{ c. ft. } 57 \text{ c. in.}$$

$$\begin{array}{r} (13) \\ \text{cub. ft.} \\ 128 \overline{) 67893} \end{array}$$

$$530 \text{ cords } 53 \text{ c. ft.}$$

$$\begin{array}{r} (14) \\ \text{sec.} \\ 60 \overline{) 3561829} \end{array}$$

$$60 \overline{) 59363} \text{ m. } 49 \text{ sec.}$$

$$24 \overline{) 989} \text{ h. } 23 \text{ m. } 49 \text{ s.}$$

$$7 \overline{) 41} \text{ d. } 5 \text{ h. } 23 \text{ m. } 49 \text{ s. } 49 \text{ bush. } 3 \text{ pecks } 1 \text{ gal. } 1 \text{ qt.}$$

$$5 \text{ wks. } 6 \text{ days. } 5 \text{ hrs. } 23 \text{ min. } 49 \text{ sec.}$$

$$\begin{array}{r} (15) \\ \text{qts.} \\ 4 \overline{) 1597} \end{array}$$

$$2 \overline{) 399} \text{ gals. } 1 \text{ qt.}$$

$$4 \overline{) 199} \text{ pks. } 1 \text{ gal. } 1 \text{ qt.}$$

$$\begin{array}{r} (16) \\ \text{c. ft.} \\ 8 \overline{) 1000} \end{array}$$

$$125 \text{ cords.}$$

| (17)            | (18)            | (19)             |
|-----------------|-----------------|------------------|
| seconds.        | sq. links.      | grs.             |
| 60)10000        | 10000)70000     | 20)11521         |
| <u>        </u> | <u>        </u> | <u>        </u>  |
| '60)166' 40''   | 7 sq. ch.       | 3)576 scr. 1 gr. |
| <u>        </u> |                 | <u>        </u>  |
| 2° 46' 40''     |                 | 8)192 dr. 1 gr.  |
|                 |                 | <u>        </u>  |
|                 |                 | 12)24 oz. 1 gr.  |
|                 |                 | <u>        </u>  |
|                 |                 | 2 lbs. 1 gr.     |

| (20)  |
|---|
| sq. ft.                                       |
| 9)26025                                       |
| <u>        </u>                               |
| 30½)2891 yds. 6 ft.                           |
| 4 ) 4   |
| <u>        </u>                               |
| 121)11564 quarter yards.                      |
| 95 per. 69 quar. yds. 6 ft. =                 |
| 40)95 per. 17 yds. 8 ft. 36 in.               |
| <u>        </u>                               |
| 2 r. 15 sq. p. 17 sq. y. 8 sq. ft. 36 sq. in. |

## EXERCISE 7—Page 53.

| (1)  | (2)                       |
|--|---------------------------|
| £3 × 400 = 1200 cents.                                       | £29 × 400 = \$116·00      |
| 7s. × 20 = 140 "   | 18s. × 20 = 3·60          |
| 1½d. = 5 far. × 5 ÷ 12 = 2½ " 3½d. = 14 far. × 5 ÷ 12 = .05½ |                           |
| <u>        </u>  | <u>        </u>           |
| £3 7s. 1½d. = 1342½ cts.                                     | £29 18s. 3½d. = \$119·65½ |

(3)

$$11½d. = 45 \text{ far.} \times 5 \div 12 = 18¾ \text{ cts.}$$

| (4)                          | (5)                            |
|------------------------------|--------------------------------|
| £69 × 400 = \$276·00         | 18s. × 20 = \$3·60             |
| 15s. × 20 = 3·00             | 8½d. = 34 far. × 5 ÷ 12 = .14½ |
| 6d. = 24 far. × 5 ÷ 12 = .10 | <u>        </u>                |
| <u>        </u>              | 18s 8½d. = \$3·74½             |
| £69 15s. 6d. = \$279·10      |                                |

(6)

$$£17 \times 400 = \$68.00$$

$$16s. \times 20 = 3.20$$

$$5\frac{1}{4}d. = 23 \text{ far.} \times 5 \div 12 = .09\frac{7}{12}$$

$$£17 \text{ } 16s. \text{ } 5\frac{1}{4}d. = \$71.29\frac{7}{12}$$

(7)

$$£87 \times 400 = \$348.00$$

(8)

$$15s. \times 20 = \$3.00$$

$$15s. \text{ } 11\frac{1}{4}d. = \$3.19\frac{7}{12}$$

(9)

$$£16 \times 400 = \$64.00$$

$$6s. \times 20 = 1.20$$

$$2d. = 8 \text{ far.} \times 5 \div 12 = .03\frac{1}{3}$$

$$£16 \text{ } 6s. \text{ } 2d. = \$65.23\frac{1}{3}$$

(10)

$$£2 \times 400 = \$8.00$$

$$9s. \times 20 = 1.80$$

$$11d. = 44 \text{ far.} \times 5 \div 12 = .18\frac{1}{3}$$

$$£2 \text{ } 9s. \text{ } 11d. = \$9.98\frac{1}{3}$$

## EXERCISE 13—Page 90.

(1)

$$36 = 12 \times 3$$

$$\$169.78$$

$$12$$

$$2037.36$$

$$3$$

$$\$6112.08$$

(2)

$$121 = 11 \times 11$$

$$796342.3$$

$$11$$

$$8759765.3$$

$$11$$

$$96357418.3$$

(3)

$$144 = 12 \times 12$$

$$\$33460$$

$$12$$

$$401520$$

$$12$$

$$\$4818240$$

(4)

$$648 = 12 \times 9 \times 6$$

$$735$$

$$12$$

$$8820$$

$$9$$

$$79380$$

$$6$$

$$476280$$

(5)

$$18 = 6 \times 3$$

$$£ \text{ } s. \text{ } d.$$

$$3 \text{ } 7 \text{ } 6$$

$$6$$

$$20 \text{ } 5 \text{ } 0$$

$$3$$

$$60 \text{ } 15 \text{ } 0$$

(6)

$$22 = 11 \times 2$$

$$£ \text{ } s. \text{ } d.$$

$$5 \text{ } 14 \text{ } 6\frac{1}{2}$$

$$11$$

$$62 \text{ } 19 \text{ } 11\frac{1}{2}$$

$$2$$

$$125 \text{ } 19 \text{ } 11$$

(7)

$$810 = 10 \times 9 \times 9$$

$$£ \text{ } s. \text{ } d.$$

$$3 \text{ } 4 \text{ } 7$$

$$10$$

$$32 \text{ } 5 \text{ } 10$$

$$9$$

$$290 \text{ } 12 \text{ } 6$$

$$9$$

$$2615 \text{ } 12 \text{ } 6$$

(8)

$$54 = 9 \times 6$$

$$\text{cwt. qrs. lbs. oz.}$$

$$11 \text{ } 3 \text{ } 14 \text{ } 7$$

$$9$$

$$107 \text{ } 0 \text{ } 4 \text{ } 15$$

$$6$$

$$642 \text{ } 1 \text{ } 4 \text{ } 10$$

$$(9) \quad 49 = 7 \times 7$$

|       |      |      |     |     |
|-------|------|------|-----|-----|
| bush. | pks. | gal. | qt. | pt. |
| 26    | 3    | 1    | 1   | 1   |
|       |      |      |     | 7   |

|     |   |   |   |   |
|-----|---|---|---|---|
| 188 | 1 | 1 | 2 | 1 |
|     |   |   |   | 7 |

|      |   |   |   |   |
|------|---|---|---|---|
| 1319 | 0 | 1 | 1 | 1 |
|------|---|---|---|---|

$$(10) \quad 63 = 9 \times 7$$

|      |      |     |     |
|------|------|-----|-----|
| yds. | qrs. | na. | in. |
| 2    | 2    | 2   | 2   |
|      |      |     | 9   |

|    |   |   |   |
|----|---|---|---|
| 24 | 0 | 2 | 0 |
|    |   |   | 7 |

|     |   |   |   |
|-----|---|---|---|
| 168 | 3 | 2 | 0 |
|-----|---|---|---|

$$(11) \quad 288 = 12 \times 12 \times 2$$

|      |      |      |      |
|------|------|------|------|
| dys. | hrs. | min. | sec. |
| 5    | 17   | 33   | 11   |
|      |      |      | 12   |

|    |    |    |    |
|----|----|----|----|
| 68 | 18 | 38 | 12 |
|    |    |    | 12 |

|     |   |    |    |
|-----|---|----|----|
| 825 | 7 | 38 | 24 |
|     |   |    | 2  |

|      |    |    |    |
|------|----|----|----|
| 1650 | 15 | 16 | 48 |
|------|----|----|----|

## EXERCISE 14—Page 92.

$$(1) \quad 83 = 3 + 10 \times 8$$

|    |    |    |    |    |    |
|----|----|----|----|----|----|
| £  | s. | d. | £  | s. | d. |
| 12 | 2  | 4  | 36 | 7  | 0  |
|    |    | 10 |    |    |    |

|     |   |   |      |    |   |
|-----|---|---|------|----|---|
| 121 | 3 | 4 | 969  | 6  | 8 |
|     |   |   | 1005 | 13 | 8 |

$$(2) \quad 999 = 10 \times 10 \times 10 - 1$$

|     |    |                 |
|-----|----|-----------------|
| £   | s. | d.              |
| 963 | 0  | 0 $\frac{1}{4}$ |
|     |    | 10              |

|      |   |                 |
|------|---|-----------------|
| 9630 | 0 | 7 $\frac{1}{2}$ |
|      |   | 10              |

|       |   |    |
|-------|---|----|
| 96300 | 6 | 3  |
|       |   | 10 |

|        |   |                 |
|--------|---|-----------------|
| 963003 | 2 | 6               |
| 963    | 0 | 0 $\frac{3}{4}$ |

|        |   |                 |
|--------|---|-----------------|
| 962040 | 2 | 5 $\frac{1}{4}$ |
|--------|---|-----------------|

$$(3) \quad 3178 = 8 + 10 \times 7 + 10 \times 10 \times 1 + 10 \times 10 \times 10 \times 3$$

|   |    |                 |    |    |    |
|---|----|-----------------|----|----|----|
| £ | s. | d.              | £  | s. | d. |
| 3 | 6  | 5 $\frac{1}{4}$ | 26 | 11 | 6  |
|   |    | 10              |    |    |    |

|    |   |                 |     |    |                 |
|----|---|-----------------|-----|----|-----------------|
| 33 | 4 | 4 $\frac{1}{2}$ | 232 | 10 | 7 $\frac{1}{2}$ |
|    |   | 10              |     |    |                 |

|     |   |    |     |   |   |
|-----|---|----|-----|---|---|
| 332 | 3 | 9  | 332 | 3 | 9 |
|     |   | 10 |     |   |   |

|      |    |   |      |    |   |
|------|----|---|------|----|---|
| 3321 | 17 | 6 | 9965 | 12 | 6 |
|------|----|---|------|----|---|

|       |    |                 |
|-------|----|-----------------|
| 10556 | 18 | 4 $\frac{1}{2}$ |
|-------|----|-----------------|

$$(4) \quad 678 = 8 + 10 \times 7 + 10 \times 10 \times 6$$

|       |     |      |       |     |      |
|-------|-----|------|-------|-----|------|
| bush. | pk. | gal. | bush. | pk. | gal. |
| 16    | 3   | 1    | 135   | 0   | 0    |
|       |     | 10   |       |     |      |

|     |   |    |      |   |   |
|-----|---|----|------|---|---|
| 168 | 3 | 0  | 1181 | 1 | 0 |
|     |   | 10 |      |   |   |

|      |   |   |       |   |   |
|------|---|---|-------|---|---|
| 1687 | 2 | 0 | 10125 | 0 | 0 |
|------|---|---|-------|---|---|

|       |   |   |
|-------|---|---|
| 11441 | 1 | 0 |
|-------|---|---|



(5)

$$247 = 7 + 10 \times 4 + 10 \times 10 \times 2$$

|    |      |      |      |              |     |      |      |                |
|----|------|------|------|--------------|-----|------|------|----------------|
| m. | fur. | rds. | yds. |              | m.  | fur. | rds. | yds.           |
| 23 | 6    | 33   | 4    | $\times 7 =$ | 166 | 7    | 36   | $0\frac{1}{2}$ |
|    |      |      |      | 10           |     |      |      |                |

---


$$238 \quad 4 \quad 17 \quad 1\frac{1}{2} \times 4 = 954 \quad 1 \quad 29 \quad 0\frac{1}{2}$$

10

---


$$2385 \quad 4 \quad 12 \quad 4 \times 2 = 4771 \quad 0 \quad 25 \quad 2\frac{1}{2}$$


---


$$5892 \quad 2 \quad 10 \quad 3\frac{1}{2}$$

(6)

$$721 = 1 + 10 \times 2 + 10 \times 10 \times 7$$

|    |      |      |      |              |    |      |      |      |
|----|------|------|------|--------------|----|------|------|------|
| S. | deg. | min. | sec. |              | S. | deg. | min. | sec. |
| 3  | 16   | 30   | 45   | $\times 1 =$ | 3  | 16   | 30   | 45   |
|    |      |      |      | 10           |    |      |      |      |

---


$$35 \quad 15 \quad 7 \quad 30 \times 2 = 71 \quad 0 \quad 15 \quad 0$$

10

---


$$355 \quad 1 \quad 15 \quad 0 \times 7 = 2485 \quad 8 \quad 45 \quad 0$$


---


$$2559 \quad 25 \quad 30 \quad 45$$

## EXERCISE 15—Page 93.

| (6)     | (7)      | (8)          | (9)         |
|---------|----------|--------------|-------------|
| 7071    | 15607    | 39948123     | 2778588     |
| 556     | 3094     | 6007         | 9867        |
| <hr/>   | <hr/>    | <hr/>        | <hr/>       |
| 42426   | 62428    | 279636861    | 19450116    |
| 35355   | 140463   | 23968873800  | 16671528    |
| 35355   | 468210   | <hr/>        | 22228704    |
| <hr/>   | <hr/>    | 239968374861 | 25007292    |
| 3931476 | 48288058 |              | <hr/>       |
|         |          |              | 27416327796 |

## EXERCISE 16—Page 95.

| (4)      | (5)       | (6)       | (7)       | (8)      |
|----------|-----------|-----------|-----------|----------|
| 3.2517   | 64.001    | 482000    | 3782.4    | 87.96    |
| .023     | 340       | .37       | .00917    | 220      |
| <hr/>    | <hr/>     | <hr/>     | <hr/>     | <hr/>    |
| 97551    | 2560040   | 3374000   | 264768    | 175920   |
| 65034    | 192003    | 1446000   | 37824     | 17592    |
| <hr/>    | <hr/>     | <hr/>     | 340416    | <hr/>    |
| .0747891 | 21760.340 | 178340.00 | <hr/>     | 19351.20 |
|          |           |           | 34.684608 |          |

## EXERCISE 17—Page 100.

| (1)                         | (2)             | (3)        | (4)                          |
|-----------------------------|-----------------|------------|------------------------------|
| $216 = 6 \times 6 \times 6$ | $\$61135.37$    | $255226$   | $176 = 11 \times 8 \times 2$ |
| $\$83469$                   | $229$           | $143$      |                              |
| $6$                         | $55021833$      | $765678$   | $203736$                     |
| $500814$                    | $12227074$      | $1020904$  | $11$                         |
| $6$                         | $12227074$      | $255226$   | $2241096$                    |
| $3004884$                   | $\$13999999.73$ | $36497318$ | $8$                          |
| $6$                         |                 |            | $17928768$                   |
| $\$18029304$                |                 |            | $2$                          |
|                             |                 |            | $35857536$                   |

| (5)        | (6)      | (7)                          | (8)               |
|------------|----------|------------------------------|-------------------|
| $116700$   | $3721$   | $297 = 11 \times 9 \times 3$ | $35 = 7 \times 5$ |
| $235$      | $73$     | $32000$                      | $9344000$         |
| $583500$   | $11163$  | $11$                         | $7$               |
| $350100$   | $26047$  | $352000$                     | $65408000$        |
| $233400$   | $271633$ | $9$                          | $5$               |
| $27424500$ |          | $3168000$                    | $327040000$       |
|            |          | $3$                          |                   |
|            |          | $9504000$                    |                   |

| (9)   | (10)                    |
|---|-------------------------|
| $749 = 9 + 10 \times 4 + 10 \times 10 \times 7$ | $999998 = 1000000 - 2$  |
| lbs. oz. drs. scr. gr.                          | lbs. oz. drs. scr. grs. |
| $123 \ 4 \ 7 \ 2 \ 17 \times 9 =$               | $1110 \ 8 \ 7 \ 1 \ 13$ |
| $10$  | $1698732$               |
| $1234 \ 1 \ 7 \ 1 \ 10 \times 4 =$              | $1000000$               |
| $4936 \ 7 \ 6 \ 0 \ 0$                          | $1698732000000$         |
| $10$  | $3397464$               |
| $12341 \ 7 \ 3 \ 0 \ 0 \times 7 =$              | $1698728602536$         |
| $86391 \ 3 \ 5 \ 0 \ 0$                         |                         |
| $92438 \ 8 \ 2 \ 1 \ 13$                        |                         |

(11)

$$\begin{array}{r}
 640 = 10 \times 8 \times 8 \\
 \text{bush. pk. gal. qt. pt.} \\
 123 \quad 1 \quad 1 \quad 1 \quad 1 \\
 \phantom{123} \phantom{1} \phantom{1} \phantom{1} 10 \\
 \hline
 1234 \quad 0 \quad 1 \quad 3 \quad 0 \\
 \phantom{1234} \phantom{0} \phantom{1} \phantom{3} 8 \\
 \hline
 9873 \quad 3 \quad 0 \quad 0 \quad 0 \\
 \phantom{9873} \phantom{3} \phantom{0} \phantom{0} 8 \\
 \hline
 78990 \quad 0 \quad 0 \quad 0 \quad 0
 \end{array}$$

(12)

$$\begin{array}{r} 89 \\ \cdot 73 \\ \hline 267 \\ 623 \\ \hline \$64.97 \end{array}$$

(13)

$$\begin{array}{r}
 1143 = 3 + 10 \times 4 + 10 \times 10 \times 1 + 10 \times 10 \times 10 \times 1 \\
 \text{yds. qrs. na. in.} \qquad \qquad \qquad \text{yds. qrs. na. in.} \\
 \begin{array}{r}
 7 \quad 3 \quad 2 \quad 1 \times 3 = \quad 23 \quad 2 \quad 3 \quad 0\frac{3}{4} \\
 \hline
 10 \\
 79 \quad 0 \quad 0 \quad 1 \times 4 = \quad 316 \quad 0 \quad 1 \quad 1\frac{3}{4} \\
 \hline
 10 \\
 790 \quad 1 \quad 0 \quad 1 \times 1 = \quad 790 \quad 1 \quad 0 \quad 1 \\
 \hline
 10 \\
 7902 \quad 3 \quad 0 \quad 1 \times 1 = \quad 7902 \quad 3 \quad 0 \quad 1 \\
 \hline
 9032 \quad 3 \quad 2 \quad 0
 \end{array}
 \end{array}$$

(14)

$$\begin{array}{r} 1634 \cdot 5789 \\ 635000 \\ \hline 81728945000 \\ 49037367 \\ 98074734 \\ \hline 1037957601 \cdot 5 \end{array}$$

(15)

$$\begin{array}{r} \$968.49 \\ 3 \cdot 4 \\ \hline 387396 \\ 290547 \\ \hline \$3292.866 \end{array}$$

\$12183.6042  
3292.866  
968.49  

---

\$16444.9602

$$\begin{array}{r} \$3292.866 \\ 3.7 \\ \hline 23050062 \\ 9878598 \\ \hline \$12183.6042 \end{array}$$

## EXERCISE 18—Page 110.

(9)

$$\begin{array}{r}
 6423)798965(124\overset{2}{\underset{6}{4}}\overset{5}{\underset{4}{2}}\overset{1}{\underset{3}{3}} \\
 \underline{6423} \\
 15666 \\
 \underline{12846} \\
 28205 \\
 \underline{25692} \\
 2513
 \end{array}$$

(10)

$$\begin{array}{r}
 \text{£ s. d.} \\
 12)176 \text{ 14 } 6 \\
 \hline
 14 \text{ 14 } 6\frac{1}{2}
 \end{array}$$

(11)

$$\begin{array}{r}
 741)56789(76\overset{1}{\underset{4}{1}}\overset{7}{\underset{1}{1}} \\
 \underline{5187} \\
 4919 \\
 \underline{4446} \\
 473
 \end{array}$$

(12)

$$\begin{array}{r}
 7894)6785158(859\overset{1}{\underset{7}{8}}\overset{2}{\underset{9}{4}}\overset{1}{\underset{2}{2}} \\
 \underline{63152} \\
 46995 \\
 \underline{39470} \\
 75258 \\
 \underline{71046} \\
 4212
 \end{array}$$

(13)

$$\begin{array}{r}
 \text{£ s. d.} \\
 317)4728 \text{ 16 } 2(14 \text{ 18 } 4\overset{5}{\underset{3}{1}}\overset{1}{\underset{7}{7}}
 \end{array}$$

$$\begin{array}{r}
 317 \\
 \underline{1558} \\
 1268 \\
 \underline{290} \\
 20 \\
 \underline{5816} \\
 317
 \end{array}$$

$$\begin{array}{r}
 \underline{2646} \\
 2536 \\
 \underline{110} \\
 12 \\
 \underline{1322} \\
 1268 \\
 \underline{54}
 \end{array}$$

$$\begin{array}{r}
 (15) \\
 6)970763 \\
 \hline
 161793 \cdot 8333+
 \end{array}$$

$$\begin{array}{r}
 (16) \\
 9)71234 \\
 \hline
 7914\frac{8}{9}
 \end{array}$$

(14)

$$429)\$97896 \cdot 64(\$228 \cdot 19\overset{3}{\underset{4}{2}}\overset{1}{\underset{3}{3}}$$

$$\begin{array}{r}
 858 \\
 \hline
 1209 \\
 \underline{858} \\
 3516 \\
 \underline{3432}
 \end{array}$$

$$\begin{array}{r}
 84 \cdot 6 \\
 42 \cdot 9 \\
 \hline
 41 \cdot 74 \\
 \underline{38 \cdot 61} \\
 3 \cdot 13
 \end{array}$$

(17)

$$\begin{array}{r}
 1209 \text{ 47600)977076(20\overset{2}{\underset{4}{7}}\overset{5}{\underset{8}{6}}\overset{8}{\underset{0}{0}} \\
 \underline{858} \\
 95200 \\
 \hline
 25076
 \end{array}$$

(18)

| lbs.     | oz. | drs. | scr. | grs. | lbs. | oz. | drs. | scr. | grs. |
|----------|-----|------|------|------|------|-----|------|------|------|
| 498)7289 | 6   | 4    | 2    | 13   | 14   | 7   | 5    | 0    | 12   |
| 498      |     |      |      |      |      |     |      |      |      |
| 2309     |     |      |      |      |      |     |      |      |      |
| 1992     |     |      |      |      |      |     |      |      |      |
| 317      |     |      |      |      |      |     |      |      |      |
| 12       |     |      |      |      |      |     |      |      |      |
| 3810     |     |      |      |      |      |     |      |      |      |
| 3486     |     |      |      |      |      |     |      |      |      |
| 324      |     |      |      |      |      |     |      |      |      |
| 8        |     |      |      |      |      |     |      |      |      |
| 2596     |     |      |      |      |      |     |      |      |      |
| 2490     |     |      |      |      |      |     |      |      |      |
| 106      |     |      |      |      |      |     |      |      |      |
| 3        |     |      |      |      |      |     |      |      |      |
| 320      |     |      |      |      |      |     |      |      |      |
| 20       |     |      |      |      |      |     |      |      |      |
| 6413     |     |      |      |      |      |     |      |      |      |
| 5976     |     |      |      |      |      |     |      |      |      |
| 437      |     |      |      |      |      |     |      |      |      |

(19)

| £       | s. | d. | s. | d. |
|---------|----|----|----|----|
| 487)157 | 16 | 7  | 6  | 5  |
|         |    |    |    |    |

20

(20)

|              |                                       |
|--------------|---------------------------------------|
| 9712)7867674 | (810 <sup>954</sup> <sub>9712</sub> ) |
| 77696        |                                       |
| 9807         |                                       |
| 9712         |                                       |
| 954          |                                       |

(21)

| m.     | fur. | rds. | m. | fur. | rds. |
|--------|------|------|----|------|------|
| 37)422 | 3    | 38   | 11 | 3    | 14   |
| 407    |      |      |    |      |      |
| 15     |      |      |    |      |      |
| 8      |      |      |    |      |      |
| 123    |      |      |    |      |      |
| 111    |      |      |    |      |      |
| 12     |      |      |    |      |      |
| 40     |      |      |    |      |      |
| 518    |      |      |    |      |      |
| 37     |      |      |    |      |      |
| 148    |      |      |    |      |      |
| 148    |      |      |    |      |      |

## EXERCISE 19—Page 112.

(1)

25=5×5

5)3766

5)753... 1

150... 3

3×5+1=16

150<sup>16</sup><sub>5</sub>

(2)

42=7×6

7)26406

6)3772... 2

628... 4

4×7+2=30

628<sup>30</sup><sub>7</sub>

(3)

96=12×8

12)25431

8)2119... 3

264... 7

7×12+3=87

264<sup>87</sup><sub>12</sub>

(4)

24=12×2

| £     | s. | d. |
|-------|----|----|
| 12)24 | 17 | 6  |

2)2 1 5<sup>1</sup><sub>2</sub>

1 0 8<sup>1</sup><sub>2</sub>

(5)

$$\begin{array}{r}
 49 = 7 \times 7 \\
 \text{£ s. d.} \\
 7)740 \ 13 \ 4 \\
 \hline
 7)105 \ 16 \ 2\frac{1}{4} \dots 1 \\
 \hline
 15 \ 2 \ 3\frac{3}{4} \dots \frac{1}{49}
 \end{array}$$

(6)

$$\begin{array}{r}
 56 = 8 \times 7 \\
 \text{£ s. d.} \\
 8)547 \ 12 \ 4 \\
 \hline
 7)68 \ 9 \ 0\frac{1}{2} \\
 \hline
 9 \ 15 \ 6\frac{1}{4} \dots \frac{40}{56}
 \end{array}$$

(7)

$$\begin{array}{r}
 35 = 7 \times 5 \\
 7)6789436 \\
 \hline
 5)969919 \dots 3 \\
 \hline
 193983 \dots 4 \\
 4 \times 7 + 3 = 31 \\
 193983\frac{31}{5}
 \end{array}$$

(8)

$$\begin{array}{r}
 147 = 7 \times 7 \times 3 \\
 7)753293 \\
 \hline
 7)107613 \dots 2 \\
 \hline
 3)15373 \dots 2 \\
 \hline
 5124 \dots 1 \\
 1 \times 7 \times 7 + 2 \times 7 + 2 = 65 \\
 5124\frac{65}{147}
 \end{array}$$

(9)

$$\begin{array}{r}
 81 = 9 \times 9 \\
 \text{lbs. oz. dwt. grs.} \\
 9)1798 \ 6 \ 11 \ 9 \\
 \hline
 9)199 \ 10 \ 1 \ 6 \dots 3 \\
 \hline
 22 \ 2 \ 9 \ 0 \dots 6 \\
 6 \times 9 + 3 = 57 \\
 22 \text{ lbs. } 2 \text{ oz. } 9 \text{ dwt. } 0\frac{57}{1} \text{ grs.}
 \end{array}$$

## EXERCISE 20—Page 114.

(1)

$$\begin{array}{r}
 \text{£ s. d.} \\
 491 \ 12 \ 0\frac{1}{4} \\
 20 \\
 \hline
 9832 \\
 12 \\
 \hline
 117984 \\
 4 \\
 \hline
 471937 \ ) \\
 \hline
 8609934 \ (18\frac{115068}{471937} \\
 471937 \\
 \hline
 3890564 \\
 3775496 \\
 \hline
 115068
 \end{array}$$

(2)

$$\begin{array}{r}
 \text{m. fur. rds.} \\
 17 \ 5 \ 27 \\
 8 \\
 \hline
 141 \\
 40 \\
 \hline
 5667 \ ) \\
 \hline
 328686 \ (58 \\
 28335 \\
 \hline
 45336 \\
 45336
 \end{array}$$

|              |    |    | (3) |               |    |     |            |      |   | (4)           |     |      |      |
|--------------|----|----|-----|---------------|----|-----|------------|------|---|---------------|-----|------|------|
| £            | s. | d. |     | £             | s. | d.  | dwt.       | grs. |   | lbs           | oz. | dwt. | grs. |
| 57           | 0  | 7½ | )   | 171           | 1  | 10½ | 5          | 9    | ) | 9             | 9   | 3    | 12   |
| 20           |    |    |     | 20            |    |     | 24         |      |   | 12            |     |      |      |
| <u>1140</u>  |    |    |     | <u>3421</u>   |    |     | <u>129</u> |      |   | <u>117</u>    |     |      |      |
| 12           |    |    |     | 12            |    |     |            |      |   | 20            |     |      |      |
| <u>13687</u> |    |    |     | <u>41062</u>  |    |     |            |      |   | <u>2343</u>   |     |      |      |
| 4            |    |    |     | 4             |    |     |            |      |   | 24            |     |      |      |
| <u>54750</u> | )  |    |     | <u>164250</u> | (3 |     |            |      |   | <u>9384</u>   |     |      |      |
|              |    |    |     | 164250        |    |     |            |      |   | 4686          |     |      |      |
|              |    |    |     |               |    |     |            |      |   | 129)56244(436 |     |      |      |
|              |    |    |     |               |    |     |            |      |   | 516           |     |      |      |
|              |    |    |     |               |    |     |            |      |   | <u>464</u>    |     |      |      |
|              |    |    |     |               |    |     |            |      |   | 387           |     |      |      |
|              |    |    |     |               |    |     |            |      |   | <u>774</u>    |     |      |      |
|              |    |    |     |               |    |     |            |      |   | 774           |     |      |      |

## EXERCISE 21—Page 115.

|                           |    |      | (5) |                         |     |      | (6)             |
|---------------------------|----|------|-----|-------------------------|-----|------|-----------------|
| a.                        | r. | per. |     | a.                      | r.  | per. |                 |
| 91                        | 0  | 6    | )   | 2366                    | 3   | 36   | 47·655 ÷ 4·5 =  |
| 4                         |    |      |     | 4                       |     |      | 45)476·55(10·59 |
| <u>364</u>                |    |      |     | <u>9467</u>             |     |      | <u>45</u>       |
| 40                        |    |      |     | 40                      |     |      | 26·5            |
| <u>14566</u>              | )  |      |     | <u>378716</u>           | (26 |      | <u>22·5</u>     |
|                           |    |      |     | 29132                   |     |      | 4·05            |
|                           |    |      |     | 87396                   |     |      | 4·05            |
|                           |    |      |     | 87396                   |     |      |                 |
|                           |    |      | (7) |                         |     |      | (8)             |
| 756·98 ÷ 76·73612 =       |    |      |     | 47·5782975 ÷ 26·175 =   |     |      |                 |
| 7673612)75698000 (9·864 + |    |      |     | 26175)47578·2975(1·8177 |     |      |                 |
| 69062508                  |    |      |     | 26175                   |     |      |                 |
| <u>6635492·0</u>          |    |      |     | <u>21403·2</u>          |     |      |                 |
| 6138889·6                 |    |      |     | 20940·0                 |     |      |                 |
| <u>496602·40</u>          |    |      |     | <u>463·29</u>           |     |      |                 |
| 460416·72                 |    |      |     | 261·75                  |     |      |                 |
| <u>36185·680</u>          |    |      |     | <u>201·547</u>          |     |      |                 |
| 30694·448                 |    |      |     | 183·225                 |     |      |                 |
| <u>5491·232</u>           |    |      |     | <u>18·3225</u>          |     |      |                 |
|                           |    |      |     | 18·3225                 |     |      |                 |

$$\begin{array}{r}
 (9) \\
 1 \div 7.6345 = \\
 76345) 10000.0 (0.1309 + \\
 \underline{7634.5} \\
 2365.50 \\
 \underline{2290.35} \\
 75.1500 \\
 68.7105
 \end{array}$$

$$\begin{array}{r}
 (10) \\
 75.347 \div 0.3829 = \\
 3829) 753470 (196.7798 + \\
 \underline{3829} \\
 37057 \\
 \underline{34461} \\
 25960 \\
 \underline{22974} \\
 2986.0 \\
 \underline{2680.3} \\
 305.70 \\
 \underline{268.03} \\
 37.670 \\
 \underline{34.461} \\
 3.2090 \\
 \underline{3.0632} \\
 .1458
 \end{array}$$

$$\begin{array}{r}
 (11) \\
 .0002 \div 0.00000008 = \\
 8) 200000 \\
 \underline{\hspace{1cm}} \\
 25000
 \end{array}$$

## EXERCISE 22—Page 116.

|                             |                              |
|-----------------------------|------------------------------|
| (1)                         | (2)                          |
| 95) \$3300000 (\$34736.8421 | 126) \$3860000 (\$30634.9206 |
| 285                         | 378                          |
| —                           | —                            |
| 450                         | 800                          |
| 380                         | 756                          |
| —                           | —                            |
| 700                         | 440                          |
| 665                         | 378                          |
| —                           | —                            |
| 350                         | 620                          |
| 285                         | 504                          |
| —                           | —                            |
| 650                         | 116.0                        |
| 570                         | 113.4                        |
| —                           | —                            |
| 80.0                        | 2.60                         |
| 76.0                        | 2.52                         |
| —                           | —                            |
| 4.00                        | .800                         |
| 3.80                        | .756                         |
| —                           | —                            |
| .200                        | .044                         |
| .190                        |                              |
| —                           |                              |
| .100                        |                              |
| .095                        |                              |
| .005                        |                              |

$$\begin{array}{r}
 (4) \\
 35781628) \$1145012096 (\$32 \\
 \underline{107344884} \\
 71563256 \\
 \underline{71563256}
 \end{array}$$



|   |                    |   |                        |
|---|--------------------|---|------------------------|
| (5)                                     |                    | (6)   | (7)                    |
| 27475271)                               | \$3764112127(\$137 | 9)\$972                                       | 108)\$972(\$9          |
|   | 27475271           | <u>      </u>                                 | 972                    |
|   | <u>      </u>      | \$108   |                        |
|   | 101658502          |   | (10)                   |
|   | 82425813           | (9)   | 1728)1000(·578 oz.     |
|   | <u>      </u>      | 792)340480(429 $\frac{89}{9}$ oz.             | 864·0                  |
|   | 192326897          | 3168  | <u>      </u>          |
|   | 192326897          | <u>      </u>                                 | 136·00                 |
| (8)                                     |                    | 2368  | 120·96                 |
| 294)\$8526(\$29                         |                    | 1584  | <u>      </u>          |
| 588                                     | (11)               |   | (12)                   |
| <u>      </u>                           | m. fur.            | 7840  | 19)4750(250lbs. 15·040 |
| 2646                                    | 33 2               | 7128  | 38                     |
| 2646                                    | 8                  | <u>      </u>                                 | <u>      </u>          |
|   | <u>      </u>      | 712   | 13·824                 |
|   | 266                | $\frac{712}{792} = \frac{89}{99}$             | 95                     |
|   | 40                 |   | 95                     |
|   | <u>      </u>      |   | (14)                   |
|   | 10640              | bush. pk. gal. qt. pt. bush. pk. gal. qt. pt. |                        |
|   | 5 $\frac{1}{2}$    | 297)729 1 1 1 1 ( 2 1 1 2 1 $\frac{2}{11}$    |                        |
|   | <u>      </u>      | 594   |                        |
|   | 53200              | <u>      </u>                                 |                        |
|   | 5320               | 135   |                        |
|   | <u>      </u>      | 4   |                        |
| 1155)58520(50 $\frac{770}{1165}$        |                    | 541   |                        |
| 5775                                    |                    | 297   |                        |
| <u>      </u>                           |                    | <u>      </u>                                 |                        |
| 770                                     |                    | 244   |                        |
| 50 $\frac{770}{1165} = 50\frac{2}{3}$ . |                    | 2   |                        |
|   |                    | <u>      </u>                                 |                        |
| (13)                                    |                    | 489   |                        |
| 978·634÷96·34762 =                      |                    | 297·  |                        |
| 9634762)97863400(10·157                 |                    | <u>      </u>                                 |                        |
| 9634762                                 |                    | 192   |                        |
| <u>      </u>                           |                    | 4   |                        |
| 1515780·0                               |                    | <u>      </u>                                 |                        |
| 963476·2                                |                    | 769   |                        |
| <u>      </u>                           |                    | 594   |                        |
| 552303·80                               |                    | <u>      </u>                                 |                        |
| 481738·10                               |                    | 175   |                        |
| <u>      </u>                           |                    | 2   |                        |
| 70565·700                               |                    | <u>      </u>                                 |                        |
| 67443·334                               |                    | 351   |                        |
| <u>      </u>                           |                    | 297   |                        |
| 3122·366                                |                    | <u>      </u>                                 |                        |
|   |                    | 54  |                        |
|   |                    | $\frac{54}{297} = \frac{2}{11}$ .             |                        |

| (15)         |         |       |                               |          |   |                                 |                               |         |             |
|--------------|---------|-------|-------------------------------|----------|---|---------------------------------|-------------------------------|---------|-------------|
| lbs. oz. dr. |         |       |                               | cwt. qr. |   | lbs. oz. dr.                    |                               |         |             |
| 9            | 7       | 8     | )                             | 179      | 3 | 4                               | 16                            | 0       |             |
| 16           |         |       |                               | 4        |   |                                 |                               |         |             |
|              |         |       |                               |          |   |                                 |                               |         | (16)        |
|              |         |       |                               |          |   | m. fur.                         | rds.                          | m.      |             |
| 151          |         |       |                               | 719      |   | 93                              | 4                             | 7       | 25000       |
| 16           |         |       |                               | 25       |   | 8                               |                               | 8       |             |
|              |         |       |                               |          |   |                                 |                               |         |             |
| 914          |         |       |                               | 3599     |   | 748                             |                               | 200000  |             |
| 151          |         |       |                               | 1438     |   | 40                              |                               | 40      |             |
|              |         |       |                               |          |   |                                 |                               |         | dys. hrs.   |
| 2424         |         |       |                               | 17979    |   | 29927                           | 29927)                        | 8000000 | (267 718295 |
|              |         |       |                               | 16       |   |                                 |                               | 59854   | 29927       |
|              |         |       |                               |          |   |                                 |                               |         |             |
|              |         |       |                               | 107890   |   |                                 |                               | 201460  |             |
|              |         |       |                               | 17979    |   |                                 |                               | 179562  |             |
|              |         |       |                               |          |   |                                 |                               |         |             |
|              |         |       |                               | 287680   |   |                                 |                               | 218980  |             |
|              |         |       |                               | 16       |   |                                 |                               | 209489  |             |
|              |         |       |                               |          |   |                                 |                               |         |             |
|              |         |       |                               | 1726080  |   |                                 |                               | 9491    |             |
|              |         |       |                               | 287680   |   |                                 |                               | 24      |             |
|              |         |       |                               |          |   |                                 |                               |         |             |
| 2424)        | 4602880 | (1898 | <sup>266</sup> <sub>303</sub> |          |   |                                 |                               | 37964   |             |
|              | 2424    |       |                               |          |   |                                 |                               | 18982   |             |
|              |         |       |                               |          |   |                                 |                               |         |             |
|              | 21788   |       |                               |          |   |                                 |                               | 227784  |             |
|              | 19392   |       |                               |          |   |                                 |                               | 209489  |             |
|              |         |       |                               |          |   |                                 |                               |         |             |
|              | 23968   |       |                               |          |   |                                 |                               | 18295   |             |
|              | 21816   |       |                               |          |   |                                 |                               |         |             |
|              |         |       |                               |          |   |                                 |                               |         |             |
|              | 21520   |       |                               |          |   |                                 |                               |         |             |
|              | 19392   |       |                               |          |   |                                 |                               |         |             |
|              |         |       |                               |          |   |                                 |                               |         |             |
|              | 2128    |       |                               |          |   | <sup>2128</sup> <sub>2424</sub> | <sup>266</sup> <sub>303</sub> |         |             |

EXERCISE 23—Page 118.

(3)

DCCIX, M̄VCCCLXXVI, M̄X̄CMXCIX, L̄XXXVMIV,  
MMM̄CMXLV̄M̄MDXCVI.

$$\begin{array}{r}
 (4) \\
 72 = 8 \times 9 \\
 \text{lbs. oz.} \\
 749 \ 10 \\
 \quad 8 \\
 \hline
 5997 \ 0 \\
 \quad 9 \\
 \hline
 53973 \ 0
 \end{array}$$

$$\begin{array}{r}
 (5) \\
 17 = 7 + 10 \times 1 \\
 \text{s. d.} \quad \quad \text{£ s. d.} \\
 4 \ 7\frac{3}{4} \times 7 = 1 \ 12 \ 6\frac{1}{4} \\
 \quad 10 \\
 \hline
 \text{£} 2 \ 6 \ 5\frac{1}{2} + 1 = 2 \ 6 \ 5\frac{1}{2} \\
 \hline
 \quad \quad \quad 3 \ 18 \ 11\frac{3}{4}
 \end{array}$$

$$\begin{array}{r}
 (6) \\
 3Q)285000000Q \quad \text{dys. hrs.} \\
 24)95000000(3958333 \ 8 \\
 \quad 72 \\
 \quad \hline
 \quad 230 \quad 365\frac{1}{4})3958333(10837 \\
 \quad 216 \quad \quad 4 \quad \quad 4 \\
 \quad \hline
 \quad 140 \quad 1461)1583332 \\
 \quad 120 \quad \quad 1461 \\
 \quad \hline
 \quad 200 \quad \quad 12233 \\
 \quad 192 \quad \quad 11688 \\
 \quad \hline
 \quad 80 \quad \quad 5453 \\
 \quad 72 \quad \quad 4383 \\
 \quad \hline
 \quad 80 \quad \quad 10702 \\
 \quad 72 \quad \quad 10227 \\
 \quad \hline
 \quad 80 \quad \quad 4)475 \text{ quarter days.} \\
 \quad 72 \quad \quad \text{days. hrs.} \\
 \quad \hline
 \quad \quad 118\frac{3}{4} = 118 \ 18 \\
 \quad \quad \text{Add} \quad 8 \\
 \quad \quad \hline
 \quad \quad 119 \ 2
 \end{array}$$

10837 yrs. 119 days, 2 hrs.

$$\begin{array}{r}
 (7) \\
 \text{£} 729 \times 400 = \$2916 \cdot 00 \\
 17\text{s.} \times 20 = \quad 3 \cdot 40 \\
 6\frac{1}{4}\text{d.} = 25 \text{ far.} \times 5 \div 12 = \quad 10\frac{5}{12} \\
 \hline
 \$2919 \cdot 50\frac{5}{12}
 \end{array}$$

$$\begin{array}{r}
 (8) \\
 \$10000 \\
 9876 \cdot 23 \\
 \hline
 \$123 \cdot 77
 \end{array}$$

(10)

(11)

$$\begin{array}{r}
 \text{in.} \\
 12) 7964327 \\
 \hline
 12) 663693-11 \left. \vphantom{\begin{array}{l} 12) 663693-11 \\ 55307-9 \end{array}} \right\} 119 \text{ in.} \\
 \hline
 55307-9 \\
 9) 55307 \text{ ft. } 119 \text{ in.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \$729.43 \\
 16.79 \\
 976.81 \\
 9987.17 \\
 429.00 \\
 129.19 \\
 \hline
 \$12268.30
 \end{array}$$

$$\begin{array}{r}
 30\frac{1}{4}) 6145 \text{ yds. } 2 \text{ ft. } 119 \text{ in.} \\
 \hline
 4 \quad 4
 \end{array}$$

$$\begin{array}{r}
 121) 24580 \\
 11) 24580 \\
 \hline
 \end{array}
 \quad
 \begin{array}{l}
 203 \text{ p. } 4\frac{1}{4} \text{ y.} = 203 \text{ p. } 4 \text{ yds. } 2 \text{ ft. } 36 \text{ in.} \\
 \text{Add} \quad \quad \quad 2 \text{ ft. } 119 \text{ in.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 11) 2234-6 \left. \vphantom{\begin{array}{l} 11) 2234-6 \\ 203-1 \end{array}} \right\} 17 \text{ qr. yds.} \\
 \hline
 203-1
 \end{array}$$

$$40) 203 \text{ p. } 4 \text{ yds. } 5 \text{ ft. } 11 \text{ in.}$$

$$4) 5 \text{ rd. } 3 \text{ p. } 4 \text{ yds. } 5 \text{ ft. } 11 \text{ in.}$$

$$1 \text{ a. } 1 \text{ r. } 3 \text{ p. } 4 \text{ yds. } 5 \text{ ft. } 11 \text{ in.}$$

(12)

$$429 = 9 + 10 \times 2 + 10 \times 10 \times 4$$

$$\begin{array}{cccc}
 \text{wks.} & \text{dys.} & \text{hrs.} & \text{min.} \\
 6 & 4 & 3 & 17 \times 9 = 10
 \end{array}
 \quad
 \begin{array}{cccc}
 \text{wks.} & \text{dys.} & \text{hrs.} & \text{min.} \\
 59 & 2 & 5 & 33
 \end{array}$$

$$\begin{array}{cccc}
 65 & 6 & 8 & 50 \times 2 = 10
 \end{array}
 \quad
 \begin{array}{cccc}
 131 & 5 & 17 & 40
 \end{array}$$

$$\begin{array}{cccc}
 659 & 0 & 16 & 20 \times 4 = 2636
 \end{array}
 \quad
 \begin{array}{cccc}
 2636 & 2 & 17 & 20 \\
 \hline
 2827 & 3 & 16 & 33
 \end{array}$$

wks.

$$52) 2827 (54 \text{ yrs. } 19 \text{ wks. } 3 \text{ dys. } 16 \text{ hrs. } 33 \text{ min.}$$

260

227

208

19 wks.

(15)

(16)

tons.

\$136

324

 $\$136 \times 4 = 544 - 95 = 449$ 

20

1902

cwt. qr. lbs. —

13 2 14 6480

2487

4 4

54 25920

25 25

284 129600

108 51840

1364 ) 648000  $(475 \frac{100}{1364} =$ 5456  $475 \frac{25}{341} \text{ hds.}$ 

(14)

78.96 10240

.00042 9548

15792 6920

31584 6820

.0331632 100

(17)

yds. qrs. na. yds. qrs. na.

3 1 2 ) 39 2 3

4 4

13 158

4 4

54 ) 635  $(11 \frac{11}{34})$ 

54

95

54

41

(18)

(19)

(21)

a. a. a. r. per.

25 732 96 3 17

197 674 4

156 — —

97 58 387

199 40

674 15497) \$7764.0 (\$0.501

7748.5

15.500

15.497

3

lbs. oz. dwt. grs.

12) 36 8 14 16

3 0 14 13  $\frac{1}{3}$ 

(20)

\$

20 \$312

75 275

97

83 \$ 37

275

(22)

a. r. per.

6 3 12

7 2 0

9 0 13

5 2 36

29 0 21

(23)

(24)

(25)

|           |                    |   |
|-----------|--------------------|---|
| 5         | lbs. oz. dwt. grs. | $£972 \times 400 = \$3888.00$   |
| 7         | 5 9 8 0            | $11s. \times 20 = 2.20$   |
| 9         | 3 2 16 16          | $11\frac{1}{4}d. = 45 \text{ far.} \times 5 \div 12 = .18\frac{1}{4}$ |
| —         | 4 6 17 0           | _____   |
| 21)294(14 | 1 8 19 22          | $\$3890.38\frac{1}{4}$  |
| 21        | _____              |   |
| —         | 15 4 1 14          |   |
| 84        |                    |   |
| 84        |                    |   |

(26)

(27)

(28)

|                         |                       |               |
|-------------------------|-----------------------|---------------|
| lbs. oz. drs. scr. grs. | 56                    | cwt. qr. lbs. |
| 179 3 3 1 14            | 25                    | 6 2 11        |
| 12                      | —                     | 5 3 16        |
| —                       | 280                   | 8 0 7         |
| 2151 oz.                | 112                   | 3 1 17        |
| 8                       | _____                 | _____ lbs.    |
| —                       | 1400                  | 24 0 1 = 2401 |
| 17211 drs.              | 2                     | _____ .15     |
| 3                       | _____                 |               |
| —                       | 2800 sq. ft. in roof. | 12005         |
| * 51634 scr.            | 6                     | 2401          |
| 20                      | _____                 | _____         |
| —                       | 16800                 | $\$360.15$    |
| 1032694 grs.            |                       |               |

(29)

(30)

|            |            |            |
|------------|------------|------------|
| 29         | \$         |            |
| 57         | 139468     | 370129     |
| —          | 98579      | 238047     |
| 203        | _____      | _____      |
| 145        | $\$238047$ | $\$132082$ |
| —          |            |            |
| 1653       |            |            |
| .15        |            |            |
| —          |            |            |
| 8265       |            |            |
| 1653       |            |            |
| —          |            |            |
| $\$247.95$ |            |            |

(31)

| £             | s.                            | d.               | £                | s.                              | d.              |
|---------------|-------------------------------|------------------|------------------|---------------------------------|-----------------|
| 9             | 19                            | 11 $\frac{3}{4}$ | 1694             | 16                              | 0 $\frac{1}{2}$ |
| 20            |                               |                  | 20               |                                 |                 |
| <u>      </u> | $\frac{3}{4} = \frac{63}{84}$ |                  | <u>      </u>    | $\frac{11}{42} = \frac{22}{84}$ |                 |
| 199           |                               |                  | 33896            |                                 |                 |
| 12            |                               |                  | 12               |                                 |                 |
| <u>      </u> |                               |                  | <u>      </u>    |                                 |                 |
| 2399          |                               |                  | 406752           |                                 |                 |
| 84            |                               |                  | 84               |                                 |                 |
| <u>      </u> |                               |                  | <u>      </u>    |                                 |                 |
| 9659          |                               |                  | 1627030          |                                 |                 |
| 19192         |                               |                  | 3254016          |                                 |                 |
| <u>      </u> |                               |                  | <u>      </u>    |                                 |                 |
| 201579        |                               |                  | 84167190 (169·49 |                                 |                 |
|               |                               |                  | 201579           |                                 |                 |
|               |                               |                  | <u>      </u>    |                                 |                 |
|               |                               |                  | 1400929          |                                 |                 |
|               |                               |                  | 1209474          |                                 |                 |
|               |                               |                  | <u>      </u>    |                                 |                 |
|               |                               |                  | 1914550          |                                 |                 |
|               |                               |                  | 1814211          |                                 |                 |
|               |                               |                  | <u>      </u>    |                                 |                 |
|               |                               |                  | 100339·0         |                                 |                 |
|               |                               |                  | 80631·6          |                                 |                 |
|               |                               |                  | <u>      </u>    |                                 |                 |
|               |                               |                  | 19707·40         |                                 |                 |
|               |                               |                  | 18142·11         |                                 |                 |
|               |                               |                  | <u>      </u>    |                                 |                 |
|               |                               |                  | 1565·29          |                                 |                 |

(34)

| cwt.          | qr.           | lbs.          |                        |
|---------------|---------------|---------------|------------------------|
| 2             | 0             | 17            |                        |
| 3             | 2             | 15            |                        |
| 2             | 1             | 20            |                        |
| 5             | 3             | 17            |                        |
| <u>      </u> | <u>      </u> | <u>      </u> | lbs.                   |
| 14            | 0             | 19            | =1419                  |
|               |               |               | ·37 $\frac{1}{2}$      |
|               |               |               | <u>      </u>          |
|               |               |               | 9933                   |
|               |               |               | 4257                   |
|               |               |               | 709 $\frac{1}{2}$      |
|               |               |               | <u>      </u>          |
|               |               |               | \$532·12 $\frac{1}{2}$ |

(32)

|                    |   |       |   |                       |
|--------------------|---|-------|---|-----------------------|
| £19                | × | 400   | = | \$76·00               |
| 19s.               | × | 20    | = | 3·80                  |
| 11 $\frac{3}{4}$ d | = | 47far | × | 5 ÷ 12 =              |
|                    |   |       |   | ·19 $\frac{7}{8}$     |
|                    |   |       |   | <u>      </u>         |
|                    |   |       |   | \$79·99 $\frac{7}{8}$ |

(33)

| cwt.          | qr.           | lbs.          | cwt.          | qr.           | lbs.          |               |
|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| 3             | 2             | 11            | 12            | 0             | 0             |               |
| 4             | 1             | 15            | 8             | 0             | 1             |               |
| <u>      </u> | <u>      </u> | <u>      </u> | <u>      </u> | <u>      </u> | <u>      </u> | lbs.          |
| 8             | 0             | 1             | 3             | 3             | 24            | =399          |
|               |               |               |               |               |               | ·15           |
|               |               |               |               |               |               | <u>      </u> |
|               |               |               |               |               |               | 1995          |
|               |               |               |               |               |               | 399           |
|               |               |               |               |               |               | <u>      </u> |
|               |               |               |               |               |               | \$59·85       |

(36)

|         |               |         |   |
|---------|---------------|---------|---|
| 43·2    | ÷             | 76·8437 | = |
| 768437) | 432000·0      | (0·562  |   |
|         | 384218·5      |         |   |
|         | <u>      </u> |         |   |
|         | 47781·50      |         |   |
|         | 46106·22      |         |   |
|         | <u>      </u> |         |   |
|         | 1675·280      |         |   |
|         | 1536·874      |         |   |
|         | <u>      </u> |         |   |
|         | 138·406       |         |   |

(37)

$$123 \cdot 4 \div \cdot 000000066 =$$

$$123400000000 \div 66$$

$$6)123400000000$$

$$11)20566666666 \cdot 666$$

$$\underline{\hspace{1cm}} \quad \cdot \cdot$$

$$1869696969 \cdot 69$$

(38)

$$\$63 \cdot 29 \quad \$2789 \cdot 27$$

$$\underline{\hspace{1cm}} \quad 17 \quad 1075 \cdot 93$$

$$44303 \quad \$1713 \cdot 34$$

$$\underline{\hspace{1cm}} \quad 6329$$

$$\underline{\hspace{1cm}} \quad \$1075 \cdot 93$$

(39)

$$\pounds 29 \times 400 = \$116 \cdot 00$$

$$6s. \times 20 = 1 \cdot 20$$

$$11 \frac{1}{2} d. = 47 far. \times 5 \div 12 = \cdot 19 \frac{7}{12}$$

$$\underline{\hspace{1cm}} \quad 117 \cdot 39 \frac{7}{12}$$

$$\$278 \cdot 43$$

$$417 \cdot 16$$

$$11 \cdot 27$$

$$2110 \cdot 40$$

$$723 \cdot 15$$

$$117 \cdot 39 \frac{7}{12}$$

$$173) 3657 \cdot 80 \frac{7}{12}$$

$$\underline{\hspace{1cm}} \quad 12 \quad \underline{\hspace{1cm}} \quad 12$$

$$2076) 43893 \cdot 67 (\$21 \cdot 1433$$

$$\underline{\hspace{1cm}} \quad 4152$$

$$2373$$

$$\underline{\hspace{1cm}} \quad 2076$$

$$297 \cdot 6$$

$$207 \cdot 6$$

$$\underline{\hspace{1cm}} \quad 90 \cdot 07$$

$$83 \cdot 04$$

$$\underline{\hspace{1cm}} \quad 7 \cdot 030$$

$$6 \cdot 228$$

$$\underline{\hspace{1cm}} \quad \cdot 8020$$

$$\cdot 6228$$

$$\underline{\hspace{1cm}} \quad \cdot 1792$$

(40)

$$2076) 491544 \frac{1}{2} (236 \frac{1}{2}$$

$$\underline{\hspace{1cm}} \quad 4152$$

$$7634$$

$$6228$$

$$\underline{\hspace{1cm}} \quad 14064$$

$$12456$$

$$\underline{\hspace{1cm}} \quad 1608$$

$$\frac{1608}{2076} = \frac{1}{3}$$



## Exercise 24.—Page 127.

| (1)                                | (2)                   | (3)        | (4)                            |
|------------------------------------|-----------------------|------------|--------------------------------|
| 2)11368                            | 2)2934                | 3)1011     | 2)1000                         |
| <u>2)5684</u>                      | <u>3)1467</u>         | <u>337</u> | <u>2)500</u>                   |
| 2)2842                             | 3)489                 | 3×337      | 2)250                          |
| <u>7)1421</u>                      | <u>163</u>            |            | <u>5)125</u>                   |
| 7)203                              | 2×3 <sup>2</sup> ×163 |            | <u>5)25</u>                    |
| <u>29</u>                          |                       |            | <u>5</u>                       |
| 2 <sup>3</sup> ×7 <sup>2</sup> ×29 |                       |            | 2 <sup>3</sup> ×5 <sup>3</sup> |
| (5)                                | (6)                   | (7)        | (8)                            |
| 2)1024                             | 2)32320               | 7)707      | 2)1118                         |
| <u>2)512</u>                       | <u>2)16160</u>        | <u>101</u> | <u>13)559</u>                  |
| 2)256                              | 2)8080                | 7×101      | <u>43</u>                      |
| <u>2)128</u>                       | <u>2)4040</u>         |            | 2×13×43                        |
| 2)64                               | 2)2020                |            |                                |
| <u>2)32</u>                        | <u>2)1010</u>         |            |                                |
| 2)16                               | 5)505                 |            |                                |
| <u>2)8</u>                         | <u>101</u>            |            |                                |
| 2)4                                | 2 <sup>6</sup> ×5×101 |            |                                |
| <u>2</u>                           |                       |            |                                |
| 2 <sup>10</sup>                    |                       |            |                                |

## EXERCISE 25—Page 128.

(1)

$$100=2^2 \times 5^2$$

1..2..4

1..5..25

1..2..4..5..10..20..25..50..100

(2)

$$810 = 3^4 \times 2 \times 5.$$

1..3..9..27..81

1..2

1..3..9..27..81..2..6..18..54..162

1..5

1..3..9..27..81..2..6..18..54..162..5..15..45..135..405..

10..30..90..270..810 =

1..2..3..5..6..9..10..15..18..27..30..45..54..81..90..135..

162..270..405..810.

(3)

$$920 = 2^3 \times 5 \times 23.$$

1..2..4..8

1..5

1..2..4..8..5..10..20..40

1..23

1..2..4..8..5..10..20..40..23..46..92..184..115..230..460..920 =

1..2..4..5..8..10..20..23..40..46..92..115..184..230..460..920.

(4)

$$25000 = 5^5 \times 2^3$$

1..5..25..125..625..3125

1..2..4..8

1..5..25..125..625..3125..2..10..50..250..1250..6250..4..20..100..

500..2500..12500..8..40..200..1000..5000..25000 =

1..2..4..5..8..10..20..25..40..50..100..125..200..250..500..625..

1000..1250..2500..3125..5000..6250..12500..25000.

---

EXERCISE 26—Page 128.

(1)

$$88200 = 2^3 \times 3^2 \times 5^2 \times 7^2$$

$$3+1=4$$

$$2+1=3$$

$$2+1=3$$

$$2+1=3$$

$$4 \times 3 \times 3 \times 3 = 108$$

(2)

$$3500 = 2^2 \times 5^3 \times 7$$

$$2+1=3$$

$$3+1=4$$

$$1+1=2$$

$$3 \times 4 \times 2 = 24$$

(3)

$$6336=2^6 \times 3^2 \times 11$$

$$6+1=7$$

$$2+1=3$$

$$1+1=2$$

$$7 \times 3 \times 2=42$$

(4)

$$824=2^3 \times 103$$

$$3+1=4$$

$$1+1=2$$

$$4 \times 2=8$$

(5)

$$49000=2^3 \times 5^3 \times 7^2$$

$$3+1=4$$

$$3+1=4$$

$$2+1=3$$

$$4 \times 4 \times 3=48$$

(6)

$$81000=2^3 \times 3^4 \times 5^3$$

$$3+1=4$$

$$4+1=5$$

$$3+1=4$$

$$4 \times 5 \times 4=80$$

(7)

$$75600=2^4 \times 3^3 \times 5^2 \times 7$$

$$4+1=5$$

$$3+1=4$$

$$2+1=3$$

$$1+1=2$$

$$5 \times 4 \times 3 \times 2=120$$

(8)

$$25600=2^{10} \times 5^2$$

$$10+1=11$$

$$2+1=3$$

$$11 \times 3=33$$

---

EXERCISE 27—Page 129.

(1)

$$21=7 \times 3$$

$$18=2 \times 3 \times 3$$

$$27=3 \times 3 \times 3$$

$$36=4 \times 3 \times 3$$

3 is common to all.

(2)

$$21=3 \times 7$$

$$77=11 \times 7$$

$$42=2 \times 3 \times 7$$

$$35=5 \times 7$$

7 is common to all.

(3)

$$26=2 \times 13$$

$$52=2 \times 2 \times 13$$

$$91=7 \times 13$$

$$143=11 \times 13$$

13 is common to all.

(4)

$$82=41 \times 2$$

$$118=59 \times 2$$

$$146=73 \times 2$$

2 is common to all.

## EXERCISE 28—Page 130.

| (1)            | (2)            | (3)           |
|----------------|----------------|---------------|
| 296)407(1      | 308)506(1      | 74)84(1       |
| <u>296</u>     | <u>308</u>     | <u>74</u>     |
| 111)296(2      | 198)308(1      | 10)74(7       |
| <u>222</u>     | <u>198</u>     | <u>70</u>     |
| 74)111(1       | 110)198(1      | 4)10(2        |
| <u>74</u>      | <u>110</u>     | <u>8</u>      |
| 37)74(2        | 88)110(1       | 2)4           |
| <u>74</u>      | <u>88</u>      | <u>-</u>      |
| G. C. M. = 37. | —              | 2             |
|                | 22)88(4        | G. C. M. = 2. |
|                | <u>88</u>      |               |
|                | G. C. M. = 22. |               |

| (4)             | (5)           |
|-----------------|---------------|
| 1825)2555(1     | 556)672(1     |
| <u>1825</u>     | <u>556</u>    |
| 730)1825(2      | 116)556(4     |
| <u>1460</u>     | <u>464</u>    |
| 365)730(2       | 92)116(1      |
| <u>730</u>      | <u>92</u>     |
| G. C. M. = 365. | —             |
|                 | 24)92(3       |
|                 | <u>72</u>     |
|                 | —             |
|                 | 20)24(1       |
|                 | <u>20</u>     |
|                 | —             |
|                 | 4)20(5        |
|                 | <u>20</u>     |
|                 | G. C. M. = 4. |

## EXERCISE 29—Page 131.

$$\begin{array}{r} (1) \\ 110)140(1 \\ \underline{110} \end{array}$$

$$\begin{array}{r} 30)110(3 \\ \underline{90} \end{array}$$

$$\begin{array}{r} 20)30(1 \\ \underline{20} \end{array}$$

$$\begin{array}{r} 10)680 \\ \underline{68} \end{array} \quad \begin{array}{r} 10)20 \\ \underline{2} \end{array}$$

Therefore 10 is their G. C. M.

$$\begin{array}{r} (3) \\ 468)922(1 \\ \underline{468} \\ 454)468(1 \\ \underline{454} \end{array}$$

$$\begin{array}{r} 14)454(32 \\ \underline{42} \end{array}$$

$$\begin{array}{r} 34 \\ \underline{28} \end{array}$$

$$\begin{array}{r} 6)14(2 \\ \underline{12} \end{array}$$

$$\begin{array}{r} 2)6 \\ \underline{-} \end{array}$$

$$\begin{array}{r} 3 \end{array}$$

375 is not divisible by 2,  
and therefore their G. C. M. is 1.

$$\begin{array}{r} (2) \\ 1326)3094(2 \\ \underline{2652} \end{array}$$

$$\begin{array}{r} 442)1326(3 \\ \underline{1326} \end{array}$$

Also 4420 is divisible by 442;  
therefore it is their G. C. M.

$$\begin{array}{r} (4) \\ 204)1190(5 \\ \underline{1020} \end{array}$$

$$\begin{array}{r} 170)204(1 \\ \underline{170} \end{array}$$

$$\begin{array}{r} 34)170(5 \\ \underline{170} \end{array}$$

$$\begin{array}{r} 17)2006(118 \\ \underline{17} \end{array}$$

$$\begin{array}{r} 30 \\ \underline{17} \end{array}$$

$$\begin{array}{r} 136 \\ \underline{136} \end{array}$$

G. C. M.=17.

## EXERCISE 30—Page 132.

(2)

$$56=2^3 \times 7$$

$$84=2^2 \times 3 \times 7$$

$$140=2^2 \times 5 \times 7$$

$$168=2^3 \times 3 \times 7$$

The greatest factors which are common are  $2^2$  and 7;  
therefore the G. C. M.= $2^2 \times 7=28$ .

(3)

$$241920 = 2^8 \times 3^3 \times 5 \times 7$$

$$380160 = 2^8 \times 3^3 \times 5 \times 11$$

$$69120 = 2^9 \times 3^3 \times 5$$

$$103680 = 2^8 \times 3^4 \times 5$$

The greatest factors which are common are  $2^8$ ,  $3^3$  and 5 ;  
therefore the G. C. M.  $= 2^8 \times 3^3 \times 5 = 34560$ .

(4)

$$10800 = 2^4 \times 3^3 \times 5^2$$

$$28040 = 2^3 \times 5 \times 701$$

$$2160 = 2^4 \times 3^3 \times 5$$

The greatest factors which are common are  $2^3$  and 5 ;  
therefore the G. C. M.  $= 2^3 \times 5 = 40$ .

---

EXERCISE 31—Page 133.

(2)

$$6 = 2 \times 3$$

$$7 = 7$$

$$42 = 2 \times 3 \times 7$$

$$9 = 3^2$$

$$10 = 2 \times 5$$

$$630 = 2 \times 3^2 \times 5 \times 7$$

$$2 \times 3^2 \times 5 \times 7 = 630.$$

(3)

$$1 = 1$$

$$2 = 2$$

$$3 = 3$$

$$4 = 2^2$$

$$5 = 5$$

$$6 = 2 \times 3$$

$$7 = 7$$

$$8 = 2^3$$

$$9 = 3^2$$

$$3^2 \times 2^3 \times 5 \times 7 = 2520.$$

(4)

$$6 = 2 \times 3$$

$$9 = 3^2$$

$$12 = 2^2 \times 3$$

$$15 = 3 \times 5$$

$$18 = 2 \times 3^2$$

$$21 = 3 \times 7$$

$$30 = 2 \times 3 \times 5$$

$$2^2 \times 3^2 \times 5 \times 7 = 1260.$$

(5)

$$670 = 2 \times 5 \times 67$$

$$100 = 2^2 \times 5^2$$

$$335 = 5 \times 67$$

$$25 = 5^2$$

$$2^2 \times 5^2 \times 67 = 6700$$

(6)

$$8 = 2^3$$

$$10 = 2 \times 5$$

$$18 = 2 \times 3^2$$

$$27 = 3^3$$

$$36 = 2^2 \times 3^2$$

$$44 = 2^2 \times 11$$

$$396 = 2^2 \times 3^2 \times 11$$

$$2^3 \times 3^3 \times 5 \times 11 = 11880.$$

## EXERCISE 32—Page 134.

| (1)  | (2)   | (3)  |
|--|---|--|
| 2)12..10..24   | 2)14..21..3..2..63  | 2)18..12..39..216..234                                   |
| 2) 6.. 5..12   | 3) 7..21..3..1..63  | 2) 9.. 6..39..108..117                                   |
| 3) 3.. 5.. 6   | 7) 7.. 7..1..1..21  | 3) 9.. 3..39.. 54..117                                   |
| 1.. 5.. 2  | 1.. 1..1..1.. 3   | 3) 3.. 1..13.. 18.. 39                                   |
| $2 \times 2 \times 3 \times 5 \times 2 = 120$                              | $2 \times 3 \times 7 \times 3 = 126$                            | 13) 1.. 1..13.. 6.. 13                                   |
|  |   | 1.. 1..1 .. 6.. 1  |
|  |   | $2 \times 2 \times 3 \times 3 \times 13 \times 6 = 2808$ |
| (4)  | (5)   |  |
| 2)8..18..15..20..70  | 2)24..16..18..20  |  |
| 2)4.. 9..15..10..35  | 2)12.. 8.. 9..10  |  |
| 3)2.. 9..15.. 5..35  | 2) 6.. 4.. 9.. 5  |  |
| 5)2.. 3.. 5.. 5..35  | 3) 3.. 2.. 9.. 5  |  |
| 2.. 3.. 1.. 1.. 7  | 1.. 2.. 3.. 5   |  |
| $2 \times 2 \times 3 \times 5 \times 2 \times 3 \times 7 = 2520$           | $2 \times 2 \times 2 \times 3 \times 2 \times 3 \times 5 = 720$ |  |
| (6)  | (7)   |  |
| 2)60..50..144..35..18  | 2)27..54..81..14..63  |  |
| 2)30..25.. 72..35.. 9  | 3)27..27..81.. 7..63  |  |
| 3)15..25.. 36..35.. 9  | 3) 9.. 9..27.. 7..21  |  |
| 3) 5..25.. 12..35.. 3  | 3) 3.. 3.. 9.. 7.. 7  |  |
| 5) 5..25.. 4..35.. 1   | 7) 1.. 1.. 3.. 7.. 7  |  |
| 1.. 5.. 4.. 7.. 1  | 1.. 1.. 3.. 1.. 1   |  |
| $2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 4 \times 7 = 25200$ | $2 \times 3 \times 3 \times 3 \times 7 \times 3 = 1134$         |  |

## EXERCISE 33—Page 136.

| (1)                                | (2)                               |
|------------------------------------|-----------------------------------|
| 300 300..200..150..50..50..75..125 | 165 20..60..15..15..210..63..27   |
| 10  2                              | 21  4.. 4 14..21.. 9              |
|                                    | 12  4.. 4 2 3                     |
| $300 \times 10 = 3000$             | $165 \times 21 \times 12 = 41580$ |

(3)

$$\begin{array}{r|rrrrr}
 144 & 12 & 132 & 144 & 60 & 96 & 1728 \\
 12 & & 11 & & 5 & 2 & 12 \\
 55 & & 11 & & 5 & & \\
 \hline
 & 144 \times 12 \times 55 = 95040.
 \end{array}$$

## EXERCISE 34—Page 138.

| (1)                 | (2)                      | (3)       | (4)                 |
|---------------------|--------------------------|-----------|---------------------|
| 12)592835           | 5)3700                   | 11)10000  | 6)1000000           |
| 12)49402.. <i>e</i> | 5)740..0                 | 11)909..1 | 6)166666.. <i>4</i> |
| 12)4116.. <i>t</i>  | 5)148..0                 | 11)82..7  | 6)27777.. <i>4</i>  |
| 12)343..0           | 5)29..3                  | 7..5      | 6)4629..3           |
| 12)28..7            | 5)5..4                   | 7571.     | 6)771..3            |
| 2..4                | 1..0                     |           | 6)128..3            |
| 2470 <i>te</i>      | 104300.                  |           | 6)21..2             |
|                     |                          |           | 3..3                |
|                     |                          |           | 33233344            |
| (5)                 | (6)                      | (7)       | (8)                 |
| 8)10000             | 12)12345654321           | 9)10000   | 2)300               |
| 8)1250..0           | 12)1028804526.. <i>9</i> | 9)1111..1 | 2)150..0            |
| 8)156..2            | 12)85733710.. <i>6</i>   | 9)123..4  | 2)75..0             |
| 8)19..4             | 12)7144475.. <i>t</i>    | 9)13..6   | 2)37..1             |
| 2..3                | 12)595372.. <i>e</i>     | 1..4      | 2)18..1             |
| 23420.              | 12)49614.. <i>4</i>      | 14641.    | 2)9..0              |
|                     | 12)4134.. <i>6</i>       |           | 2)4..1              |
|                     | 12)344.. <i>6</i>        |           | 2)2..0              |
|                     | 12)28.. <i>8</i>         |           | 1..0                |
|                     | 2..4                     |           | 100101100           |
|                     | 248664 <i>et</i> 69.     |           |                     |



## EXERCISE 35—Page 139.

| (1)       | (2)     | (3)        |
|-----------|---------|------------|
| IX        | V       | IV         |
| 8)37704   | 7)444   | 9)1212201  |
| 8)4311..5 | 7)32..5 | 9)23121..0 |
| 8)480..1  | 2..3    | 9)1101..0  |
| 8)54..4   | 235.    | 9)21..0    |
| 6..1      |         | 1..0       |
| 61415.    |         | 10000.     |

## EXERCISE 36—Page 140.

| (1)      | (2)       | (3)     | (4)      |
|----------|-----------|---------|----------|
| IV       | III       | IX      | VI       |
| 20212331 | 101202220 | 1522365 | 33233344 |
| 4        | 3         | 9       | 6        |
| —        | —         | —       | —        |
| 8        | 3         | 14      | 21       |
| 4        | 3         | 9       | 6        |
| —        | —         | —       | —        |
| 34       | 10        | 128     | 128      |
| 4        | 3         | 9       | 6        |
| —        | —         | —       | —        |
| 137      | 32        | 1154    | 771      |
| 4        | 3         | 9       | 6        |
| —        | —         | —       | —        |
| 550      | 96        | 10389   | 4629     |
| 4        | 3         | 9       | 6        |
| —        | —         | —       | —        |
| 2203     | 290       | 93507   | 27777    |
| 4        | 3         | 9       | 6        |
| —        | —         | —       | —        |
| 8815     | 872       | 841568  | 166666   |
| 4        | 3         |         | 6        |
| —        | —         |         | —        |
| 35261    | 2618      |         | 1000000  |
|          | 3         |         |          |
|          | 7854      |         |          |

(6)

$$\begin{array}{r}
 \text{IX} \\
 3)132713 \\
 \hline
 3)40834..0 \\
 \hline
 3)13271..1 \\
 \hline
 3)4083..1 \\
 \hline
 3)1327..0 \\
 \hline
 3)408..1 \\
 \hline
 3)132..2 \\
 \hline
 3)40..2 \\
 \hline
 3)13..0 \\
 \hline
 3)4..0 \\
 \hline
 1..1
 \end{array}$$

$$\begin{array}{r}
 \text{IX} \\
 12)132713 \\
 \hline
 12)10207..9 \\
 \hline
 12)682..t \\
 \hline
 12)51..8 \\
 \hline
 3..t
 \end{array}$$

$$\begin{array}{r}
 \text{IX} \\
 8)132713 \\
 \hline
 8)14757..1 \\
 \hline
 8)1652..0 \\
 \hline
 8)184..6 \\
 \hline
 8)21..5 \\
 \hline
 2..3
 \end{array}$$

| IX            | III           |               | XII        | VIII          |
|---------------|---------------|---------------|------------|---------------|
| 132713 =      | 11002210110 = |               | 3t8t9 =    | 235601        |
| 9             | 3             |               | 12         | 8             |
| <hr/>         | <hr/>         |               | <hr/>      | <hr/>         |
| 12            | 4             | 332           | 46         | 19            |
| 9             | 3             | 3             | 12         | 8             |
| <hr/>         | <hr/>         | <hr/>         | <hr/>      | <hr/>         |
| 110           | 12            | 997           | 560        | 157           |
| 9             | 3             | 3             | 12         | 8             |
| <hr/>         | <hr/>         | <hr/>         | <hr/>      | <hr/>         |
| 997           | 36            | 2991          | 6730       | 1262          |
| 9             | 3             | 3             | 12         | 8             |
| <hr/>         | <hr/>         | <hr/>         | <hr/>      | <hr/>         |
| 8974          | 110           | 8974          | 80769 den. | 10096         |
| 9             | 3             | 3             |            | 8             |
| <hr/>         | <hr/>         | <hr/>         |            | <hr/>         |
| 80769 denary. | 332           | 26923         |            | 80769 denary. |
|               |               | 3             |            |               |
|               |               | <hr/>         |            |               |
|               |               | 80769 denary. |            |               |

(7)

| XII                 | XII                 | XII                 | XII                  |
|---------------------|---------------------|---------------------|----------------------|
| 9) <u>t2t290</u>    | 6) <u>t2t290</u>    | 4) <u>t2t290</u>    | 2) <u>t2t290</u>     |
| 9) <u>117978..0</u> | 6) <u>185856..0</u> | 4) <u>268683..0</u> | 2) <u>515146..0</u>  |
| 9) <u>1624t..2</u>  | 6) <u>34e4 e..0</u> | 4) <u>78180..3</u>  | 2) <u>268683..0</u>  |
| 9) <u>2032..4</u>   | 6) <u>69 t9..5</u>  | 4) <u>1e050..0</u>  | 2) <u>134341..1</u>  |
| 9) <u>284..2</u>    | 6) <u>1179..3</u>   | 4) <u>5913..0</u>   | 2) <u>78180..1</u>   |
| 9) <u>37..1</u>     | 6) <u>233..3</u>    | 4) <u>1533..3</u>   | 2) <u>3 t0 t0..0</u> |
| <u>4..7</u>         | 6) <u>46..3</u>     | 4) <u>439..3</u>    | 2) <u>1e050..0</u>   |
|                     | 6) <u>9..0</u>      | 4) <u>10e..1</u>    | 2) <u>e626..0</u>    |
|                     | <u>1..3</u>         | 4) <u>32..3</u>     | 2) <u>5913..0</u>    |
|                     |                     | 4) <u>9..2</u>      | 2) <u>2 t67. 1</u>   |
|                     |                     | <u>2..1</u>         | 2) <u>1533..1</u>    |
|                     |                     |                     | 2) <u>877..1</u>     |
|                     |                     |                     | 2) <u>439..1</u>     |
|                     |                     |                     | 2) <u>21 t..1</u>    |
|                     |                     |                     | 2) <u>10 e..0</u>    |
|                     |                     |                     | 2) <u>65..1</u>      |
|                     |                     |                     | 2) <u>32..1</u>      |
|                     |                     |                     | 2) <u>17..0</u>      |
|                     |                     |                     | 2) <u>9..1</u>       |
|                     |                     |                     | 2) <u>4..1</u>       |
|                     |                     |                     | 2) <u>2..0</u>       |
|                     |                     |                     | <u>1..0</u>          |

(Continued on next page.)

(7 continued.)

| XII  | IX     | VI      | IV     | II    |
|--|--------|---------|--------|-------|
| $22290 = 4712420 = 130333500 = 21231330030 = 1001101101111100001100$ |        |         |        |       |
| 12   | 9      | 6       | 4      | 2     |
| —  | —      | —       | —      | —     |
| 122  | 43     | 9       | 9      | 2     |
| 12   | 9      | 6       | 4      | 2     |
| —  | —      | —       | —      | —     |
| 1474   | 388    | 54      | 38     | 4     |
| 12   | 9      | 6       | 4      | 2     |
| —  | —      | —       | —      | —     |
| 17690  | 3494   | 327     | 155    | 9     |
| 12   | 9      | 6       | 4      | 2     |
| —  | —      | —       | —      | —     |
| 212289   | 31450  | 1965    | 621    | 19    |
| 12   | 9      | 6       | 4      | 2     |
| —  | —      | —       | —      | —     |
| 2547468  | 283052 | 11793   | 2487   | 38    |
|  | 9      | 6       | 4      | 2     |
| —  | —      | —       | —      | —     |
| 2547468  | 70763  | 9951    | 77     | 39804 |
|  | 6      | 4       | 2      | 2     |
| —  | —      | —       | —      | —     |
|  | 424578 | 39804   | 155    | 79608 |
|  | 6      | 4       | 2      | 2     |
| —  | —      | —       | —      | —     |
| 2547468  | 159216 | 310     | 159216 |       |
|  | 4      | 2       | 2      |       |
| —  | —      | —       | —      | —     |
|  | 636867 | 621     | 318433 |       |
|  | 4      | 2       | 2      |       |
| —  | —      | —       | —      | —     |
| 2547468  | 1243   | 636867  |        |       |
|  |        | 2       |        |       |
| —  | —      | —       | —      | —     |
|  |        | 1273734 |        |       |
|  |        | 2       |        |       |
| —  | —      | —       | —      | —     |
|  |        | 2547468 |        |       |

## EXERCISE 37—Page 142.

| (1)      | (2)                             | (3)    | (4)      |
|----------|---------------------------------|--------|----------|
| VI       | XII                             | III    | VIII     |
| 252      | 62te)32e75721(62te              | 201210 | 57264    |
| 252      | 31556                           | 102221 | 675      |
| <hr/>    | <hr/>                           | <hr/>  | <hr/>    |
| 544      | 161e7                           | 21212  | 354604   |
| 2224     | 1059t                           |        | 513354   |
| 544      | <hr/>                           |        | 434070   |
| <hr/>    | 58192                           |        | <hr/>    |
| 122024   | 53512                           |        | 51117344 |
|          | <hr/>                           |        |          |
|          | 58801                           |        |          |
|          | 58801                           |        |          |
|          | <hr/>                           |        |          |
| (5)      | (6)                             | (7)    | (8)      |
| II       | VII                             | VII    | XII      |
| 101      | 2143)142613(50·5254+            | 65432  | 7t348    |
| 1001     | 14111                           | 43210  | 5e6t4    |
| 1111     | <hr/>                           | 1444   | <hr/>    |
| 1011     | 1503·0                          | 65001  | 1t864    |
| 1000     | 1411·1                          | 54321  |          |
| 1111     | <hr/>                           | <hr/>  |          |
| 10101    | 61·60                           | 326041 |          |
| <hr/>    | 43·16                           |        |          |
| 1010100  | <hr/>                           |        |          |
|          | 15·410                          |        |          |
|          | 14·111                          |        |          |
|          | <hr/>                           |        |          |
|          | 1·2660                          |        |          |
|          | 1·1635                          |        |          |
|          | <hr/>                           |        |          |
|          | ·1022                           |        |          |
|          | <hr/>                           |        |          |
| (9)      | (10)                            |        |          |
| XII      | II                              |        |          |
| 34t7     | 100101)1010100001(1001010010101 |        |          |
| 6666     | 100101                          |        |          |
| <hr/>    | <hr/>                           |        |          |
| 18536    | 101000                          |        |          |
| 18536    | 100101                          |        |          |
| 18536    | <hr/>                           |        |          |
| 18536    | 111                             |        |          |
| <hr/>    |                                 |        |          |
| 1t36e296 |                                 |        |          |

## EXERCISE 38—Page 146.

| (1)   |      |        |       |        |       |       | (2)    |     |        |              |
|-------|------|--------|-------|--------|-------|-------|--------|-----|--------|--------------|
| 4 ft. | 7'   | 6''    | 10''' |        |       |       | 19 ft. | 10' | 3''    |              |
| 9     | 7    | 11     | 11    |        |       |       | 11     | 2   | 7      |              |
| <hr/> |      |        |       |        |       |       | <hr/>  |     |        |              |
|       |      | 4      | 2     | 11'''' | 3'''' | 2'''' |        | 11  | 6      | 11'''' 9'''' |
|       | 4    | 2      | 11    | 3      | 2     |       | 3      | 3   | 8      | 6            |
| 2     | 8    | 4      | 11    | 10     |       |       | 218    | 4   | 9      |              |
| 41    | 8    | 1      | 6     |        |       |       | <hr/>  |     |        |              |
| 44    | 9    | 1      | 8     | 0      | 5     | 2     | 222    | 8   | 0      | 5 9          |
| (3)   |      |        |       |        |       |       | (4)    |     |        |              |
| 9''   | 7''' | 4''''  |       |        |       |       | 9½ in. | =   | 9' 9'' |              |
| 7     | 3    | 11'''' |       |        |       |       | 9' 9'' |     |        |              |
| <hr/> |      |        |       |        |       |       | 5      | 7   | 4'''   |              |
|       |      |        | 8'''' | 9''''  | 8'''' | 8'''' | <hr/>  |     |        |              |
|       | 2    | 4      | 10    | 0      |       |       |        | 3   | 3''''  | 0''''        |
| 5     | 7    | 3      | 4     |        |       |       | 5      | 8   | 3      |              |
| <hr/> |      |        |       |        |       |       | 4      | 0   | 9      |              |
| 5     | 10   | 4      | 11    | 8      | 8     |       | <hr/>  |     |        |              |
|       |      |        |       |        |       |       | 4      | 6   | 8      | 6            |

| (5)   |    |      |      |        |
|-------|----|------|------|--------|
| 7 ft. | 4' | 11'' |      |        |
| 3     | 2  | 2    |      |        |
| <hr/> |    |      |      |        |
|       | 1  | 2    | 9''' | 10'''' |
| 1     | 2  | 9    | 10   |        |
| 22    | 2  | 9    |      |        |
| <hr/> |    |      |      |        |
| 23    | 6  | 9    | 7    | 10     |

## EXERCISE 39—Page 147.

EXERCISES TO PAGE 114.

| (1)             | (2)    | (3)             |
|-----------------|--------|-----------------|
| 15 ft.          |        | 10 ft.          |
| 1    2'         | XII    | 5               |
| <hr/>           | 45·6   | <hr/>           |
| 2    6          | t·3    | 50 sq.ft.       |
| 15              | <hr/>  | 7               |
| <hr/>           | 1146   | —c'rds.c.ft.    |
| 17    6         | 3870   | 128)350(2    94 |
| 8               | <hr/>  | 256             |
| <hr/>           | 398·46 | <hr/>           |
| 11    8    0''= | 2      | 94 cub.ft.      |

$11\frac{2}{3}$  cub.ft.=11 cub.ft 1152 cub.in. ——— cub.ft.  
 774·90=1096 9'  
 XII  
 774=1096 com. scale.

(4)

4 ft.

 $5\frac{1}{4}$ 

---

20

1

---

21 sq. ft.

70

---

128)1470( $11\frac{3}{4}$  cords

128

---

190

128

---

62

 $\frac{62}{128} = \frac{31}{64}$ 

(5)

xii

4.78

9.6

---

23 to

3590

---

38.0t

2.e

---

34492

7418

---

cub.ft.

t8.652=128 6' 5' 2'''

t8 duoden.=128 den.

(6)

25 ft.=300 in.

20 " =240 "

2 ft. 6 in.= 30 "

8

4

---

32

2

---

64=8×8

390

240

---

72000

30

---

8)2160000

---

8)270000

---

33750

## EXERCISE 40—Page 149.

(1)

£93×400

=\$372.00

14s.×20

= 2.80

 $7\frac{1}{2}$ d.=30 f.×5÷12 = .12½

£276×400

=\$1104.00

19s.×20

= 3.80

 $10\frac{1}{2}$ d.=42 f.×5÷12 = .17½

£93 14s. 7½d.

=\$374.92½

£275×400

=\$1100.00

4s.×20

= .80

 $11\frac{1}{4}$ d.=47 f.×5÷12 = .19½

£275 4s. 11¼d.

=\$1100.99½

£276 19s. 10½d.

=\$1107.97½

\$729.18

710.50

166.78

374.92½

1107.97½

497.81

1100.99½

---

\$4688.16½

(2)

576=6+10×7+10×10×5

m.fur.per.yds.ft.in.

m.fur.per.yds.ft.in.

47 6 17 4 2 7×6=

286 6 27 1 2 0

10

478 0 18 4 1 10×7= 3346 3 11 4 2 4

10

4780 4 28 2 0 4×5=23902 7 21 4 3 2

---

27536 1 21 0 1 6

(3)

$$243000 = 2^3 \times 3^5 \times 5^3$$

$$3+1=4$$

$$4+1=6$$

$$3+1=4$$

$$4 \times 6 \times 4 = 96$$

(4)

| V           | VIII        |
|-------------|-------------|
| 8)4234434   | 5)713427    |
| 8)241110..4 | 5)133721..2 |
| 8)13423..1  | 5)22303..2  |
| 8)1024..1   | 5)3532..1   |
| 8)32..3     | 5)570..2    |
| 2..1        | 5)113..1    |
|             | 5)17..0     |
|             | 3..0        |

| VIII   | V        |
|--------|----------|
| 713427 | 30012122 |
| 213114 | 4234434  |
| 500313 | 20222133 |

(5)

$$79 \cdot 342 \div .00006378 =$$

|                               |
|-------------------------------|
| 6378)7934200000(1243994.98275 |
| 6378                          |
| 15562                         |
| 12756                         |
| 28060                         |
| 25512                         |
| 25480                         |
| 19134                         |
| 63460                         |
| 57402                         |
| 60580                         |
| 57402                         |
| 31780                         |
| 25512                         |
| 6268.0                        |
| 5740.2                        |
| 527.80                        |
| 510.24                        |
| 17.560                        |
| 12.756                        |
| 4.8040                        |
| 4.4646                        |
| .33940                        |
| .31890                        |
| .02050                        |



(7)

|    |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 40 | 5.  | 7.  | 9.  | 11. | 13. | 15. | 17. | 19. | 21. | 23. | 25. | 27. | 29. | 31. | 33. | 35. | 37. | 39. | 41. | 43. | 45. | 47. | 49. |
| 21 | 7.  | 9.  | 11. | 13. | 15. | 17. | 19. | 21. | 23. | 25. | 27. | 29. | 31. | 33. | 35. | 37. | 39. | 41. | 43. | 45. | 47. | 49. |     |
| 33 | 11. | 13. | 15. | 17. | 19. | 21. | 23. | 25. | 27. | 29. | 31. | 33. | 35. | 37. | 39. | 41. | 43. | 45. | 47. | 49. |     |     |     |
| 10 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |

$$40 \times 21 \times 33 \times 10 = 277200.$$

(9)

$$9999993000 = 10000000000 - 7000.$$

$$64276 \cdot 3427 \times 10000000000 = 642763427000000$$

$$64276 \cdot 3427 \times 7000 = 449934398 \cdot 9$$

$$642762977065601 \cdot 1$$

(10)

$$\begin{array}{r}
 \text{IX} \\
 5)78263 \\
 \hline
 5)15230 \dots 3 \\
 \hline
 5)2760 \dots 0 \\
 \hline
 5)511 \dots 4 \\
 \hline
 5)102 \dots 0 \\
 \hline
 5)17 \dots 3 \\
 \hline
 3 \dots 1
 \end{array}$$

$$\begin{array}{r}
 \text{IX} \\
 11)78263 \\
 \hline
 11)6430 \dots 3 \\
 \hline
 11)526 \dots 6 \\
 \hline
 11)43 \dots 0 \\
 \hline
 3 \dots 6
 \end{array}$$

$$\begin{array}{r}
 \text{IX} \\
 7)78263 \\
 \hline
 7)11160 \dots 3 \\
 \hline
 7)1407 \dots 5 \\
 \hline
 7)177 \dots 3 \\
 \hline
 7)23 \dots 4 \\
 \hline
 3 \dots 0
 \end{array}$$

$$\begin{array}{r}
 \text{V} \\
 7)3130403 \\
 \hline
 7)214200 \dots 3 \\
 \hline
 7)13220 \dots 5 \\
 \hline
 7)1101 \dots 3 \\
 \hline
 7)41 \dots 4 \\
 \hline
 3 \dots 0
 \end{array}$$

$$\begin{array}{r}
 \text{XI} \\
 7)36063 \\
 \hline
 7)5640 \dots 3 \\
 \hline
 7)884 \dots 5 \\
 \hline
 7)128 \dots 3 \\
 \hline
 7)1 \dots 4 \\
 \hline
 3 \dots 0
 \end{array}$$

(12)

(13)

$$\begin{aligned}
 £672 \times 400 &= \$2688.00 \\
 7s. \times 20 &= 1.40 \\
 7d. = 28f. \times 5 \div 12 &= .11\frac{2}{3} \\
 \hline
 £672 \ 7s. \ 7d. &= \$2689.51\frac{2}{3}
 \end{aligned}$$

(13 continued.)

$$81) 37800(466$$

324

540

486

540

486

54)81(1

54

27)54(2

54

Therefore G. C. M. = 27.

(17)

(18)

(19)

|                         |  |          |       |      |        |       |       |        |       |
|-------------------------|--|----------|-------|------|--------|-------|-------|--------|-------|
| £ s. d.                 | 2)276000                                 |          |       |      |        |       |       |        |       |
| 178 16 4 $\frac{3}{4}$  | —————                                    | 6 ft. 2' | 7''   | 9''' | 10'''' |       |       |        |       |
| 97 15 11 $\frac{1}{2}$  | 2)138000                                 | 13 11    | 11 11 | 7    |        |       |       |        |       |
| 693 19 11 $\frac{3}{4}$ | —————                                    |          |       |      |        |       |       |        |       |
| 216 11 9 $\frac{1}{4}$  | 2)69000                                  |          | 3     | 7    | 6''''  | 6'''' | 8'''' | 10'''' |       |
| 678 14 7 $\frac{1}{2}$  | —————                                    |          | 5     | 8    | 5      | 2     | 0     | 2      |       |
| 197 13 11 $\frac{3}{4}$ | 2)34500                                  | 5        | 8     | 5    | 2      | 0     | 2     |        |       |
| 117 6 5                 | —————                                    | 5        | 8     | 5    | 2      | 0     | 2     |        |       |
| 91 1 1 $\frac{3}{4}$    | 2)17250                                  | 80 10    | 5     | 7    | 10     |       |       |        |       |
| 2272 0 3 $\frac{1}{4}$  | 3)8625                                   | 87       | 1     | 1    | 3      | 0     | 10    | 8      | 10 10 |
|                         | —————                                    |          |       |      |        |       |       |        |       |
|                         | 5)2875                                   |          |       |      |        |       |       |        |       |
|                         | —————                                    |          |       |      |        |       |       |        |       |
|                         | 5)575                                    |          |       |      |        |       |       |        |       |
|                         | —————                                    |          |       |      |        |       |       |        |       |
|                         | 5)115                                    |          |       |      |        |       |       |        |       |
|                         | —————                                    |          |       |      |        |       |       |        |       |
|                         | 23                                       |          |       |      |        |       |       |        |       |
|                         | 2 <sup>5</sup> × 3 × 5 <sup>3</sup> × 23 |          |       |      |        |       |       |        |       |

| (20)                    | (21)    | (22)  |
|-------------------------|---------|-------|
| XII                     | IV      | VIII  |
| 713t96)7te9·047(·011436 | 3333333 | 10000 |
| 713t·96                 | 4       | 8     |
| <hr/>                   | <hr/>   | <hr/> |
| 97t·2 t7                | 15      | 8     |
| 713·t96                 | 4       | 8     |
| <hr/>                   | <hr/>   | <hr/> |
| 266·4110                | 63      | 64    |
| 245·3720                | 4       | 8     |
| <hr/>                   | <hr/>   | <hr/> |
| 21·05e00                | 255     | 512   |
| 19·3e846                | 4       | 8     |
| <hr/>                   | <hr/>   | <hr/> |
| 3·862760                | 1023    | 4096  |
| 3·67e490                | 4       |       |
| <hr/>                   | <hr/>   |       |
| ·1 t3290                | 4095    |       |
|                         | 4       |       |
|                         | <hr/>   |       |
|                         | 16383   |       |

(23)

$$74002702 \div 144 = 513907 \text{ ft. } 94 \text{ in.}$$

$$513907 \text{ ft. } \div 9 = 57100 \text{ yards } 7 \text{ ft.}$$

$$57100 \text{ yds. } \div 30\frac{1}{4} = 1887 \text{ per. } 18\frac{1}{4} \text{ yds.}$$

$$1887 \text{ per. } 18 \text{ yds. } 2 \text{ ft. } 36 \text{ in.}$$

$$\text{Add } 7 \text{ ft. } 94 \text{ in.}$$

$$40)1887 \text{ per. } 19 \text{ yds. } 0 \text{ ft. } 130 \text{ in.}$$

$$4)47 \text{ r. } 7 \text{ per. } 19 \text{ yds. } 0 \text{ ft. } 130 \text{ in.}$$

$$11 \text{ a. } 3 \text{ r. } 7 \text{ per. } 19 \text{ yds. } 0 \text{ ft. } 130 \text{ in.}$$

(24)

$$\begin{array}{r|l} 1728 & 240 \dots 780 \dots 1620 \dots 1728 \\ 65 & 5 \dots 65 \dots 15 \\ 3 & 8 \end{array}$$

$$1728 \times 65 \times 3 = 336960.$$

(25)

6 children will have 6 children's shares

4 women will have  $4 \times 2 = 8$  " "3 men will have  $3 \times 5 \times 2 = 30$  " "

3 men 4 w'n &amp; 6 chi'n will have 44 children's sha.

4) \$7894.16

11) \$1973.54

\$179.41  $\frac{3}{11}$  = child's share.\$179.41  $\frac{3}{11} \times 2 = \$358.82 \frac{6}{11}$  = woman's share.\$358.82  $\frac{6}{11} \times 5 = \$1794.12 \frac{8}{11}$  = man's share.

(26)

(27)

| II         | II         | yds. qrs. na. in.   | yds. qrs. na. in.             |
|------------|------------|---------------------|-------------------------------|
| 1111111111 | 1000000000 | 7 1 1 1 ) 729 3 3 1 |                               |
| 2          | 2          | 4                   | 4                             |
| —          | —          | —                   | —                             |
| 3          | 2          | 29                  | 2919                          |
| 2          | 2          | 4                   | 4                             |
| —          | —          | —                   | —                             |
| 7          | 4          | 117                 | 11679                         |
| 2          | 2          | 2 $\frac{1}{4}$     | 2 $\frac{1}{4}$               |
| —          | —          | —                   | —                             |
| 15         | 8          | 235                 | 23359                         |
| 2          | 2          | 29 $\frac{1}{4}$    | 2919 $\frac{3}{4}$            |
| —          | —          | —                   | —                             |
| 31         | 16         | 264 $\frac{1}{4}$   | 26278 $\frac{3}{4}$           |
| 2          | 2          | 4                   | 4                             |
| —          | —          | —                   | —                             |
| 63         | 32         | 1057 )              | 105115 (99 $\frac{472}{1057}$ |
| 2          | 2          |                     | 9513                          |
| —          | —          |                     | —                             |
| 127        | 64         |                     | 9985                          |
| 2          | 2          |                     | 9513                          |
| —          | —          |                     | —                             |
| 255        | 128        |                     | 472                           |
| 2          | 2          |                     |                               |
| —          | —          |                     |                               |
| 511        | 256        |                     |                               |
| 2          | 2          |                     |                               |
| —          | —          |                     |                               |
| 1023       | 512        |                     |                               |

| (28)          | (29)           | (30)               |
|---------------|----------------|--------------------|
| 762·4978      | 723426         | lbs. oz. drs. scr. |
| 63·423        | 938·9126141    | 129 0 0 0          |
| <hr/>         | <hr/>          | 63 4 7 2           |
| 22874934      | 722487·0873859 | <hr/>              |
| 15249956      |                | 65 7 0 1           |
| 30499912      |                |                    |
| 22874934      |                |                    |
| 45749868      |                |                    |
| <hr/>         |                |                    |
| 48359·8979694 |                |                    |

(31)

$$1064 = 2^3 \times 7 \times 19.$$

1..2..4..8

1..7

1..2..4..8..7..14..28..56

1..19

1..2..4..8..7..14..28..56..19..38..76..152..133..266..532..1064=

1..2..4..7..8..14..19..28..38..56..76..133..152..266..532..1064

(32)

30 ft. 6 in. = 366 in. 366

20 ft. 11 in. = 251 in. 251

2 ft. 7 in. = 31 in. —

366

1830

732

— in.

31)91866(2963 $\frac{1}{3}$ 

62

—

298

279

—

196

186

—

106

93

—

13

$$2963\frac{1}{3} \div 36 = 82\frac{5}{6} \text{ yds.}$$

## EXERCISE 46—Page 158.

(1)

$$\frac{2}{5}, \frac{5}{7}, \frac{8}{9}, \frac{3}{5}, \frac{5}{18} = \frac{2 \times 7 \times 9 \times 5 \times 18}{5 \times 7 \times 9 \times 5 \times 18}, \frac{5 \times 5 \times 9 \times 5 \times 18}{5 \times 7 \times 9 \times 5 \times 18}, \frac{8 \times 5 \times 7 \times 5 \times 18}{5 \times 7 \times 9 \times 5 \times 18},$$

$$\frac{3 \times 5 \times 7 \times 9 \times 18}{5 \times 7 \times 9 \times 5 \times 18} = \frac{11340}{28350}, \frac{20250}{28350}, \frac{25200}{28350}, \frac{17010}{28350}, \frac{7875}{28350}$$

(2)

$$\frac{8}{17}, \frac{12}{13}, \frac{5}{14} = \frac{8 \times 13 \times 14}{11 \times 13 \times 14}, \frac{12 \times 11 \times 14}{11 \times 13 \times 14}, \frac{5 \times 11 \times 13}{11 \times 13 \times 14} = \frac{1456}{2002}, \frac{1848}{2002}, \frac{715}{2002}$$

(3)

$$\frac{6}{7}, \frac{4}{11}, \frac{5}{13}, \frac{1}{2} = \frac{6 \times 11 \times 13 \times 7 \times 2}{7 \times 11 \times 13 \times 7 \times 2}, \frac{4 \times 7 \times 13 \times 7 \times 2}{7 \times 11 \times 13 \times 7 \times 2},$$

$$\frac{5 \times 7 \times 11 \times 7 \times 2}{7 \times 11 \times 13 \times 7 \times 2}, \frac{4 \times 7 \times 11 \times 13 \times 2}{7 \times 11 \times 13 \times 7 \times 2}, \frac{1 \times 7 \times 11 \times 13 \times 7}{7 \times 11 \times 13 \times 7 \times 2} =$$

$$\frac{12012}{14014}, \frac{5096}{14014}, \frac{5390}{14014}, \frac{8008}{14014}, \frac{7007}{14014}$$

(4)

$$\frac{6}{11}, \frac{4}{7}, \frac{8}{13} = \frac{6 \times 7 \times 13}{11 \times 7 \times 13}, \frac{4 \times 11 \times 13}{11 \times 7 \times 13}, \frac{8 \times 11 \times 7}{11 \times 7 \times 13} = \frac{546}{1001}, \frac{572}{1001}, \frac{616}{1001}$$

(5)

$$\frac{5}{6}, \frac{4}{7}, \frac{4}{5}, \frac{2}{11} = \frac{5 \times 7 \times 5 \times 11}{6 \times 7 \times 5 \times 11}, \frac{4 \times 6 \times 5 \times 11}{6 \times 7 \times 5 \times 11}, \frac{4 \times 6 \times 7 \times 11}{6 \times 7 \times 5 \times 11},$$

$$\frac{2 \times 6 \times 7 \times 5}{6 \times 7 \times 5 \times 11} = \frac{1925}{2310}, \frac{1320}{2310}, \frac{1848}{2310}, \frac{420}{2310}$$

(6)

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{7}{7} = \frac{1 \times 3 \times 5 \times 7}{2 \times 3 \times 5 \times 7}, \frac{2 \times 2 \times 5 \times 7}{2 \times 3 \times 5 \times 7}, \frac{3 \times 2 \times 3 \times 7}{2 \times 3 \times 5 \times 7},$$

$$\frac{2 \times 2 \times 3 \times 5}{2 \times 3 \times 5 \times 7} = \frac{105}{210}, \frac{140}{210}, \frac{126}{210}, \frac{60}{210}.$$

## EXERCISE 47—Page 159.

(1)

$$\frac{1}{5}, \frac{2}{8}, \frac{4}{6}, \frac{3}{4}, \frac{7}{15}.$$

The least common multiple of 5, 8, 6, 4, 15 is 120.

The multiplier for both terms of the first fraction is  $\frac{120}{5} = 24$ ; for the second  $\frac{120}{8} = 15$ ; for the third  $\frac{120}{6} = 20$ ; for the fourth  $\frac{120}{4} = 30$ ; for the fifth  $\frac{120}{15} = 8$ .

Multiplying by these numbers, we obtain  $\frac{96}{120}, \frac{45}{120}, \frac{80}{120}, \frac{90}{120}$ , and  $\frac{56}{120}$ .

(2)

$$\frac{6}{11}, \frac{2}{3}, \frac{4}{7}, \frac{18}{77}, \frac{19}{33}.$$

The least common multiple of 11, 3, 7, 77 and 33 is 231.

The multiplier for both terms of the first fraction is  $\frac{231}{11} = 21$ ; for the second,  $\frac{231}{3} = 77$ ; for the third,  $\frac{231}{7} = 33$ ; for the fourth,  $\frac{231}{77} = 3$ ; and for the fifth,  $\frac{231}{33} = 7$ .

Multiplying by these numbers, we obtain  $\frac{126}{231}, \frac{154}{231}, \frac{132}{231}, \frac{54}{231}$ , and  $\frac{133}{231}$ .

(3)

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}, \frac{13}{15}, \frac{7}{16}, \frac{37}{80}.$$

The least common multiple of 2, 3, 5, 6, 8, 10, 15, 16 and 80 is 240.

The multiplier for both terms of the first fraction is  $\frac{240}{2} = 120$ ; for the second,  $\frac{240}{3} = 80$ ; for the third,  $\frac{240}{5} = 48$ ; for the fourth,  $\frac{240}{6} = 40$ ; for the fifth,  $\frac{240}{8} = 30$ ; for the sixth,  $\frac{240}{10} = 24$ ; for the seventh,  $\frac{240}{15} = 16$ ; for the eighth,  $\frac{240}{16} = 15$ ; and for the ninth,  $\frac{240}{80} = 3$ .

Multiplying by these numbers, we obtain  $\frac{120}{240}, \frac{160}{240}, \frac{144}{240}, \frac{900}{240}, \frac{210}{240}, \frac{216}{240}, \frac{208}{240}, \frac{105}{240}$ , and  $\frac{111}{240}$ .

(4)

$$\frac{3}{5}, \frac{7}{15}, \frac{6}{25}, \frac{11}{30}, \frac{13}{45}, \frac{23}{60}.$$

The least common multiple of 5, 10, 25, 30, 45, and 60 is 900.

The multiplier for both terms of the first fraction is  $\frac{900}{5} = 180$ ; for the second,  $\frac{900}{10} = 90$ ; for the third,  $\frac{900}{25} = 36$ ; for the fourth,  $\frac{900}{30} = 30$ ; for the fifth,  $\frac{900}{45} = 20$ ; and for the sixth,  $\frac{900}{60} = 15$ .

Multiplying by these numbers, we obtain  $\frac{540}{900}$ ,  $\frac{630}{900}$ ,  $\frac{216}{900}$ ,  $\frac{330}{900}$ ,  $\frac{260}{900}$ , and  $\frac{345}{900}$ .

(5)

$$\frac{19}{20}, \frac{7}{30}, \frac{11}{40}, \frac{1}{50}.$$

The last common multiple of 20, 30, 40 and 50 is 600.

The multiplier for both terms of the first fraction is  $\frac{600}{20} = 30$ ; for the second,  $\frac{600}{30} = 20$ ; for the third,  $\frac{600}{40} = 15$ ; and for the fourth,  $\frac{600}{50} = 12$ .

Multiplying by these numbers, we obtain  $\frac{570}{600}$ ,  $\frac{140}{600}$ ,  $\frac{165}{600}$  and  $\frac{12}{600}$ .

(6)

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{11}{12}, \frac{15}{16}, \frac{23}{24}.$$

The least common multiple of 2, 3, 4, 6, 8, 12, 16, and 24 is 48.

The multiplier for both terms of the first fraction is  $\frac{48}{2} = 24$ ; for the second,  $\frac{48}{3} = 16$ ; for the third,  $\frac{48}{4} = 12$ ; for the fourth,  $\frac{48}{6} = 8$ ; for the fifth,  $\frac{48}{8} = 6$ ; for the sixth,  $\frac{48}{12} = 4$ ; for the seventh,  $\frac{48}{16} = 3$ ; and for the eighth,  $\frac{48}{24} = 2$ .

Multiplying by these numbers, we obtain  $\frac{24}{48}$ ,  $\frac{32}{48}$ ,  $\frac{36}{48}$ ,  $\frac{40}{48}$ ,  $\frac{42}{48}$ ,  $\frac{44}{48}$ ,  $\frac{45}{48}$ , and  $\frac{46}{48}$ .

(7)

$$\frac{5}{7}, \frac{11}{12}, \frac{2}{15}, \frac{7}{27}, \frac{9}{35}, \frac{17}{40}.$$

The least common multiple of 7, 12, 15, 27, 35 and 40 is 7560.

The multiplier for both terms of the first fraction is  $\frac{7560}{7} = 1080$ ; for the second,  $\frac{7560}{12} = 630$ ; for the third,  $\frac{7560}{15} = 504$ ; for the fourth,  $\frac{7560}{27} = 280$ ; for the fifth,  $\frac{7560}{35} = 216$ ; for the sixth,  $\frac{7560}{40} = 189$ .

Multiplying by these numbers, we obtain  $\frac{5400}{7560}$ ,  $\frac{6930}{7560}$ ,  $\frac{1008}{7560}$ ,  $\frac{2240}{7560}$ ,  $\frac{1944}{7560}$ , and  $\frac{3213}{7560}$ .



(8)

$$\frac{14}{15}, \frac{7}{8}, \frac{4}{3}, \frac{11}{12}, \frac{6}{11}, \frac{12}{20}, \frac{6}{7}, \frac{39}{35}.$$

The least common multiple of 15, 8, 3, 12, 11, 20, 7, and 35 is 9240.

The multiplier for both terms of the first fraction is  $\frac{9240}{15} = 616$ ; for the second,  $\frac{9240}{8} = 1155$ ; for the third,  $\frac{9240}{3} = 3080$ ; for the fourth,  $\frac{9240}{12} = 770$ ; for the fifth,  $\frac{9240}{11} = 840$ ; for the sixth,  $\frac{9240}{20} = 462$ ; for the seventh,  $\frac{9240}{7} = 1320$ ; for the eighth,  $\frac{9240}{35} = 264$ .

Multiplying by these numbers, we obtain  $\frac{8624}{9240}, \frac{8085}{9240}, \frac{12320}{9240}, \frac{8470}{9240}, \frac{5040}{9240}, \frac{8778}{9240}, \frac{7920}{9240}$ , and  $\frac{7656}{9240}$ .

## EXERCISE 48—Page 160.

(1)

$$\frac{1}{7} \text{ of } \frac{3}{5} \text{ of } \frac{6}{11} \text{ of } \frac{35}{72} = \frac{4 \times 3 \times 6 \times 35}{7 \times 5 \times 11 \times 72} = \frac{2520}{27720} = \frac{1}{11}.$$

(2)

$$\frac{2}{3} \text{ of } \frac{4}{9} \text{ of } \frac{6}{7} \text{ of } \frac{81}{100} \text{ of } \frac{25}{24} = \frac{2 \times 4 \times 6 \times 81 \times 25}{3 \times 9 \times 7 \times 100 \times 24} = \frac{97200}{453600} = \frac{1}{4}.$$

(3)

$$\frac{21}{35} \text{ of } \frac{6}{11} \text{ of } \frac{77}{36} = \frac{21 \times 6 \times 77}{35 \times 11 \times 36} = \frac{7}{15}.$$

(4)

$$\frac{2}{5} \text{ of } \frac{4}{7} \text{ of } \frac{3}{11} \text{ of } \frac{13}{17} = \frac{2 \times 4 \times 3 \times 13}{5 \times 7 \times 11 \times 17} = \frac{312}{6545}.$$

## EXERCISE 49—Page 161.

(1)

$$\frac{5}{9} \text{ of } \frac{6}{7} \text{ of } \frac{2}{3} \text{ of } \frac{3}{16} = \frac{5 \times 6 \times 2 \times 3}{9 \times 7 \times 3 \times 16} = \frac{5 \times \overset{2}{\cancel{6}} \times \cancel{2} \times \cancel{3}}{\cancel{9} \times 7 \times \cancel{3} \times \underset{8}{\underset{4}{16}}} = \frac{5}{3 \times 7 \times 4} = \frac{5}{84}.$$

(2)

$$\frac{2}{3} \text{ of } \frac{5}{9} \text{ of } \frac{18}{132} \text{ of } \frac{6}{11} \text{ of } \frac{11}{13} \text{ of } \frac{13}{17} = \frac{2 \times 5 \times 18 \times 6 \times 11 \times 13}{3 \times 9 \times 132 \times 11 \times 13 \times 17} =$$

$$\frac{2 \times 5 \times \overset{2}{\cancel{18}} \times \overset{2}{\cancel{6}} \times \cancel{11} \times \cancel{13}}{\cancel{3} \times \cancel{9} \times \underset{33}{\cancel{132}} \times \cancel{11} \times \cancel{13} \times 17} = \frac{2 \times 5}{33 \times 17} = \frac{10}{561}.$$

(3)

$$\frac{2}{7} \text{ of } \frac{4}{11} \text{ of } 5\frac{1}{2} = \frac{2 \times 4 \times 11}{7 \times 11 \times 2} = \frac{\cancel{2} \times 4 \times \cancel{11}}{7 \times \cancel{11} \times \cancel{2}} = \frac{4}{7}.$$

(4)

$$\frac{1}{9} \text{ of } \frac{8}{13} \text{ of } \frac{117}{200} \text{ of } \frac{50}{169} \text{ of } \frac{13}{17} \text{ of } \frac{13}{6} = \frac{1 \times 8 \times 117 \times 50 \times 13 \times 13}{9 \times 13 \times 200 \times 169 \times 17 \times 6} =$$

$$\frac{1 \times \overset{2}{\cancel{8}} \times \overset{9}{\cancel{117}} \times \cancel{50} \times \cancel{13} \times \cancel{13}}{\cancel{9} \times \cancel{13} \times \underset{4}{\underset{13}{200}} \times \underset{13}{169} \times 17 \times \underset{3}{6}} = \frac{1}{17 \times 3} = \frac{1}{51}.$$

(5)

$$\frac{3}{11} \text{ of } \frac{4}{7} \text{ of } \frac{9}{19} \text{ of } \frac{33}{47} \text{ of } \frac{38}{72} \text{ of } \frac{47}{7} = \frac{3 \times 4 \times 9 \times 33 \times 38 \times 47}{11 \times 7 \times 19 \times 47 \times 72 \times 7} =$$

$$\frac{3 \times \cancel{4} \times \overset{3}{\cancel{9}} \times \cancel{33} \times \overset{2}{\cancel{38}} \times \cancel{47}}{\cancel{11} \times 7 \times \cancel{19} \times \cancel{47} \times \underset{18}{\underset{2}{72}} \times 7} = \frac{3 \times 3}{7 \times 7} = \frac{9}{49}.$$

(6)

$$\frac{4}{7} \text{ of } \frac{3}{11} \text{ of } \frac{154}{1} = \frac{4 \times 3 \times 154}{7 \times 11 \times 1} = \frac{4 \times 3 \times \overset{2}{\cancel{14}} \overset{11}{\cancel{14}}}{\cancel{7} \times \cancel{11} \times 1} = \frac{2 \times 4 \times 3}{1} = 24.$$

## EXERCISE 50—Page 162.

(1)

$$\frac{\frac{14}{45}}{1\frac{7}{25}} = \frac{\frac{14}{45}}{\frac{42}{25}} = \frac{14 \times 25}{45 \times 42} = \frac{\overset{5}{\cancel{14}} \times \cancel{25}}{\cancel{45} \times \cancel{42}} = \frac{5}{9 \times 3} = \frac{5}{27}.$$

(2)

$$\frac{1\frac{1}{2}}{7\frac{7}{18}} = \frac{1\frac{1}{2}}{1\frac{43}{18}} = \frac{11 \times 18}{12 \times 143} = \frac{11 \times \overset{3}{\cancel{18}}}{\cancel{12} \times \cancel{143}} = \frac{3}{2 \times 13} = \frac{3}{26}.$$

(3)

$$\frac{15\frac{3}{5}}{7\frac{1}{5}} = \frac{\frac{78}{5}}{\frac{39}{5}} = \frac{78 \times 5}{5 \times 39} = \frac{\overset{2}{\cancel{78}} \times \cancel{5}}{\cancel{5} \times \cancel{39}} = 2.$$

(4)

$$\frac{11\frac{2}{3}}{12\frac{5}{6}}, \frac{3\frac{1}{4}}{9}, \frac{\frac{2}{3}}{\frac{3}{5}} = \frac{\frac{35}{3}}{\frac{68}{6}}, \frac{1\frac{3}{4}}{\frac{2}{1}}, \frac{\frac{2}{3}}{\frac{3}{5}} = \frac{35 \times 5}{3 \times 68}, \frac{13 \times 1}{9 \times 4}, \frac{2 \times 5}{7 \times 3} = \frac{175}{204}, \frac{13}{36}, \frac{10}{21}.$$

(5)

$$\frac{\frac{7}{12}}{15\frac{1}{4}}, \frac{5\frac{7}{8}}{16}, \frac{2\frac{3}{8}}{3\frac{3}{7}} = \frac{\frac{7}{12}}{\frac{63}{4}}, \frac{\frac{47}{8}}{16}, \frac{\frac{12}{5}}{\frac{24}{7}} = \frac{7 \times 4}{12 \times 63}, \frac{\overset{2}{47 \times 16}}{8 \times 3}, \frac{12 \times 7}{5 \times \underset{2}{24}} = \frac{1}{3 \times 9}, \frac{47 \times 2}{3}, \frac{7}{5 \times 2} = \frac{1}{27}, 31\frac{1}{3}, 7\frac{7}{10}.$$

(6)

$$\frac{16\frac{2}{3}}{11\frac{2}{3}}, \frac{6\frac{1}{2}}{13}, \frac{17}{18\frac{1}{3}}, \frac{21\frac{3}{8}}{10\frac{2}{7}}, \frac{\frac{1}{2}}{4\frac{2}{3}} = \frac{\frac{50}{3}}{\frac{35}{3}}, \frac{\frac{31}{6}}{\frac{13}{1}}, \frac{\frac{17}{1}}{\frac{55}{3}}, \frac{\frac{108}{3}}{\frac{72}{7}}, \frac{\frac{1}{2}}{\frac{23}{6}} = \frac{10}{50 \times 3}, \frac{3}{3 \times 35 \times 7}$$

$$\frac{31 \times 1}{5 \times 13}, \frac{17 \times 3}{55 \times 1}, \frac{108 \times 7}{72 \times 5}, \frac{1 \times 5}{2 \times 23} = \frac{10}{7}, \frac{31}{65}, \frac{51}{55}, \frac{21}{10}, \frac{5}{46} = 1\frac{3}{7}, \frac{31}{65}, \frac{51}{55}, 2\frac{1}{10}, \frac{5}{46}$$

## EXERCISE 51—Page 163.

(1)

$$\frac{\frac{4}{5}}{\frac{1}{16}} \text{ of } \frac{1}{\frac{16}{4}} = \frac{1}{\frac{1}{10}} \text{ of a lb.}$$

(2)

$$\frac{\frac{2}{3}}{\frac{2}{3}} \text{ of } \frac{\frac{3}{7}}{\frac{12}{6}} \text{ of } \frac{1}{\frac{12}{6}} \text{ of } \frac{1}{20} = \frac{1}{7 \times 6 \times 20} = £\frac{1}{340}$$

(3)

$$\frac{\frac{2}{9}}{\frac{2}{9}} \text{ of } \frac{\frac{35}{4}}{\frac{2}{2}} \text{ of } \frac{1}{\frac{7}{7}} = \frac{5}{9 \times 2} = 1\frac{5}{9} \text{ wk.}$$

(4)

$$\frac{\frac{5}{11}}{\frac{5}{11}} \text{ of } \frac{81}{5} \text{ of } \frac{1}{4} \text{ of } \frac{1}{5} = \frac{81}{11 \times 4 \times 5} = \frac{81}{220} \text{ Eng. Ell.}$$

(5)

$$\frac{3}{7} \text{ of } \frac{4}{11} \text{ of } \frac{1}{5\frac{1}{2}} = \frac{3}{7} \text{ of } \frac{4}{11} \text{ of } \frac{2}{11} = \frac{24}{847} \text{ per.}$$

(6)

$$\frac{2}{3} \text{ of } \frac{4}{7} \text{ of } 21 - \frac{1}{14} \text{ of } \frac{1}{8} = \frac{2 \times 4 \times 295 \times 1}{3 \times 7 \times 14 \times 8} = \frac{295}{294} = 1\frac{1}{294} \text{ c.}$$

(7)

$$\frac{3}{19} \text{ of } \frac{4}{17} \text{ of } 9 - \frac{1}{2} \text{ of } \frac{1}{40} \text{ of } \frac{1}{4} = \frac{3 \times 4 \times 19 \times 1 \times 1}{19 \times 17 \times 2 \times 40 \times 4} = \frac{3}{17 \times 2 \times 40} = \frac{3}{1360} \text{ a.}$$

## EXERCISE 52—Page 164.

(1)

$$\frac{14}{79} \text{ of } \frac{4}{1} \text{ of } \frac{2}{1} \text{ of } \frac{4}{1} = 4\frac{1}{79} \text{ qt.}$$

(2)

$$\frac{2}{3} \text{ of } \frac{4}{1} \times \frac{2}{1} \times \frac{4}{1} \times \frac{5}{1} \times \frac{3}{2} = \frac{2 \times 4 \times 4 \times 5}{3} = 1\frac{2}{3} \text{ q.}$$

(3)

$$\frac{7}{3} \times \frac{2}{1} \times \frac{2}{1} \times \frac{4}{1} \times \frac{2}{1} \times \frac{2}{1} \times \frac{3}{2} = \frac{7 \times 2 \times 2 \times 4 \times 2}{3} = 2\frac{2}{3} \text{ q.}$$

(4)

$$\frac{17}{22} \times \frac{6}{11} \times \frac{8}{1} \times \frac{3}{1} = \frac{17 \times 6 \times 8 \times 3}{11} = 24\frac{1}{11} \text{ scr.}$$

(5)

$$\frac{1}{5000} \times \frac{2}{3} \times \frac{3}{4} \times \frac{6}{11} \times \frac{2}{7} \times \frac{2}{1} \times \frac{4}{1} = \frac{2 \times 6 \times 2 \times 2 \times 4}{625 \times 7} = \frac{192}{4375} \text{ dr.}$$

## EXERCISE 53—Page 164.

(1)

bush. pk. gal. qt. pt.

$$\begin{array}{r} 11) 3 \quad 0 \quad 0 \quad 0 \quad 0 \\ \hline 1 \quad 0 \quad 0 \quad 1 \frac{5}{11} \end{array}$$

lbs. oz. dr.

$$\begin{array}{r} 7) 6 \quad 0 \quad 0 \\ \hline 13 \quad 11 \frac{3}{7} \end{array}$$

(2)

yds. qr. na. in.

$$13) 7 \quad (2 \quad 0 \quad 1 \frac{1}{4})$$

4

—

28 qrs.

26

—

2

4

—

8 na.

2  $\frac{1}{4}$ 

—

18

13

—

5

(4)

fur. per. yds. ft. in.

$$9) 8 \quad 0 \quad 0 \quad 0 \quad 0$$

$$\hline 35 \quad 3 \quad 0 \quad 2$$

£ s. d.

$$7) 4 \quad 0 \quad 0$$

$$\hline 11 \quad 5 \frac{1}{7}$$

(3)

lbs. oz. dwt. grs.

$$\begin{array}{r} 9) 8 \quad 0 \quad 0 \quad 0 \\ \hline 10 \quad 13 \quad 8 \end{array}$$

sq. m. a. r. pr. yds. ft. in.

$$113) 11 \quad (62 \quad 1 \quad 8 \quad 4 \quad 2 \quad 79 \frac{1}{113})$$

640

7040 a.

678

260

226

34

4

136 r.

113

23

40

920 per.

904

16

30  $\frac{1}{4}$ 

480

4

484 yds.

484 yds.

452

32

9

288 ft.

226

62

144

248

248

62

8928 in.

791

1018

1017

1

## EXERCISE 54—Page 165.

(1)

6 bus. 1 pk. 1 gal. 1 qt. 1 pt. = 411 pts.

50 bush. = 3200 pts.

And the required fraction is  $\frac{411}{3200}$ .

(2)

35 per. 9 ft. 2 in. = 7040 in.

1 fur. = 7920 in.

The required fraction is  $\frac{7040}{7920} = \frac{88}{99} = \frac{8}{9}$ .

(3)

7 hrs. 12 min. = 432 min.

1 day = 1440 min.

Therefore the fraction is  $\frac{432}{1440} = \frac{3}{10}$ .

(4)

2 sq. yds. 2 ft. 120 in. = 3000 in.

3 sq. per.  $13\frac{1}{4}$  yds. 1 ft. 72 in. = 135000 in.And the fraction is  $\frac{3000}{135000} = \frac{1}{45}$ .

(5)

7 oz. 7 drs. 2 scr. 14 grs. = 3834 grs.

21 lbs. = 120960 grs.

The fraction is  $\frac{3834}{120960} = \frac{426}{13440} = \frac{71}{2240}$ .

(6)

9 min. 48 sec. = 588 sec.

1 day = 86400 sec.

The required fraction is  $\frac{588}{86400} = \frac{49}{7200}$ .

(7)

16 bush. 1 pk. 1 pt. = 1041 pts.

69 bush. = 4416 pts.

Therefore the fraction is  $\frac{1041}{4416} = \frac{347}{1440}$ .

(8)

$$3 \text{ qrs. } 3\frac{1}{9} \text{ na.} = 15\frac{1}{9} = 1\frac{16}{9} \text{ na.}$$

$$1 \text{ Eng. ell} = 20 \text{ na.}$$

$$\text{And the fraction is } \frac{1\frac{16}{9}}{2\frac{0}{1}} = \frac{1\frac{16}{9}}{1\frac{36}{9}} = \frac{34}{45}.$$

(9)

$$13 \text{ dwt. } 7 \text{ grs.} = 319 \text{ grs.}$$

$$1 \text{ lb. Troy} = 5760 \text{ grs.}$$

$$\text{The required fraction is } \frac{319}{5760}.$$

(10)

$$4800 \text{ cub. ft.}$$

$$54 \text{ cords} = 6912 \text{ cub. ft.}$$

$$\text{Therefore the fraction is } \frac{4800}{6912} = \frac{100}{144} = \frac{25}{36}.$$

## EXERCISE 55—Page 167.

(1)

$$\frac{11}{13} + \frac{10}{13} + \frac{9}{13} = \frac{30}{13} = 2\frac{4}{13}.$$

(2)

$$\frac{1}{12} + \frac{6}{12} + \frac{7}{12} + \frac{9}{12} + \frac{11}{12} + \frac{5}{12} = \frac{39}{12} = 3\frac{3}{12} = 3\frac{1}{4}.$$

(3)

$$4\frac{3}{7} + 11\frac{4}{7} + 16\frac{2}{7} + 21\frac{3}{7} + 19\frac{6}{7} = 4 + 11 + 16 + 21 + 19 + (\frac{3}{7} + \frac{4}{7} + \frac{2}{7} + \frac{3}{7} + \frac{6}{7}) = 71 + \frac{18}{7} = 73\frac{4}{7}.$$

(4)

$$16\frac{2}{3} + 11\frac{1}{3} + 18\frac{4}{3} + 17\frac{2}{3} + 112\frac{2}{3} = 16 + 11 + 18 + 17 + 112 + (\frac{2}{3} + \frac{1}{3} + \frac{4}{3} + \frac{2}{3} + \frac{2}{3}) = 174 + \frac{11}{3} = 174 + 3\frac{2}{3} = 177\frac{2}{3}.$$

(5)

$$4\frac{1}{4} + 1\frac{1}{4} + \frac{7}{11} = 4 + 1 + (\frac{1}{4} + \frac{1}{4} + \frac{7}{11}) = 5 + (\frac{3}{11} + \frac{14}{11} + \frac{7}{11}) = 5 + \frac{24}{11} = 6\frac{2}{11}.$$

(6)

$$\frac{1}{2} + \frac{3}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \frac{7}{8} + \frac{8}{9}.$$

These fractions reduced to their least common denominator become  $\frac{1260}{2520} + \frac{1680}{2520} + \frac{1890}{2520} + \frac{2016}{2520} + \frac{2100}{2520} + \frac{2160}{2520} + \frac{2205}{2520} + \frac{2240}{2520} = \frac{15551}{2520} = 6\frac{131}{2520}.$



(7)

$\frac{3}{4} + \frac{5}{6} + \frac{4}{5}$  when reduced to their least common denominator become  $\frac{45}{60} + \frac{50}{60} + \frac{48}{60} = \frac{143}{60} = 2\frac{23}{60}$ .

(8)

$$\frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \frac{3}{8} + \frac{8}{11}.$$

These fractions when reduced to their least common denominator become  $\frac{7320}{9240} + \frac{7700}{9240} + \frac{7920}{9240} + \frac{3465}{9240} + \frac{6720}{9240} = \frac{33197}{9240} = 3\frac{5477}{9240}$ .

(9)

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}.$$

These fractions when reduced to their least common denominator become  $\frac{210}{420} + \frac{140}{420} + \frac{105}{420} + \frac{84}{420} + \frac{70}{420} + \frac{60}{420} = \frac{669}{420} = \frac{223}{140} = 1\frac{83}{140}$ .

(10)

$$16\frac{3}{4} + 47\frac{2}{3} + 21\frac{17}{33} + \frac{7}{18} + 19\frac{1}{2} = 16 + 47 + 21 + 19 + (\frac{3}{4} + \frac{2}{3} + \frac{17}{33} + \frac{7}{18} + \frac{1}{2}).$$

$$16 + 47 + 21 + 19 = 103.$$

$$\frac{3}{4} + \frac{2}{3} + \frac{17}{33} + \frac{7}{18} + \frac{1}{2} = \frac{186}{1782} + \frac{396}{1782} + \frac{918}{1782} + \frac{693}{1782} + \frac{891}{1782} = \frac{3384}{1782} = \frac{376}{198} = \frac{188}{99} = 1\frac{89}{99}.$$

$$103 + 1\frac{89}{99} = 104\frac{89}{99}.$$

(11)

$$17\frac{1}{2} + 43\frac{2}{3} + 168\frac{4}{9} + 207\frac{8}{27} + 506\frac{125}{126} = 17 + 43 + 168 + 207 + 506 + (\frac{1}{2} + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{125}{126}).$$

$$17 + 43 + 168 + 207 + 506 = 941.$$

$$\frac{1}{2} + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{125}{126} = \frac{63}{126} + \frac{54}{126} + \frac{56}{126} + \frac{48}{126} + \frac{125}{126} = \frac{346}{126} = \frac{173}{63} = 2\frac{47}{63}.$$

$$941 + 2\frac{47}{63} = 943\frac{47}{63}.$$

(12)

$$6\frac{2}{3} + 11\frac{1}{4} + \frac{9}{5} + 16\frac{7}{6} + \frac{1}{2} + \frac{5}{27} + 17\frac{1}{12} = 6 + 11 + 16 + 17 + (\frac{2}{3} + \frac{1}{4} + \frac{9}{5} + \frac{7}{6} + \frac{1}{2} + \frac{5}{27} + \frac{1}{12}).$$

$$6 + 11 + 16 + 17 = 50.$$

$$\frac{2}{3} + \frac{1}{4} + \frac{9}{5} + \frac{7}{6} + \frac{1}{2} + \frac{5}{27} + \frac{1}{12} = \frac{352}{336} + \frac{84}{336} + \frac{594}{336} + \frac{385}{336} + \frac{168}{336} + \frac{65}{336} + \frac{28}{336} = \frac{1993}{336}.$$

$$50 + 3\frac{1993}{336} = 53\frac{1993}{336}.$$

(13)

$$\frac{1}{5} + \frac{2}{3} + \frac{7}{9} + 68\frac{1}{4} = 68 + (\frac{1}{5} + \frac{2}{3} + \frac{7}{9} + \frac{1}{4}).$$

$$\frac{1}{5} + \frac{2}{3} + \frac{7}{9} + \frac{1}{4} = \frac{36}{180} + \frac{120}{180} + \frac{140}{180} + \frac{45}{180} = \frac{341}{180} = 1\frac{161}{180}.$$

$$68 + 1\frac{161}{180} = 69\frac{161}{180}.$$

(14)

$$173\frac{3}{2} + 8\frac{5}{7} + 91\frac{11}{13} = 173 + 8 + 91 + (\frac{3}{2} + \frac{5}{7} + \frac{11}{13}).$$

$$173 + 8 + 91 = 272.$$

$$\frac{3}{2} + \frac{5}{7} + \frac{11}{13} = \frac{39}{264} + \frac{260}{264} + \frac{308}{264} = \frac{659}{264} = 1\frac{395}{264}.$$

$$272 + 1\frac{395}{264} = 273\frac{395}{264}.$$

(15)

$$1\frac{1}{16} + 2\frac{3}{24} + 3\frac{2}{24} + 4\frac{3}{30} = 1 + 2 + 3 + 4 + (\frac{1}{16} + \frac{3}{24} + \frac{2}{24} + \frac{3}{30}).$$

$$1 + 2 + 3 + 4 = 10.$$

$$\frac{1}{16} + \frac{3}{24} + \frac{2}{24} + \frac{3}{30} = \frac{6750}{7200} + \frac{9000}{7200} + \frac{6000}{7200} + \frac{6000}{7200} = \frac{27520}{7200} = 3\frac{328}{400}.$$

$$10 + 3\frac{328}{400} = 13\frac{328}{400}.$$

(16)

$$\frac{1}{8} + \frac{3}{12} + \frac{4}{48} + \frac{5}{24} + \frac{7}{16} + \frac{2}{3} + \frac{1}{2} + \frac{5}{6} = \frac{6}{48} + \frac{12}{48} + \frac{4}{48} + \frac{10}{48} + \frac{21}{48} + \frac{32}{48} + \frac{8}{48} + \frac{40}{48} = \frac{140}{48} = 3\frac{5}{12}.$$

(17)

$$7 + 11\frac{1}{2} + 18 + 26\frac{2}{7} + 79\frac{4}{11} = 7 + 11 + 18 + 26 + 79 + (\frac{1}{2} + \frac{2}{7} + \frac{4}{11}).$$

$$7 + 11 + 18 + 26 + 79 = 141.$$

$$\frac{1}{2} + \frac{2}{7} + \frac{4}{11} = \frac{77}{154} + \frac{66}{154} + \frac{56}{154} = \frac{199}{154} = 1\frac{45}{154}.$$

$$141 + 1\frac{45}{154} = 142\frac{45}{154}.$$

(18)

$$\frac{4}{5} \text{ of } \frac{3}{7} \text{ of } 2\frac{1}{2} = \frac{18}{5} = 3\frac{3}{5}. \quad \frac{2}{3} + 7\frac{2}{11} + 3\frac{3}{5} = 10 + (\frac{2}{3} + \frac{2}{11} + \frac{3}{5}).$$

$$\frac{2}{3} + \frac{2}{11} + \frac{3}{5} = \frac{110}{165} + \frac{30}{165} + \frac{99}{165} = \frac{239}{165} = 1\frac{74}{165}.$$

$$10 + 1\frac{74}{165} = 11\frac{74}{165}.$$

(19)

$$\frac{4\frac{1}{2}}{\frac{7}{18}} = \frac{\frac{13}{3}}{\frac{7}{18}} = \frac{13 \times 18}{3 \times 7} = \frac{7^8}{7} = 11\frac{1}{2}.$$

$$\frac{1}{2} \text{ of } \frac{36}{11} \text{ of } \frac{4}{15} \text{ of } \frac{1}{4} = \frac{1 \times 36 \times 4 \times 11}{2 \times 11 \times 15 \times 4} = \frac{1^8}{15} = 1\frac{3}{15}.$$

$$\frac{20\frac{3}{4}}{\frac{7\frac{6}{11}}{\frac{8\frac{3}{11}}{11}}} = \frac{\frac{83}{4}}{\frac{83}{11}} = \frac{83 \times 11}{4 \times 83} = \frac{11}{4} = 2\frac{3}{4}.$$

$$11\frac{1}{2} + 1\frac{3}{15} + 2\frac{3}{4} = 11 + 1 + 2 + (\frac{1}{2} + \frac{3}{15} + \frac{3}{4}) = 14 + (\frac{1}{2} + \frac{3}{15} + \frac{3}{4}).$$

$$\frac{1}{2} + \frac{3}{15} + \frac{3}{4} = \frac{60}{120} + \frac{24}{120} + \frac{90}{120} = \frac{159}{120} = \frac{53}{40} = 1\frac{13}{40}.$$

$$14 + 1\frac{13}{40} = 15\frac{13}{40}.$$

(20)

$$3\frac{5}{8} + 11\frac{1}{6} + 14\frac{33}{48} = 3 + 11 + 14 + (\frac{5}{8} + \frac{1}{6} + \frac{33}{48}) = 28 + (\frac{5}{8} + \frac{1}{6} + \frac{33}{48}).$$

$$\frac{5}{8} + \frac{1}{6} + \frac{33}{48} = \frac{30}{48} + \frac{8}{48} + \frac{33}{48} = \frac{71}{48} = 1\frac{23}{48}. \quad 28 + 1\frac{23}{48} = 29\frac{23}{48}.$$

(21)

$$\frac{1}{2} \text{ of } \frac{3}{4} = \frac{3}{8}, \quad \frac{2}{3} \text{ of } \frac{6}{7} = \frac{4}{7}, \quad \frac{3}{5} \text{ of } \frac{7}{9} = \frac{7}{15}, \quad \frac{2}{9} \text{ of } \frac{37}{20} = \frac{37}{90}, \quad \frac{9}{2} \text{ of } \frac{1}{2} \text{ of } \frac{1}{2} \text{ of } \frac{1}{5} \text{ of } \frac{1}{3} = \frac{1}{360}.$$

$$\frac{3}{8} + \frac{4}{7} + \frac{7}{15} + \frac{37}{90} + \frac{1}{360} = \frac{1630}{1680} + \frac{960}{1680} + \frac{784}{1680} + \frac{504}{1680} + \frac{1}{1680} = \frac{2941}{1680} = 1\frac{261}{1680}.$$

(22)

$$41\frac{1}{2} + 105\frac{2}{3} + 300\frac{3}{4} + 241\frac{3}{5} + 472\frac{1}{4} = 41 + 105 + 300 + 241 + 472 + (\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{3}{5} + \frac{1}{4}).$$

$$41 + 105 + 300 + 241 + 472 = 1159.$$

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{3}{5} + \frac{1}{4} = \frac{90}{180} + \frac{120}{180} + \frac{135}{180} + \frac{108}{180} + \frac{45}{180} = \frac{498}{180} = \frac{83}{30} = 2\frac{23}{30}.$$

$$1159 + 2\frac{23}{30} = 1161\frac{23}{30}.$$

(23)

$$92\frac{5}{4} + 37\frac{8}{9} + 7\frac{1}{6} = 92 + 37 + 7 + (\frac{5}{4} + \frac{8}{9} + \frac{1}{6}) = 136 + (\frac{5}{4} + \frac{8}{9} + \frac{1}{6}).$$

$$\frac{5}{4} + \frac{8}{9} + \frac{1}{6} = \frac{225}{396} + \frac{336}{396} + \frac{66}{396} = \frac{627}{396} = \frac{157}{99} = 1\frac{58}{99}.$$

$$136 + 1\frac{58}{99} = 137\frac{58}{99}.$$

(24)

$$\frac{10\frac{2}{5}}{\frac{2\frac{2}{5}}{\frac{1\frac{2}{5}}{\frac{5}{5}}}} = \frac{\frac{5\frac{3}{5}}{\frac{1\frac{2}{5}}{\frac{5}{5}}}}{\frac{5}{5}} = \frac{53 \times 5}{5 \times 12} = \frac{53}{12} = 4\frac{5}{12}. \quad \frac{1}{3} \text{ of } \frac{7}{8} = \frac{7}{24}.$$

$$21\frac{1}{2} + 35\frac{1}{8} + 4\frac{5}{12} + \frac{7}{24} = 21 + 35 + 5 + (\frac{1}{2} + \frac{1}{8}) = 61\frac{5}{8}.$$

(25)

$$\frac{1}{4} \text{ of } \frac{1}{3} = \frac{1}{12} = 10\frac{1}{12}. \quad \frac{1}{5} \text{ of } \frac{3}{8} \text{ of } \frac{1}{1} = \frac{3}{40} = 15\frac{1}{40}.$$

$$\frac{1}{16} = 6\frac{1}{16}. \quad \frac{1}{3} \text{ of } \frac{1}{2} \text{ of } \frac{1}{8} \text{ of } \frac{1}{7} = \frac{1}{168} = 1\frac{1}{168}.$$

$$10\frac{1}{12} + 6\frac{1}{16} + 15\frac{1}{40} + 1\frac{1}{168} = 10 + 6 + 15 + 1 + (\frac{1}{12} + \frac{1}{16} + \frac{1}{40} + \frac{1}{168}).$$

$$\frac{1}{12} + \frac{1}{16} + \frac{1}{40} + \frac{1}{168} = \frac{140}{1680} + \frac{105}{1680} + \frac{42}{1680} + \frac{10}{1680} = \frac{297}{1680} = 1\frac{1}{1680}.$$

$$32 + 2\frac{1}{1680} = 34\frac{1}{1680}.$$

Exercise 56—Page 169.

(1)

|                           | oz. | dr. | scr. | grs.              |
|---------------------------|-----|-----|------|-------------------|
| $\frac{1}{11}$ of a lb. = | 4   | 2   | 2    | $14\frac{6}{11}$  |
| $\frac{3}{7}$ of an oz. = |     | 3   | 1    | $5\frac{5}{7}$    |
| $\frac{1}{11}$ of a dr. = |     |     | 1    | $1\frac{3}{11}$   |
| $\frac{5}{6}$ of a scr. = |     |     |      | $16\frac{2}{3}$   |
|                           | 4   | 6   | 2    | $18\frac{17}{33}$ |

(3)

|                           | in.            |
|---------------------------|----------------|
| $\frac{1}{7}$ of a yd. =  | $5\frac{1}{7}$ |
| $\frac{1}{7}$ of a ft. =  | $1\frac{5}{7}$ |
| $\frac{1}{7}$ of an in. = | $\frac{1}{7}$  |
|                           | 7              |

(5)

|                            | day | hrs. | min. |
|----------------------------|-----|------|------|
| $\frac{1}{4}$ of a week =  | 1   | 18   | 0    |
| $\frac{1}{3}$ of a day =   |     | 8    | 0    |
| $\frac{1}{4}$ of an hour = |     |      | 12   |
|                            | 2   | 2    | 12   |

(2)

|                                 | qr. | na. | in.              |
|---------------------------------|-----|-----|------------------|
| $\frac{3}{5}$ of a yard =       | 2   | 1   | $1\frac{7}{10}$  |
| $\frac{1}{7}$ of an Eng. ell. = |     | 2   | $1\frac{1}{4}$   |
| $\frac{1}{9}$ of a qr. =        |     | 3   | $0\frac{2}{9}$   |
|                                 | 3   | 3   | $1\frac{39}{40}$ |

(4)

|                            | fur. | per. | yds. | ft. | in.               |
|----------------------------|------|------|------|-----|-------------------|
| $\frac{7}{11}$ of a mile = | 5    | 3    | 3    | 1   | 6                 |
| $\frac{1}{13}$ of a fur. = |      | 12   | 1    | 2   | $0\frac{2}{13}$   |
| $\frac{9}{22}$ of a yd. =  |      |      |      | 1   | $2\frac{8}{11}$   |
|                            | 5    | 16   | 0    | 0   | $3\frac{93}{143}$ |

(6)

|                         | s. | d.              |
|-------------------------|----|-----------------|
| $\frac{1}{7}$ of a £ =  | 2  | $10\frac{3}{7}$ |
| $\frac{2}{9}$ of a s. = |    | $2\frac{2}{3}$  |
| $\frac{1}{2}$ d. =      |    | $7\frac{1}{2}$  |
|                         | 3  | $1\frac{31}{4}$ |

|                              | (7) | £ | s.    | d.                |
|------------------------------|-----|---|-------|-------------------|
| $\frac{5}{8}$ of 21s.        | =   |   | 13    | $1\frac{1}{2}$    |
| $\frac{5}{8}$ of 5s.         | =   |   | 3     | $1\frac{1}{2}$    |
| $\frac{5}{8}$ of £3 12s. 6d. | =   | 2 | 5     | $3\frac{3}{4}$    |
| $\frac{7}{8}$ of a £         | =   |   | 10    | $9\frac{3}{4}$    |
| $\frac{1}{2}$ d.             | =   |   |       | $\frac{4}{8}$     |
|                              |     |   | <hr/> | <hr/>             |
|                              |     |   | 3     | 12 $4\frac{1}{2}$ |

## EXERCISE 57—Page 171.

(1)

$$\frac{3}{4} - \frac{7}{20} = \frac{15}{20} - \frac{7}{20} = \frac{8}{20} = \frac{2}{5}.$$

(2)

$$\begin{aligned} \frac{7}{17} \text{ of } \frac{3}{4} \text{ of } \frac{9}{11} &= \frac{3 \times 48}{17 \times 11} = \frac{144}{187}. \quad \frac{905}{1496} + \frac{144}{187} = \frac{905}{1496} + \frac{1152}{1496} = \\ \frac{2057}{1496} &= \frac{1561}{1496} = 1\frac{3}{8}. \\ \frac{8\frac{3}{4}}{6\frac{1}{11}} &= \frac{\frac{35}{4}}{1\frac{7}{11}} = \frac{35 \times 11}{4 \times 70} = \frac{11}{4 \times 2} = \frac{11}{8} = 1\frac{3}{8}. \\ 1\frac{3}{8} - 1\frac{3}{8} &= 0. \end{aligned}$$

(3)

$$\begin{aligned} 982\frac{17}{40} - 29\frac{9}{20} &= 982\frac{340}{1740} - 29\frac{653}{1740} = 981 + 1\frac{340}{1740} - \\ 29\frac{653}{1740} &= 981\frac{2080}{1740} - 29\frac{653}{1740} = 952\frac{1427}{1740}. \end{aligned}$$

(4)

$$\begin{aligned} 69\frac{1}{11} - 18\frac{86}{146} &= 69\frac{146}{146} - 18\frac{386}{146} = 68 + 1\frac{146}{146} - 18\frac{386}{146} = \\ 68\frac{1592}{146} - 18\frac{386}{146} &= 50\frac{3206}{146} = 50\frac{1603}{73}. \end{aligned}$$

(5)

$$100\frac{1}{2} - 9\frac{5}{8} = 100\frac{4}{8} - 9\frac{5}{8} = 99 + 1\frac{4}{8} - 9\frac{5}{8} = 99\frac{12}{8} - 9\frac{5}{8} = 90\frac{7}{8}$$

(6)

$$\frac{1}{2} \text{ of } \frac{37}{4} = \frac{37}{8} = 4\frac{5}{8}. \quad 6\frac{1}{2} - 4\frac{5}{8} = 6\frac{4}{8} - 4\frac{5}{8} = 5 + 1\frac{4}{8} - 4\frac{5}{8} = 5\frac{10}{8} - 4\frac{5}{8} = 1\frac{5}{8}.$$

(7)

$$\begin{aligned} 611\frac{13}{191} - 610\frac{98}{199} &= 611\frac{8557}{38009} - 610\frac{37818}{38009} = 610 + 1\frac{8557}{38009} - \\ 610\frac{37818}{38009} &= 610\frac{46566}{38009} - 610\frac{37818}{38009} = \frac{8748}{38009}. \end{aligned}$$

E

(8)

$$\frac{5}{9} \text{ of } \frac{2}{3} = \frac{10}{9}. \quad \frac{1}{8} + \frac{1}{9} = \frac{9}{72} + \frac{8}{72} = \frac{17}{72}. \quad \frac{6}{7} \text{ of } \frac{1}{48} = \frac{1}{8}.$$

$$\frac{10}{9} - \frac{17}{72} = \frac{80}{72} - \frac{17}{72} = \frac{63}{72}.$$

(9)

|                        |   |                 |
|------------------------|---|-----------------|
|                        | oz.   | dr.             |
| $\frac{2}{3}$ of a lb. | = 10  | $10\frac{2}{3}$ |
| $\frac{8}{9}$ of a dr. | =   | $\frac{8}{9}$   |
|                        | <hr style="width: 100px; margin: 0 auto;"/> |                 |
|                        | 10  | $9\frac{2}{3}$  |

(10)

$$24\frac{1}{24} - 21\frac{1}{24} = 24\frac{7}{24} - 21\frac{8}{24} =$$

$$23 + 1\frac{7}{24} - 21\frac{8}{24} = 23\frac{7}{24} -$$

$$21\frac{8}{24} = 2\frac{67}{24}.$$

(11)

|                          |                            |      |      |     |     |
|--------------------------|----------------------------|------|------|-----|-----|
|                          | fur.                       | per. | yds. | ft. | in. |
| $\frac{2}{3}$ of a mile  | = 1                        | 31   | 0    | 1   | 10  |
| $\frac{1}{17}$ of a fur. | =                          | 25   | 2    | 1   | 6   |
|                          | <hr style="width: 100%;"/> |      |      |     |     |
|                          | 1                          | 5    | 3    | 1   | 10  |

(12)

$$\frac{2}{3} \text{ of } 13\frac{5}{2} = 17\frac{5}{2} = 7\frac{1}{2}. \quad \frac{1}{16} \text{ of } 57 = 3\frac{5}{8} = 13\frac{5}{4}.$$

$$7\frac{1}{2} - 13\frac{5}{4} = 7\frac{1}{2} - 13\frac{5}{4} = 6 + 1\frac{1}{2} - 13\frac{5}{4} = 6\frac{2}{4} - 13\frac{5}{4} = 53\frac{3}{4}.$$

(13)

$$\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } \frac{3}{4} \text{ of } \frac{3}{4} \text{ of } \frac{62}{33} = \frac{1 \times 3 \times 2 \times 33 \times 62 \times 5}{2 \times 7 \times 9 \times 4 \times 33 \times 6} = \frac{155}{282}.$$

$$12\frac{319}{1764} + \frac{155}{282} = 12\frac{319}{1764} + \frac{1085}{1764} = 12\frac{1404}{1764} = 12\frac{351}{441} = 12\frac{3}{49}.$$

$$\frac{17\frac{9}{11}}{11} = \frac{196}{11 \times 33} = \frac{21}{2} = 10\frac{1}{2}.$$

$$12\frac{3}{49} - 10\frac{1}{2} = 12\frac{3}{49} - 10\frac{49}{98} = 2\frac{29}{98}.$$

(14)

$$3\frac{1}{2} + 8\frac{1}{2} + 5\frac{1}{2} + 6\frac{1}{2} = 3 + 8 + 5 + 6 + (\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}) =$$

$$22 + (\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}).$$

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{15}{80} + \frac{20}{80} + \frac{36}{80} + \frac{90}{80} = \frac{161}{80}.$$

$$22 + \frac{161}{80} = 22\frac{161}{80}.$$

$$3\frac{1}{30} + 2\frac{5}{6} + 16\frac{1}{4} = 3 + 2 + 16 + (\frac{1}{30} + \frac{5}{6} + \frac{1}{4}) = 21 +$$

$$(\frac{1}{30} + \frac{5}{6} + \frac{1}{4}).$$

$$\frac{1}{30} + \frac{5}{6} + \frac{1}{4} = \frac{18}{80} + \frac{60}{80} + \frac{15}{80} = \frac{83}{80} = 1\frac{3}{80}.$$

$$22\frac{161}{80} + 1\frac{3}{80} = 22\frac{164}{80} = 22\frac{41}{20} = 11\frac{21}{10} = 12\frac{11}{10}.$$

(15)

$$\begin{array}{rcl}
 & & \text{r. per. yds. ft. in.} \\
 \frac{4}{11} \text{ of an acre} & = & 1 \quad 18 \quad 5 \quad 4 \quad 72 \\
 \frac{4}{9} \text{ of a per.} & = & \quad \quad 13 \quad 4 \\
 \hline
 & & 1 \quad 17 \quad 22 \quad 2 \quad 108
 \end{array}$$

(16)

$$\begin{aligned}
 16\frac{1}{2} - 9\frac{1}{2} &= 16\frac{19}{33} - 9\frac{98}{33} = 15 + 1\frac{19}{33} - 9\frac{98}{33} = 15\frac{152}{33} - 9\frac{98}{33} = 6\frac{54}{33}. \\
 169\frac{17}{100} - 83\frac{17}{26} &= 169\frac{221}{300} - 83\frac{850}{300} = 168 + 1\frac{221}{300} - 83\frac{850}{300} = 168\frac{1521}{300} - 83\frac{850}{300} = 85\frac{671}{300}.
 \end{aligned}$$

## EXERCISE 58—Page 173.

(1)

$$\frac{7}{12} \text{ of } \frac{5}{6} = \frac{7 \times 5}{12 \times 6} = \frac{35}{72}.$$

(2)

$$\frac{5}{8} \times \frac{4}{5} = \frac{1}{2}.$$

(3)

$$\frac{4}{15} \times \frac{5}{24} = \frac{1}{18}.$$

(4)

$$\frac{7}{8} \times \frac{5}{6} \times \frac{7}{16} = \frac{245}{768}.$$

(5)

$$\frac{14}{1} \times \frac{241}{16} \times \frac{32}{9} = \frac{14 \times 241 \times 2}{9} = \frac{6748}{9} = 749\frac{7}{9}.$$

(6)

$$\frac{3}{10} \times \frac{7}{4} \times \frac{9}{11} \times \frac{11}{12} = \frac{3 \times 7 \times 9}{2 \times 4 \times 4} = \frac{189}{32} = 5\frac{29}{32}.$$

(7)

$$\frac{4}{5} \times \frac{3}{11} \times \frac{9}{17} \times \frac{182}{200} \times \frac{5}{9} = \frac{3 \times 182}{11 \times 17 \times 25} = \frac{546}{4675}.$$

(8)

$$\frac{\overset{3}{6}}{\underset{2}{7}} \times \frac{11}{\underset{4}{8}} \times \frac{\overset{3}{6}}{\underset{3}{33}} \times \frac{\overset{3}{21}}{1} \times \frac{3}{5} \times \frac{5}{1} = \frac{3 \times 3 \times 3}{2} = \frac{27}{2} = 13\frac{1}{2}.$$

(9)

$$\frac{\overset{2}{6}}{\underset{3}{9}} \times \frac{3}{5} \times \frac{6}{11} \times \frac{4}{19} \times \frac{11}{\overset{11}{209}} = \frac{2 \times 6 \times 4}{5} = \frac{48}{5} = 9\frac{3}{5}.$$

(10)

$$\frac{13}{2} \times \frac{80}{7} \times \frac{\overset{2}{180}}{11} \times \frac{2}{13} \times \frac{7}{80} \times \frac{1}{90} = \frac{2}{11}.$$

(11)

$$\frac{4}{7} \times \frac{3}{11} \times \frac{9}{\underset{4}{16}} \times \frac{\overset{7}{77}}{1} \times \frac{3}{7} \times \frac{8}{13} \times \frac{\overset{13}{51}}{1} \times \frac{167}{\underset{3}{24}} = \frac{3 \times 9 \times 167}{4} = \frac{4503}{4} = 1127\frac{1}{4}.$$

(12)

$$\frac{\frac{1}{7}}{\frac{8}{1}} \times \frac{\frac{8}{7}}{\frac{19}{2}} \times \frac{\frac{64}{9}}{\frac{8}{9}} \times \frac{\frac{19}{4}}{\frac{101}{14}} \times \frac{3}{27} \times \frac{9}{8} =$$

$$\frac{1}{7 \times 8} \times \frac{8 \times 2}{7 \times 19} \times \frac{\overset{8}{64} \times 9}{9 \times 8} \times \frac{\overset{2}{19} \times 14}{101 \times \underset{2}{4}} \times \frac{1}{9} \times \frac{9}{8} = \frac{1}{7 \times 101} = \frac{1}{707}.$$

(13)

$$\frac{1}{4} \times \frac{\overset{2}{8}}{1} \times \frac{2}{7} \times \frac{19}{1} = \frac{2 \times 2 \times 19}{7} = \frac{76}{7} = 10\frac{6}{7}.$$



(14)

$$\frac{9}{10} \times \frac{7}{5} \times \frac{11}{15} \times \frac{32}{11} = \frac{9 \times 7 \times 32}{5} = 20\frac{16}{5} = 403\frac{1}{5}.$$

(15)

$$\frac{27}{4} \times \frac{7}{8} \times \frac{4}{5} \times \frac{4}{7} = 2\frac{7}{10} = 2\frac{7}{10}.$$

(16)

$$\frac{11}{3} \times \frac{13}{8} \times \frac{15}{1} = \frac{11 \times 13 \times 15}{8} = 21\frac{45}{8} = 268\frac{1}{8}.$$

(17)

$$\frac{1}{3} \times \frac{7}{4} \times \frac{3}{19} \times \frac{19}{2} \times \frac{94}{11} \times \frac{16}{17} \times \frac{49}{8} \times \frac{4}{5} \times \frac{27}{31} \times \frac{31}{2} \times \frac{191}{188} = \frac{7 \times 49 \times 27 \times 191}{2 \times 11 \times 17} = 17\frac{68851}{374} = 4729\frac{205}{374}.$$

(18)

$$\frac{27}{37\frac{1}{2}} \times \frac{87\frac{2}{3}}{98\frac{1}{8}} \times \frac{7}{2\frac{1}{2}} \times \frac{81\frac{5}{11}}{128} = \frac{27}{189} \times \frac{785}{9 \times 785} \times \frac{7}{8 \times 7} \times \frac{896}{11 \times 128} = \frac{5}{3 \times 11} = \frac{5}{33}.$$

(19)

$$\frac{\$}{11} \times \frac{1}{7} \times \frac{3}{5} \times \frac{17}{19} = \frac{3 \times 17}{11 \times 7} = \$\frac{51}{77}.$$

(20)

$$\begin{aligned}
 & \frac{75\frac{3}{8}}{6\frac{1}{11}} \times \frac{\frac{3}{4} \text{ of } 8\frac{1}{4} \times \frac{1}{15} \text{ of } 28}{\frac{2}{11} \text{ of } 6\frac{3}{8} \times \frac{1}{17} \text{ of } 24} \times \frac{7\frac{1}{2}}{15} \times \frac{\frac{3}{4}}{\frac{5}{7}} \times 14\frac{3}{7} \times \frac{100}{121} \times \\
 & \quad \frac{4}{5\frac{1}{3}} \times \frac{\frac{5}{7}}{9} = \\
 & \frac{603}{67} \times \frac{\frac{3}{4} \times \frac{33}{4} \times \frac{1}{15} \times \frac{28}{1}}{\frac{2}{11} \times \frac{51}{8} \times \frac{1}{17} \times \frac{24}{1}} \times \frac{\frac{36}{5}}{1\frac{5}{1}} \times \frac{\frac{3}{4}}{\frac{5}{7}} \times \frac{101}{7} \times \frac{100}{121} \times \\
 & \quad \frac{\frac{1}{16}}{\frac{1}{3}} \times \frac{\frac{5}{7}}{\frac{9}{1}} = \\
 & \frac{9 \times 808 \times 11}{8 \times 67} \times \frac{11 \times 4 \times 3 \times 33 \times 28 \times 11 \times 8 \times 17}{7 \times 4 \times 15 \times 2 \times 51 \times 24} \times \frac{9 \times 36}{5 \times 15} \times \frac{8 \times 7}{4 \times 5} \times \\
 & \quad \frac{101}{7} \times \frac{100}{121} \times \frac{4 \times 3}{16} \times \frac{5}{7 \times 9} = \\
 & \quad \frac{11 \times 9 \times 101}{5 \times 7 \times 16} = \frac{9999}{560} = 17\frac{479}{560}.
 \end{aligned}$$

EXERCISE 59—Page 174.

(1)

$$\frac{4\frac{3}{8} \text{ of } 4 \text{ days, } 5 \text{ hours,}}{36} = \frac{180 \text{ d. } 23 \text{ h.}}{36} = 5 \text{ d. } 0 \text{ h. } 38 \text{ min. } 20 \text{ sec.}$$

(2)

$$\frac{1}{4}\frac{3}{2} \text{ of } £29 = \frac{£29 \times 13}{42} = \frac{£377}{42} = £8 \text{ } 19\text{s. } 6\frac{2}{3}\text{d.}$$

(3)

$$\frac{7}{9} \text{ of } 186 \text{ a. } 3 \text{ r.} = \frac{186 \text{ a. } 3 \text{ r.} \times 7}{9} = \frac{1307 \text{ a. } 1 \text{ r.}}{9} = 145 \text{ a. } 1 \text{ r.}$$

(4)

$$\frac{1}{4} \text{ of } \frac{2}{7} \text{ of } \frac{1}{30} \text{ of } \frac{4}{2} \text{ of } 24 \text{ h. } 30 \text{ m.} = \frac{1}{15} \text{ of } 24 \text{ h. } 30 \text{ m.} = 1 \text{ h. } 38 \text{ m.}$$

(5)

$$\begin{array}{l} \frac{3}{7} \text{ of } \frac{4}{9} \text{ of } \frac{2}{10} \text{ of } \frac{7}{9} \text{ of } 33 \text{ bu. } 2 \text{ p. } 1 \text{ ga.} = \frac{7}{90} \text{ of } 33 \text{ bu. } 2 \text{ p. } 1 \text{ ga.} = \\ 33 \text{ bu. } 2 \text{ p. } 1 \text{ ga.} \times 7 \quad 235 \text{ b. } 1 \text{ p. } 1 \text{ g.} \\ \hline \frac{\quad}{90} = \frac{\quad}{90} = 2 \text{ b. } 2 \text{ p. } 0 \text{ g. } 3 \text{ q } 1 \frac{1}{4} \text{ p.} \end{array}$$

## EXERCISE 60—Page 175.

(1)

$$\frac{1}{2} \text{ of } \frac{3}{5} \div \frac{2}{4} \text{ of } \frac{3}{4} = \frac{1}{2} \times \frac{3}{5} \times \frac{4}{3} \times \frac{4}{35} = \frac{2 \times 4}{5 \times 35} = \frac{8}{175}.$$

(2)

$$\frac{15}{22} \div \frac{2}{5} \div \frac{5}{11} = \frac{15}{22} \times \frac{5}{2} \times \frac{11}{5} = \frac{5}{2 \times 3} = \frac{5}{6}.$$

(3)

$$82\frac{1}{17} \div 26\frac{5}{11} = \frac{155}{17} \times \frac{41}{119} = \frac{155 \times 41}{17 \times 119} = \frac{6355}{2023} = 3\frac{286}{2023}.$$

(4)

$$2\frac{1}{2} \div \frac{3}{4} + \frac{5}{8} = \frac{5}{2} \div \frac{3}{4} = \frac{5}{2} \times \frac{4}{3} = \frac{5 \times 4}{3} = \frac{20}{3} = 6\frac{2}{3}.$$

(5)

$$1\frac{1}{2} \div \frac{1}{7} \text{ of } 2\frac{1}{2} \text{ of } 16 \text{ of } \frac{3}{4} \text{ of } \frac{1}{70} = \frac{7}{\frac{4}{2}} \times \frac{7}{1} \times \frac{4}{11} \times \frac{1}{16} \times \frac{4}{35} \times$$

$$\frac{\frac{2}{70}}{1} = \frac{7 \times 7}{2 \times 11} = \frac{49}{22} = 2\frac{5}{22}.$$

(6)

$$2\frac{1}{2} \div (\frac{5}{2} \div \frac{6}{32} \text{ of } 9) = \frac{7}{3} \div (\frac{5}{9} \text{ of } \frac{3}{2} \text{ of } \frac{1}{9}) = \frac{7}{3} \times \frac{9}{5} \times \frac{3}{\frac{32}{16}} \times$$

$$\frac{\frac{3}{9}}{1} = \frac{7 \times 9 \times 3 \times 3}{5 \times 16} = \frac{567}{80} = 7\frac{7}{80}.$$

(7)

$$48\frac{1}{2} \div \frac{2}{9} + \frac{3}{8} \text{ of } 6 = \frac{97}{2} \div \frac{2}{9} + \frac{3}{4} = \frac{97}{2} \div \frac{88}{36} = \frac{97}{2} \times \frac{18}{88} =$$

$$\frac{97 \times 18}{89} = \frac{1746}{89} = 19\frac{55}{89}.$$

(8)

$$6\frac{1}{2} \div \frac{3}{8} \text{ of } \frac{9}{10} + \frac{8}{17} = \frac{13}{2} \div \frac{27}{80} + \frac{8}{17} = \frac{13}{2} \div \frac{859}{880} = \frac{13}{2} \times$$

$$\frac{425}{859} = \frac{13 \times 425}{859} = \frac{5525}{859} = 6\frac{371}{859}.$$

(9)

$$\frac{2}{3} \times \frac{10}{3} \div \frac{9}{4} \times \frac{25}{4} = \frac{2}{3} \times \frac{10}{3} \times \frac{4}{9} \times \frac{4}{25} = \frac{4 \times 4}{3 \times 5} = \frac{16}{15} = 1\frac{1}{15}.$$

(10)

$$\frac{\frac{67}{9}}{\frac{35}{3}} \div \frac{\frac{3}{7}}{\frac{33}{8}} = \frac{67 \times 3}{9 \times 35} \div \frac{3 \times 8}{7 \times 33} = \frac{67 \times 3}{3 \times 35} \times \frac{7 \times 33}{3 \times 8} =$$

$$\frac{67 \times 11}{3 \times 5 \times 8} = \frac{737}{120} = 6\frac{17}{20}.$$

(11)

$$\frac{5}{9} \text{ of } \frac{80}{11} \div \frac{4}{11} \text{ of } \frac{12}{7} = \frac{5}{9} \times \frac{80}{11} \times \frac{11}{4} \times \frac{7}{12} = \frac{5 \times 10 \times 7}{9 \times 61} = \frac{350}{549}.$$

(12)

$$\frac{15}{28} \text{ of } \frac{10}{3} \text{ of } \frac{3}{4} \text{ of } \frac{7}{5} \div \frac{5}{6} \text{ of } \frac{3}{8} \text{ of } \frac{3}{4} \text{ of } \frac{5}{1} = \frac{45}{28} \times \frac{10}{13} \times \frac{3}{4} \times \frac{7}{5} \times \frac{6}{5} \times$$

$$\frac{28}{3} \times \frac{4}{3} \times \frac{1}{5} = \frac{3 \times 2 \times 7 \times 6}{13 \times 5} = \frac{252}{65} = 3\frac{7}{5}.$$

(13)

$$\frac{\frac{7}{4}}{\frac{9}{2}} \div \frac{\frac{7}{3}}{\frac{9}{4}} = \frac{7 \times 2}{4 \times 9} \div \frac{7 \times 4}{3 \times 9} = \frac{7 \times 2}{4 \times 9} \times \frac{3 \times 9}{7 \times 4} = \frac{3}{2 \times 4} = \frac{3}{8}.$$

(14)

$$\frac{\frac{3}{25}}{\frac{21}{2}} \div \frac{\frac{21}{35}}{\frac{3}{25}} = \frac{3}{25} \div \frac{21 \times 2}{5 \times 35} = \frac{3}{25} \times \frac{5 \times 35}{21 \times 2} = 1.$$

(15)

$$\frac{11}{8} \times \frac{1}{9} \div \frac{3}{7} \times \frac{107}{135} \times \frac{13}{7} = \frac{113}{8} \times \frac{1}{9} \times \frac{7}{3} \times \frac{13}{107} \times$$

$$\frac{17}{2 \times 135} = \frac{113 \times 2 \times 17}{9 \times 3 \times 107} = \frac{3842}{2889} = 1\frac{953}{2889}.$$



## EXERCISE 62—Page 178.

(1)

$$\begin{array}{r}
 12\frac{1}{4} \\
 \hline
 7 \\
 \hline
 3\frac{1}{4}
 \end{array}
 =
 \begin{array}{r}
 4\frac{9}{4} \\
 \hline
 7\frac{1}{1} \\
 \hline
 1\frac{3}{4}
 \end{array}
 =
 \begin{array}{r}
 7 \\
 \hline
 4 \\
 \hline
 1\frac{3}{4}
 \end{array}
 =
 \begin{array}{r}
 7 \\
 \hline
 13 \\
 \hline
 7\frac{1}{3}
 \end{array}
 =
 \begin{array}{r}
 7 \\
 \hline
 13 \\
 \hline
 7\frac{1}{3}
 \end{array}
 =
 \begin{array}{r}
 7 \\
 \hline
 13 \\
 \hline
 7\frac{1}{3}
 \end{array}
 =
 \begin{array}{r}
 7 \times 35 \\
 \hline
 13 \times 27
 \end{array}$$

$$\begin{array}{r}
 9 \\
 \hline
 3 \\
 \hline
 7 \\
 \hline
 5
 \end{array}
 =
 \begin{array}{r}
 9 \\
 \hline
 3 \\
 \hline
 7 \\
 \hline
 5
 \end{array}
 =
 \begin{array}{r}
 27 \\
 \hline
 2 \\
 \hline
 7 \\
 \hline
 5
 \end{array}
 =
 \begin{array}{r}
 243 \\
 \hline
 14 \\
 \hline
 5 \\
 \hline
 9
 \end{array}
 =
 \begin{array}{r}
 243 \\
 \hline
 70 \\
 \hline
 9 \\
 \hline
 2
 \end{array}
 =
 \begin{array}{r}
 27 \\
 \hline
 35 \\
 \hline
 13 \times 27
 \end{array}$$

$$\begin{array}{r}
 4\frac{1}{2} \\
 \hline
 \frac{2}{3} \text{ of } 32 \\
 \hline
 7\frac{1}{3}
 \end{array}
 =
 \begin{array}{r}
 9\frac{1}{2} \\
 \hline
 3\frac{1}{2} \\
 \hline
 7\frac{1}{2}
 \end{array}
 =
 \begin{array}{r}
 64 \\
 \hline
 3 \\
 \hline
 7\frac{1}{3}
 \end{array}
 =
 \begin{array}{r}
 64 \\
 \hline
 7 \\
 \hline
 7\frac{1}{3}
 \end{array}
 =
 \begin{array}{r}
 64 \\
 \hline
 7 \\
 \hline
 7\frac{1}{3}
 \end{array}
 = 3.$$

$$\begin{array}{r}
 9\frac{1}{3} \\
 \hline
 3\frac{1}{2} \\
 \hline
 7\frac{1}{2}
 \end{array}
 =
 \begin{array}{r}
 28 \\
 \hline
 3 \\
 \hline
 7\frac{1}{2}
 \end{array}
 =
 \begin{array}{r}
 64 \\
 \hline
 8 \\
 \hline
 7\frac{1}{2}
 \end{array}
 =
 \begin{array}{r}
 64 \\
 \hline
 21 \\
 \hline
 7\frac{1}{2}
 \end{array}
 = 3.$$

$$\begin{array}{r}
 7 \\
 \hline
 8 \\
 \hline
 7 \times 35 \\
 \hline
 13 \times 27
 \end{array}
 \times
 \begin{array}{r}
 3 \\
 \hline
 1 \\
 \hline
 735 \\
 \hline
 351
 \end{array}
 =
 \begin{array}{r}
 735 \\
 \hline
 351
 \end{array}
 = 2\frac{11}{17}.$$

(2)

$$\begin{array}{r}
 \frac{1}{3} \\
 \hline
 7 \\
 \hline
 6\frac{1}{2}
 \end{array}
 =
 \begin{array}{r}
 \frac{1}{3} \\
 \hline
 7\frac{1}{1} \\
 \hline
 1\frac{3}{2}
 \end{array}
 =
 \begin{array}{r}
 \frac{1}{21} \\
 \hline
 1\frac{3}{2} \\
 \hline
 1\frac{3}{2}
 \end{array}
 =
 \begin{array}{r}
 2 \\
 \hline
 21 \times 13 \\
 \hline
 8
 \end{array}
 =
 \begin{array}{r}
 8 \\
 \hline
 7 \times 13 \times 19
 \end{array}$$

$$\begin{array}{r}
 9\frac{1}{2} \\
 \hline
 3 \\
 \hline
 \frac{1}{2}
 \end{array}
 =
 \begin{array}{r}
 19 \\
 \hline
 2 \\
 \hline
 \frac{1}{2}
 \end{array}
 =
 \begin{array}{r}
 19 \\
 \hline
 2 \\
 \hline
 \frac{1}{2}
 \end{array}
 =
 \begin{array}{r}
 19 \\
 \hline
 2 \\
 \hline
 \frac{1}{2}
 \end{array}
 =
 \begin{array}{r}
 19 \\
 \hline
 2 \\
 \hline
 \frac{1}{2}
 \end{array}
 =
 \begin{array}{r}
 19 \\
 \hline
 2 \\
 \hline
 \frac{1}{2}
 \end{array}$$

$$\begin{array}{r}
 \frac{5}{6} \\
 \hline
 7 \\
 \hline
 42
 \end{array}
 =
 \begin{array}{r}
 5 \\
 \hline
 42
 \end{array}
 =
 \begin{array}{r}
 8 \\
 \hline
 7 \times 13 \times 19
 \end{array}
 \div
 \begin{array}{r}
 5 \\
 \hline
 42
 \end{array}
 =
 \begin{array}{r}
 8 \\
 \hline
 7 \times 13 \times 19
 \end{array}
 \times
 \begin{array}{r}
 42 \\
 \hline
 5
 \end{array}
 = 12\frac{1}{3}.$$

(3)

$$\begin{array}{r}
 12\frac{1}{2} \\
 \hline
 5\frac{1}{4} \\
 \hline
 3\frac{3}{4} \\
 \hline
 5\frac{1}{2} \\
 \hline
 2\frac{1}{4} \\
 \hline
 5 \\
 \hline
 4\frac{1}{2} \\
 \hline
 3\frac{3}{4} \\
 \hline
 16\frac{2}{3} \\
 \hline
 \frac{1}{2}
 \end{array}
 =
 \begin{array}{r}
 2\frac{5}{2} \\
 \hline
 2\frac{1}{4} \\
 \hline
 1\frac{5}{4} \\
 \hline
 1\frac{1}{2} \\
 \hline
 \frac{9}{4} \\
 \hline
 \frac{5}{1} \\
 \hline
 \frac{9}{2} \\
 \hline
 1\frac{5}{4} \\
 \hline
 5\frac{0}{3} \\
 \hline
 \frac{1}{2}
 \end{array}
 =
 \begin{array}{r}
 5\frac{0}{21} \\
 \hline
 1\frac{5}{22} \\
 \hline
 \frac{9}{20} \\
 \hline
 \frac{9}{2} \\
 \hline
 1\frac{5}{4} \\
 \hline
 10\frac{0}{3}
 \end{array}
 = 2\frac{20}{63}.$$
  

$$\begin{array}{r}
 220 \\
 \hline
 63
 \end{array}
 \div \frac{8}{9} = \frac{220}{63} \times \frac{9}{8} = \frac{55}{14} = 3\frac{13}{14}.$$

## EXERCISE 63—Page 180.

(1)

$$\begin{array}{l}
 \frac{800}{2000} = \frac{2}{5}. \quad \frac{420}{2000} = \frac{21}{100}. \quad \frac{100}{2000} = \frac{1}{20}. \quad \frac{160}{2000} = \frac{2}{25}. \\
 \frac{35}{2000} = \frac{7}{400}.
 \end{array}$$

(2)

$$\frac{2}{5} \text{ of } \frac{5}{4} \text{ of } \frac{8}{5} \text{ of } \frac{1}{4} \text{ of } \frac{5}{2} \text{ of } \frac{4}{5} = \frac{2}{5} \times \frac{5}{4} \times \frac{8}{5} \times \frac{1}{4} \times \frac{5}{2} \times \frac{4}{5} = \frac{2}{5}.$$

(3)

$$6\frac{7}{8} \times 65\frac{3}{4} \text{ cts.} = \frac{55}{8} \times 2\frac{63}{4} \text{ cts.} = 14\frac{165}{32} \text{ cts.} = \$4.52\frac{1}{2}.$$



(4)

$$\frac{3}{8} + \frac{1}{17} = \frac{51}{136} + \frac{8}{136} = \frac{59}{136}.$$

(5)

$$\frac{1}{3} + \frac{1}{10} + \frac{1}{8} + \frac{1}{6} = \frac{40}{120} + \frac{12}{120} + \frac{15}{120} + \frac{20}{120} = \frac{87}{120} = \frac{29}{40}.$$

$$1 \text{ or } \frac{40}{40} - \frac{29}{40} = \frac{11}{40}.$$

(6)

$$\frac{5\frac{1}{2} - 2\frac{1}{8}}{3\frac{3}{4} + \frac{9}{20}} \text{ of } \frac{4\frac{1}{2} + 5\frac{1}{2}}{4\frac{1}{20}} \text{ of } \frac{2\frac{3}{8} + 1\frac{3}{8}}{7\frac{1}{24} - 2\frac{1}{4}} = \frac{5\frac{3}{40} - 2\frac{5}{40}}{3\frac{15}{20} + \frac{9}{20}} \text{ of } \frac{4\frac{25}{80} + 5\frac{38}{80}}{\frac{81}{20}} \text{ of}$$

$$\frac{2\frac{9}{16} + 1\frac{10}{16}}{7\frac{9}{24} - 2\frac{6}{24}} = \frac{3\frac{27}{40}}{4\frac{4}{20}} \text{ of } \frac{10\frac{3}{80}}{\frac{81}{20}} \text{ of } \frac{4\frac{1}{18}}{5\frac{13}{24}} = \frac{1\frac{17}{40}}{\frac{84}{20}} \text{ of } \frac{5\frac{13}{60}}{\frac{81}{20}} \text{ of } \frac{\frac{64}{18}}{\frac{133}{24}} =$$

$$\frac{7}{8} \times \frac{2 \times 57}{5 \times 9} \times \frac{8 \times 64}{5 \times 133} = \frac{2 \times 64}{5 \times 3 \times 5} = \frac{128}{75} = 1\frac{53}{75}$$

(7)

$$1670\frac{7}{8} \times 12\frac{3}{4} \text{ cts.} = 217\frac{17}{3} \times 5\frac{1}{4} \text{ cts.} = 1107\frac{567}{2} \text{ cts.} = \$212.99\frac{1}{2}.$$

(8)

$\frac{3}{8}$  of the longer =  $\frac{3}{4}$  of the shorter; therefore  $\frac{1}{3}$  of the longer =  $\frac{1}{2}$  of  $\frac{3}{4}$  =  $\frac{3}{8}$  of the shorter.

Hence the longer =  $\frac{3}{8} \times 3 = \frac{9}{8}$  of the shorter.

The whole tree = longer + shorter =  $\frac{9}{8} + \frac{8}{8}$  of shorter =  $1\frac{7}{8}$  of the shorter.

If 136 ft. =  $1\frac{7}{8}$  of the shorter,  $\frac{1}{17}$  of 136 = 8 =  $\frac{1}{8}$  of the shorter.

Hence shorter =  $8 \times 8 = 64$  ft.; and longer =  $136 - 64 = 72$  ft.

(9)

$$97\frac{1}{4} + 127\frac{3}{8} + 500\frac{3}{8} + 333\frac{1}{3} = 97\frac{30}{20} + 127\frac{45}{20} + 500\frac{45}{20} + 333\frac{40}{20} = 1057\frac{63}{20} = 1058\frac{13}{20}.$$

$$\$1000 + \$1375\frac{1}{2} + \$6831 + \$4013\frac{3}{8} = \$1000 + \$1375\frac{8}{16} + \$6831 + \$4013\frac{3}{8} = \$13219\frac{11}{16} = \$13219.68\frac{1}{2}.$$

(10)

$$12\frac{5}{6} + \frac{8}{15} = 13\frac{11}{30}. \quad 8\frac{3}{4} + 1\frac{1}{10} = 9\frac{7}{10}. \quad 13\frac{1}{10} - 9\frac{7}{10} = 3\frac{4}{10} = \frac{21}{50}.$$

$$7\frac{5}{12} - 6\frac{1}{2} = \frac{1}{12}. \quad \frac{211}{50} \times \frac{2}{3} \times \frac{1}{12} = \frac{2321}{1600} = 14\frac{81}{1600}.$$

$$\frac{2}{3} \div \frac{12}{7} = \frac{2}{3} \times \frac{7}{12} = \frac{7}{18}. \quad \frac{5}{8} \div \frac{35}{11} = \frac{5}{8} \times \frac{11}{35} = \frac{11}{56}. \quad \frac{7}{8} - \frac{11}{56} = \frac{27}{56}.$$

(11)

$$19\frac{7}{8} \times \$6\frac{3}{4} = \frac{159}{8} \times \$\frac{27}{4} = \$\frac{4323}{32} = \$134.15\frac{3}{4}.$$

(12)

$$376\frac{1}{8} \times \$75\frac{3}{8} = \frac{4779}{8} \times \$\frac{603}{8} = \frac{4081737}{64} = \$28387.06\frac{1}{4}.$$

(13)

$$147\frac{2}{3} + 320\frac{1}{8} = 147\frac{10}{12} + 320\frac{3}{8} = 467\frac{13}{12}. \quad 467\frac{13}{12} - 156\frac{1}{4} =$$

$$467\frac{13}{12} - 156\frac{3}{6} = 311\frac{10}{12}.$$

(14)

$$\begin{array}{l} 7 (1\frac{1}{2} \text{ of } \frac{3}{4}) \\ \hline \frac{1}{6} \left( \frac{3}{3\frac{1}{2}} \text{ of } 7 \right) \end{array} \div 7\frac{7}{8} = \frac{1 \times \frac{3}{2} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{2} \times 7} \div \frac{63}{8} = \frac{7 \times 3 \times 3}{1 \times 2 \times 4} \times \frac{8}{63} =$$

$$\begin{array}{l} 7 \times 3 \times 3 \\ \hline 1 \times 2 \times 4 \end{array}$$

$$\frac{7 \times 3 \times 3}{1 \times 2 \times 4} \times \frac{2}{3} = 1. \quad \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}{1 \quad 1 \quad 1} = \frac{\frac{6}{12} + \frac{4}{12} + \frac{3}{12}}{1 \quad 1 \quad 1} =$$

$$\frac{13}{12} = \frac{13}{12} \quad \frac{13}{12} \quad \frac{13}{12} \quad 2535$$

$$\frac{2+13+9}{288+188+380} = \frac{244}{546} = \frac{122}{273} = 1\frac{359}{276}.$$

(15)

$$17\frac{1}{2} \div 7\frac{7}{8} = \frac{123}{8} \div \frac{53}{8} = \frac{123}{53} \times \frac{7}{8} = \frac{123}{83} = 2\frac{17}{83}.$$

(16)

$$3\frac{2}{3} + 4\frac{1}{4} + 4\frac{1}{6} = 3\frac{10}{12} + 4\frac{3}{4} + 4\frac{2}{6} = 13\frac{13}{12} = 7\frac{1}{6}.$$

$$7\frac{1}{2} - 5\frac{5}{6} = 7\frac{3}{6} - 5\frac{5}{6} = 2\frac{1}{6} = \frac{8}{24}.$$

$$94\frac{1}{8} + 93\frac{1}{9} = 94\frac{9}{72} + 93\frac{8}{72} = 187\frac{17}{72} = 134\frac{81}{72}.$$

$$7\frac{23}{60} \times \frac{85}{42} \div \frac{13481}{72} = \frac{798}{60} \times \frac{85}{42} \times \frac{72}{13481} = \frac{1}{7}.$$

(17)

$$2\frac{2}{3} + \frac{1}{6} + 4 = 2\frac{10}{12} + \frac{2}{12} + 4 = 7\frac{7}{6} = 11\frac{1}{6}.$$

$$2 \div 1\frac{1}{6} = 2 \times \frac{6}{7} = \frac{12}{7}. \quad 1\frac{2}{3} - \frac{7}{9} = \frac{15}{9} - \frac{7}{9} = \frac{8}{9}.$$

$$\frac{15}{66} + \frac{8}{9} = \frac{88}{604}. \quad 5\frac{1}{2} - 4\frac{1}{2} = 4\frac{3}{10} - 4\frac{5}{10} = \frac{7}{10}.$$

$$\frac{583}{72} \times \frac{7}{10} = \frac{583}{10}.$$

(18)

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}. \quad 1\frac{1}{3} + 2\frac{3}{4} = 4\frac{1}{12} = \frac{49}{12}. \quad 2\frac{1}{4} - 1\frac{1}{2} = \frac{1}{4} = \frac{1}{4}.$$

$$3\frac{1}{10} - \frac{2}{7} = 2\frac{1}{7} = \frac{187}{70}. \quad \frac{5}{6} \times \frac{49}{12} \times \frac{4}{7} \times \frac{187}{70} = \frac{187}{3 \times 12} = \frac{187}{36} = 5\frac{7}{36}.$$

$$1\frac{3}{4} \div 2\frac{1}{2} = \frac{7}{4} \times \frac{2}{5} = \frac{7}{10}. \quad 5\frac{1}{2} \div 3\frac{1}{8} = \frac{11}{2} \times \frac{8}{25} = \frac{44}{25} = 1\frac{19}{25}.$$

$$\frac{7}{10} + 1\frac{9}{25} = 2\frac{23}{50}.$$

(19)

$$1 - (\frac{1}{3} + \frac{1}{2}) = \frac{1}{6}. \quad \frac{2}{3} \text{ of } \frac{1}{3} = \frac{1}{6}. \quad \frac{1}{3} - \frac{1}{5} = \frac{2}{15}. \quad \frac{1}{6} + \frac{2}{15} = \frac{3}{10}.$$

$$\frac{1}{3} - \frac{2}{30} = \frac{1}{10}. \quad \frac{1}{30} \text{ of } \$40000 = \$1333.33\frac{1}{3}.$$

EXERCISE 66—Page 183.

(1)

$$\begin{array}{r} \frac{1}{2} = 2) 1 \\ \hline .5 \end{array} \quad \begin{array}{r} \frac{3}{8} = 8) 3 \\ \hline .375 \end{array}$$

(2)

$$\begin{array}{r} \frac{2}{5} = 25) 9 \\ \hline .36 = \frac{36}{100} \end{array} \quad \begin{array}{r} \frac{1}{4} = 4) 1 \\ \hline .25 = \frac{25}{100} \end{array}$$

(3)

75)73 ( $\cdot 9733+$

67.5

5.50

5.25

 $\cdot 250$  $\cdot 225$ 250

225

25

123)574( $\cdot 666+$

492

82.0

73.8

8.20

7.38

 $\cdot 820$  $\cdot 738$ 

82

34)15 ( $\cdot 44117+$

13.6

1.40

1.36

40

34

60

34

260

238

22

(4)

7)6

 $\cdot 857142+$ 

12)5

 $\cdot 4166+$ 

9)4

 $\cdot 44444+$ 

(5)

112)17 ( $\cdot 15178571428+$

11.2

5.80

5.60

 $\cdot 200$  $\cdot 112$ 880

784

960

896

640

560

800

1296)718 ( $\cdot 554012+$

648.0

70.00

64.80

5.200

5.184

1600

1296

3040

2592

448

64

## EXERCISE 67—Page 184.

(1)

$$12) 1.0 \text{ in.}$$

$$3) 2.083333 \text{ ft.}$$

$$5\frac{1}{2}) 3.694444 \text{ yd.}$$

$$\begin{array}{r} 2 \\ 2 \end{array}$$

$$11) 7.388888$$

$$40) .671717 \text{ per.}$$

$$.01679 + \text{fur.}$$

(2)

$$12) 17.0 \text{ grs.}$$

$$2) 1.41666666$$

$$20) 3.70833333 \text{ dwt.}$$

$$12) .18541666 \text{ oz.}$$

$$.01545138 + \text{lb.}$$

(3)

$$20) 7.0 \text{ grs.}$$

$$3) 2.35 \text{ scr.}$$

$$8) .7833333 \text{ dr.}$$

$$12) .0979166 \text{ oz.}$$

$$.0081597 + \text{lb.}$$

(4)

$$12) 9.0 \text{ in}$$

$$3) 2.75 \text{ ft.}$$

$$5\frac{1}{2}) 2.91666$$

$$\begin{array}{r} 2 \\ 2 \end{array}$$

$$11) 5.83333$$

$$40) 35.53030 \text{ per.}$$

$$8) 5.88825 \text{ fur.}$$

$$.73603 + \text{mile.}$$

(5)

$$4) 2.0 \text{ na.}$$

$$4) 3.5 \text{ qr.}$$

$$.875 \text{ yd.}$$

(7)

$$60) 21.0 \text{ sec.}$$

$$60) 55.35 \text{ min.}$$

$$12) 12.9225 \text{ hr.}$$

$$2) 1.076875$$

$$.5384375 \text{ day.}$$

(6)

$$13\text{s. } 4\text{d.} = 160\text{d.}$$

$$5\text{s.} = 60\text{d.}$$

$$\frac{60}{160} = \frac{3}{8} = .375$$

(8)

$$\frac{2}{3} \text{ of } \frac{1}{2} \text{ of } 6\frac{1}{2}\text{d.} = 2\frac{1}{8}\text{d. and } \pounds\frac{1}{3} = 80\text{d. } \frac{2}{3} \text{ of } \frac{1}{2} \text{ of } 1 \text{ mil.} = 12672 \text{ in.}$$

$$2\frac{1}{8}\text{d.} = \frac{1}{80} \text{ of } \frac{2}{3} \text{ of } \pounds\frac{1}{3} = \frac{2}{240} \text{ of } \pounds\frac{1}{3}.$$

$$27 \div 2240 = 0.012053.$$

$$3\frac{1}{2}) 12672$$

$$\begin{array}{r} 2 \\ 2 \end{array}$$

$$7) 25344$$

$$3620.571428 +$$

(10)

$$\frac{1}{8} \text{ of } \frac{7}{8} \text{ of } \frac{1}{4} \text{ lbs.} = \frac{1}{30} \text{ lb.} = 110\frac{1}{8} \text{ drs.} = 166\frac{4}{5} \text{ drs.}$$

$$\frac{3}{4} \text{ of an oz.} = 12 \text{ drs. } 166\frac{4}{5} \div 12 = 13\frac{8}{5}$$

$$180)1664(9\cdot2444+$$

(11)

$$\begin{array}{r} 1620 \\ \hline \end{array}$$

$$2)1\cdot0 \text{ pts.}$$

$$\begin{array}{r} 440 \\ \hline \end{array}$$

$$4)1\cdot5 \text{ qt.}$$

$$\begin{array}{r} 360 \\ \hline \end{array}$$

$$2)1\cdot375 \text{ gal.}$$

$$\begin{array}{r} 800 \\ \hline \end{array}$$

$$4)3\cdot6875 \text{ pk.}$$

$$\begin{array}{r} 720 \\ \hline \end{array}$$

$$\cdot921875 \text{ bush.}$$

$$\begin{array}{r} 800 \\ \hline \end{array}$$

$$\begin{array}{r} 720 \\ \hline \end{array}$$

$$\begin{array}{r} 800 \\ \hline \end{array}$$

$$\begin{array}{r} 720 \\ \hline \end{array}$$

$$\begin{array}{r} 80 \\ \hline \end{array}$$

## EXERCISE 68—Page 186.

(1)

(2)

(3)

$$\begin{array}{r} \cdot3945 \\ \hline \end{array}$$

$$\begin{array}{r} \cdot3965 \\ \hline \end{array}$$

$$\begin{array}{r} \cdot309153 \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \hline \end{array}$$

$$\begin{array}{r} 20 \\ \hline \end{array}$$

$$\begin{array}{r} 15780 \\ \hline \end{array}$$

$$\begin{array}{r} 3\cdot1720 \text{ fur.} \\ \hline \end{array}$$

$$\begin{array}{r} 6\cdot183060 \text{ dwt.} \\ \hline \end{array}$$

$$\begin{array}{r} 7890 \\ \hline \end{array}$$

$$\begin{array}{r} 40 \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ \hline \end{array}$$

$$\begin{array}{r} 9\cdot4680 \text{ hrs.} \\ \hline \end{array}$$

$$\begin{array}{r} 6\cdot8800 \text{ per.} \\ \hline \end{array}$$

$$\begin{array}{r} 732240 \\ \hline \end{array}$$

$$\begin{array}{r} 60 \\ \hline \end{array}$$

$$\begin{array}{r} 5\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 366120 \\ \hline \end{array}$$

$$\begin{array}{r} 28\cdot0800 \text{ min.} \\ \hline \end{array}$$

$$\begin{array}{r} 44000 \\ \hline \end{array}$$

$$\begin{array}{r} 4\cdot393440 \text{ grs.} \\ \hline \end{array}$$

$$\begin{array}{r} 60 \\ \hline \end{array}$$

$$\begin{array}{r} 4400 \\ \hline \end{array}$$

$$\begin{array}{r} 4\cdot8000 \text{ sec.} \\ \hline \end{array}$$

$$\begin{array}{r} 4\cdot8400 \text{ yds.} \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$\begin{array}{r} 2\cdot5200 \text{ ft.} \\ \hline \end{array}$$

$$\begin{array}{r} 12 \\ \hline \end{array}$$

$$\begin{array}{r} 6\cdot2400 \text{ in.} \\ \hline \end{array}$$

(4)

$$22.75 = 22\frac{75}{100} = 22\frac{3}{4}.$$

$$£2 \text{ 2s. 6d.} \times 22\frac{3}{4} = £48 \text{ 6s. } 10\frac{1}{2}\text{d.}$$

$$7 \text{ b. 1 p. 1 g. 1 qt.} = 237 \text{ qts.}$$

$$11.17825 \times 237 = 2649.24525 \text{ qt.} =$$

$$82 \text{ b. 3 p. 0 g. 1 q. 0.4905 pts}$$

(5)

(6)

$$\begin{array}{r} .2057 \\ 12 \\ \hline \end{array}$$

$$\begin{array}{r} 2.4684 \text{ oz.} \\ 20 \\ \hline \end{array}$$

$$\begin{array}{r} 9.3680 \text{ dwt.} \\ 24 \\ \hline \end{array}$$

$$\begin{array}{r} 14720 \\ 7360 \\ \hline \end{array}$$

$$8.8320 \text{ grs.}$$

(7)

$$1 \text{ f. 36 p. 2 y. 5 in.} = 15125 \text{ in.}$$

$$15125 \times .176 = 2662 \text{ in.} =$$

$$13 \text{ per. 2 yds. 1 ft. 4 in.}$$

(8)

$$\begin{array}{r} .625 \\ 3 \\ \hline \end{array}$$

$$\begin{array}{r} 1.875 \text{ mil.} \\ 8 \\ \hline \end{array}$$

$$7.000 \text{ fur.}$$

(9)

$$\begin{array}{r} .015625 \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} .062500 \text{ pk.} \\ 2 \\ \hline \end{array}$$

$$\begin{array}{r} .125000 \text{ gal.} \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} .500000 \text{ qt.} \\ 2 \\ \hline \end{array}$$

$$1.000000 \text{ pt.}$$

(10)

$$\begin{array}{r} .9378 \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 3.7512 \text{ r.} \\ 40 \\ \hline \end{array}$$

$$\begin{array}{r} 30.0480 \text{ per.} \\ 30\frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 14400 \\ 120 \\ \hline \end{array}$$

$$\begin{array}{r} 1.4520 \text{ yd.} \\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 4.0680 \text{ ft.} \\ 144 \\ \hline \end{array}$$

$$\begin{array}{r} 2720 \\ 2720 \\ 680 \\ \hline \end{array}$$

$$9.7920 \text{ in.} = 9\frac{99}{100} \text{ in.}$$

(11)

$$1 \text{ sq. yd. 3 ft. 72 in.} = 1800 \text{ in.}$$

$$.2775 \times 1800 = 499.5 \text{ in.} =$$

$$3 \text{ ft. } 67\frac{1}{2} \text{ in.}$$

## EXERCISE 71—Page 191.

(1)

$$\dot{\cdot}8 = \frac{8}{9}.$$

$$\dot{\cdot}05 = \frac{5}{99}.$$

$$\dot{\cdot}342 = \frac{342}{999} = \frac{38}{111}.$$

$$\dot{\cdot}7004 = \frac{7004}{9999}.$$

$$\dot{\cdot}002003 = \frac{2003}{999999}.$$

(2)

$$\dot{\cdot}\dot{\cdot}19 = \frac{19}{99}.$$

$$\dot{\cdot}1067 = \frac{1067}{9999} = \frac{97}{909}.$$

$$\dot{\cdot}11115 = \frac{11115}{99999} = \frac{1235}{11111}.$$

$$\dot{\cdot}704103 = \frac{704103}{999999} = \frac{334704}{333333}.$$

(3)

$$\dot{\cdot}102 = \frac{102}{999} = \frac{34}{333}.$$

$$\dot{\cdot}0013 = \frac{13}{9999}.$$

$$\dot{\cdot}00007103 = \frac{7103}{99999999}.$$

$$\dot{\cdot}01020304 = \frac{1020304}{99999999}.$$

$$\dot{\cdot}987654321 = \frac{987654321}{999999999} = \frac{109739369}{111111111}.$$

## EXERCISE 72—Page 192.

(1)

$$\begin{array}{r} \dot{\cdot}8325 \\ 83 \\ \hline \end{array}$$

$$\frac{8325}{9900} = \frac{111}{1280}$$

$$\begin{array}{r} \dot{\cdot}147658 \\ 147 \\ \hline \end{array}$$

$$\frac{147658}{999000}$$

$$\begin{array}{r} \dot{\cdot}4320075. \\ 432 \\ \hline \end{array}$$

$$\frac{4320075}{9999000} = \frac{1139881}{3333000}.$$

(2)

$$\begin{array}{r} \dot{\cdot}875 \cdot 4965 \\ 49 \\ \hline \end{array}$$

$$\frac{8754965}{9900} = 875\frac{1225}{2475}$$

$$\begin{array}{r} \dot{\cdot}301 \cdot 82756 \\ 82 \\ \hline \end{array}$$

$$\frac{30182756}{99900} = 301\frac{9186}{1100} = 301\frac{1531}{1880}.$$



(3)

$$\begin{array}{r}
 . \\
 \cdot 083 \\
 8 \\
 \hline
 \frac{75}{900} = 1\frac{1}{2}
 \end{array}
 \qquad
 \begin{array}{r}
 . \\
 \cdot 123456 \\
 123 \\
 \hline
 \frac{123333}{999000} = \frac{41111}{333000}
 \end{array}
 \qquad
 \begin{array}{l}
 \frac{714285}{9999990} = \frac{79365}{111110} = \frac{7215}{101010} = \frac{1443}{20202} = \\
 \frac{481}{6734} = \frac{1}{14}
 \end{array}$$

(4)

$$\begin{array}{r}
 . \\
 \cdot 7034 \\
 703 \\
 \hline
 \frac{6331}{9000}
 \end{array}
 \qquad
 \begin{array}{r}
 . \\
 \cdot 96432 \\
 96 \\
 \hline
 \frac{96336}{99900} = \frac{10704}{11100} = \frac{3568}{3700} = \frac{892}{925}
 \end{array}$$

$$\begin{array}{r}
 . \\
 \cdot 00207 \\
 2 \\
 \hline
 \frac{205}{99000} = \frac{41}{19800}
 \end{array}
 \qquad
 \begin{array}{r}
 . \\
 \cdot 143271 \\
 1432 \\
 \hline
 \frac{141839}{990000}
 \end{array}$$

EXERCISE 73—Page 194.

(1)

Dissimilar.                      Similar.                      Similar and Coterminous.

$$\begin{array}{rclclcl}
 \cdot 9 & = & \cdot 99999 & = & \cdot 9999999999 \\
 6 \cdot 327 & = & 6 \cdot 327272 & = & 6 \cdot 3272727272 \\
 19 \cdot 43 & = & 19 \cdot 43000 & = & 19 \cdot 4300000000 \\
 27 \cdot 0278 & = & 27 \cdot 027878 & = & 27 \cdot 0278787878 \\
 \cdot 0347123 & = & \cdot 0347123 & = & \cdot 0347123123 \\
 & & & & 2 \text{ carried.}
 \end{array}$$

$$\text{Sum,} = 53 \cdot 8198638274$$

|                       |   | (2)               |   |                                    |
|-----------------------|---|-------------------|---|------------------------------------|
| Dissimilar.           |   | Similar.          |   | Similar and Coterminous.           |
| $7.\ddot{4}2\ddot{7}$ | = | $7.427\ddot{2}7$  | = | $7.4272\ddot{7}2727272\ddot{7}$    |
| $9.1234$              | = | $9.123423$        | = | $9.123423423423423$                |
| $17.2987643$          | = | $17.2987643$      | = | $17.298764376437643$               |
| $18.\ddot{6}7$        | = | $18.676\ddot{7}6$ | = | $18.67676\ddot{7}6767676\ddot{7}6$ |
|                       |   |                   |   | 2 carried                          |

$$\text{Sum,} = 52.526228203901471$$

(3)

| Dissimilar.   |   | Similar.           |   | Similar and Coterminous.     |
|---------------|---|--------------------|---|------------------------------|
| $4.\ddot{9}5$ | = | $4.959\ddot{5}95$  | = | $4.95959\ddot{5}959\ddot{5}$ |
| $7.164$       | = | $7.164164\ddot{1}$ | = | $7.164164164\ddot{1}$        |
| $4.7123$      | = | $4.7123123$        | = | $4.7123123123$               |
| $.97317$      | = | $.97317$           | = | $.973177777\ddot{7}$         |
|               |   |                    |   | 2 carried.                   |

$$\text{Sum,} = 17.8092502138$$

(4)

| Dissimilar. |   | Similar.      |   | Similar and Coterminous |
|-------------|---|---------------|---|-------------------------|
| $1.5$       | = | $1.5000$      | = | $1.500000000$           |
| $99.083$    | = | $99.0830$     | = | $99.083000000$          |
| $.162$      | = | $.162162$     | = | $.162162162$            |
| $.814$      | = | $.814814$     | = | $.814814814$            |
| $2.93$      | = | $2.93939$     | = | $2.939393939$           |
| $3.769230$  | = | $3.769230769$ | = | $3.769230769$           |
| $97.26$     | = | $97.2666$     | = | $97.266666666$          |
| $134.09$    | = | $134.09090$   | = | $134.090909090$         |
|             |   |               |   | 3 carried.              |

$$\text{Sum,} = 339.626177443$$

## EXERCISE 74—Page 195.

(1)

| Dissimilar.            |   | Similar.                       |   | Similar and Coterminous.       |
|------------------------|---|--------------------------------|---|--------------------------------|
| $729 \cdot 342\dot{7}$ | = | $729 \cdot 342\dot{7}4\dot{2}$ | = | $729 \cdot 342\dot{7}4\dot{2}$ |
| $93 \cdot 126$         | = | $93 \cdot 1260$                | = | $93 \cdot 126000$              |
|                        |   |                                |   | <hr/>                          |
|                        |   |                                |   | $636 \cdot 216\dot{7}4\dot{2}$ |

(2)

| Dissimilar.            |   | Similar.                       |   | Similar and Coterminous.            |
|------------------------|---|--------------------------------|---|-------------------------------------|
| $1 \cdot 43729\dot{1}$ | = | $1 \cdot 437291\dot{3}\dot{7}$ | = | $1 \cdot 43729137291\dot{3}\dot{7}$ |
| $\cdot 0071\dot{3}$    | = | $\cdot 0071\dot{3}$            | = | $\cdot 0071\dot{3}1313131\dot{3}$   |
|                        |   |                                |   | <hr/>                               |
|                        |   |                                |   | $1 \cdot 430160059782\dot{4}$       |

(3)

| Dissimilar.           |   | Similar.              |   | Similar and Coterminous.       |
|-----------------------|---|-----------------------|---|--------------------------------|
| $1 \cdot 1275\dot{4}$ | = | $1 \cdot 1275\dot{4}$ | = | $1 \cdot 1275475475475\dot{4}$ |
| $\cdot 4738\dot{4}$   | = | $\cdot 47384\dot{7}$  | = | $\cdot 4738473847384\dot{7}$   |
|                       |   |                       |   | <hr/>                          |
|                       |   |                       |   | $\cdot 6537001628090\dot{7}$   |

(4)

| Dissimilar.                 |   | Similar.                    |   | Similar and Coterminous.    |
|-----------------------------|---|-----------------------------|---|-----------------------------|
| $42 \cdot 1876\dot{3}$      | = | $42 \cdot 187633\dot{3}$    | = | $42 \cdot 187633333\dot{3}$ |
| $17 \cdot 000000843\dot{2}$ | = | $17 \cdot 000000843\dot{2}$ | = | $17 \cdot 000000843\dot{2}$ |
|                             |   |                             |   | <hr/>                       |
|                             |   |                             |   | $25 \cdot 187632490\dot{0}$ |

## EXERCISE 75—Page 196.

(1)

$$2.\dot{9} = 2\frac{9}{9} = 3. \quad 7.25 \times 3 = 21.75.$$

(2)

$$\dot{2}97 = \frac{297}{999} = \frac{1}{3} \text{ and } 7.72 = 7\frac{72}{100} = 7\frac{18}{25} = \frac{193}{25}.$$

$$\frac{1}{3} \times \frac{193}{25} = \frac{193}{75} = 2.29513.$$

(3)

$$\dot{8}18 = \frac{818}{990} = \frac{9}{11} \text{ and } .77 = \frac{77}{100}. \quad \frac{9}{11} \times \frac{77}{100} = \frac{63}{100} = .63$$

(4)

$$1.\dot{7}35 = 1\frac{735}{990} = 1\frac{164}{220} = \frac{859}{110} \text{ and } .47053 = \frac{47053}{100000} = \frac{3529}{75000}.$$

$$\frac{859}{110} \times \frac{3529}{75000} = \frac{3031411}{37125000} = .81654168350$$

(5)

$$4.\dot{7}22 = 4\frac{722}{990} = 4\frac{13}{18} = \frac{85}{9} \text{ and } .198 = \frac{198}{999} = \frac{2}{11}.$$

$$\frac{85}{9} \times \frac{2}{11} = \frac{170}{99} = .935.$$

## Exercise 76—Page 196.

(1)

$$\dot{0}82 = \frac{82}{999} \text{ and } \dot{1}23 = \frac{123}{999} = \frac{41}{333}.$$

$$\frac{82}{999} \div \frac{41}{333} = \frac{82}{999} \times \frac{333}{41} = \frac{2}{3} = .6.$$

(2)

$$389.\dot{1}85 = 389\frac{185}{999} = \frac{388796}{999} \text{ and } 15.\dot{7} = 15\frac{7}{9} = \frac{142}{9}.$$

$$\frac{388796}{999} \div \frac{142}{9} = \frac{388796}{999} \times \frac{9}{142} = \frac{2738}{111} = 24.\dot{6}.$$

(3)

$$.81654168350 = \frac{81654168350}{99999999999} = \frac{10206760837}{12499987500}.$$

$$.47053 = \frac{47053}{100000} = \frac{19587}{22500}.$$

$$\frac{10206760837}{12499987500} \div \frac{19587}{22500} = \frac{10206760837}{12499987500} \times \frac{22500}{19587} = \frac{10206760837}{681680780} =$$

$$1.735.$$

(4)

$$\cdot\ddot{45} = \frac{45}{99} = \frac{5}{11} \text{ and } \cdot\dot{118881} = \frac{118881}{999999} = \frac{17}{143}.$$

$$\frac{5}{11} \div \frac{17}{143} = \frac{5}{11} \times \frac{143}{17} = \frac{65}{17} = 3.8235294117647058.$$

## EXERCISE 77.

(1)

$$\frac{1}{2} \text{ of } \frac{3}{7} \text{ of } \frac{4}{5} \text{ of } 14 = \frac{1}{2} \times \frac{3}{7} \times \frac{4}{5} \times 14 = \frac{4}{5} = .8.$$

(2)

$$\cdot\dot{67} = \frac{67}{90} \text{ and } 2\cdot\ddot{13} = 2\frac{13}{99} = \frac{211}{99}.$$

$$\frac{67}{90} \times \frac{211}{99} = \frac{12877}{8910} = 1.4445566778 +$$

(3)

wk.

$$\cdot 678125 = 4 \text{ days } 17 \text{ hours } 55 \text{ minutes } 30 \text{ seconds.}$$

7

$$4.746875 \text{ days.}$$

24

$$2987500$$

$$1493750$$

$$17.925000 \text{ hours.}$$

60

$$55.500000 \text{ minutes.}$$

60

$$30.000000 \text{ seconds.}$$

(4)

..

$$\cdot 92437$$

92

$$\cdot 92437 = \frac{92437}{999999} = \frac{18488}{199999}.$$

(5)

|             |  |          |  |                          |
|-------------|--|----------|--|--------------------------|
| Dissimilar. |  | Similar. |  | Similar and Coterminous. |
|-------------|--|----------|--|--------------------------|

$$67\cdot2\ddot{3}\ddot{4} = 67\cdot23434\ddot{3}\ddot{4} = 67\cdot234343434\ddot{3}\ddot{4}$$

$$98\cdot7\ddot{1}\ddot{3} = 98\cdot71371\ddot{3}\ddot{7}\ddot{1} = 98\cdot7137137137\ddot{1}\ddot{3}$$

$$91\cdot0347123\ddot{4} = 91\cdot0347123\ddot{4} = \underline{\underline{91\cdot0347123423\ddot{4}}}$$

$$\text{Sum,} = 256\cdot9827694903\ddot{9}$$

|             |  |                          |
|-------------|--|--------------------------|
| Dissimilar. |  | Similar and Coterminous. |
|-------------|--|--------------------------|

$$256\cdot9827694903\ddot{9} = 256\cdot9827694903\ddot{9}$$

$$100\cdot12345678\ddot{9} = \underline{\underline{100\cdot1234567894\ddot{5}}}$$

$$\text{Difference} = 156\cdot8593127009\ddot{4}$$

(6)

$$12) 9 \text{ in.}$$

$$3) 2\cdot75 \text{ ft.}$$

$$5\frac{1}{2}) 2\cdot916 \text{ yds.}$$

$$\begin{array}{r} 2 \quad \quad 2 \\ \hline \end{array}$$

$$11) 5\cdot833$$

$$40) 36\cdot5303 \text{ rds.}$$

$$8) 5\cdot913257 \text{ fur.}$$

$$\underline{\underline{\cdot739157196 \text{ miles.}}}$$

(7)

$$17\cdot42857\ddot{1} \text{ sq. ft.} = 17\frac{3}{7}\frac{8571}{9999} \text{ sq. ft.} = 17\frac{3}{7} \text{ sq. ft.} = 17 \text{ sq. ft. } 61\frac{1}{2} \text{ in}$$

$$100\cdot8 \text{ sq. in.} = \underline{\underline{100\frac{4}{5}}}$$

$$\text{Difference,} = 16 \text{ sq. ft. } 104\frac{2}{3} \text{ in.}$$

(8)

$$\begin{array}{r} \cdot 91789772 \\ 917897 \\ \hline \end{array}$$

$$\cdot 91789772 \text{ of } 2 \text{ a.} = \frac{99871875}{99000000} \times \frac{2}{1} \text{ a.} = \frac{99871875}{49500000} = \frac{3231}{1760} = 1 \text{ a. } 3 \text{ r. } 13 \text{ per. } 22 \text{ yds.}$$

(9)

$$\begin{array}{r} 11 \cdot 287 \\ 2 \\ \hline \end{array}$$

$$11 \cdot 287 = \frac{11287}{990} = 11\frac{19}{66}. \quad 1 \cdot 0428571 = \frac{1428571}{9999990} = 1\frac{3}{70}.$$

(10)

$$47 \cdot 345 = \frac{47345}{1000} \text{ and } 1 \cdot 76 = 1\frac{19}{25} = \frac{176}{100}.$$

$$\frac{47345}{1000} \div \frac{176}{100} = \frac{47345}{1000} \times \frac{100}{176} = \frac{937431}{35000} = 26 \cdot 7837428571.$$

(11)

Dissimilar.                      Similar.      Similar and Coterminous.

$$85 \cdot 62 = 85 \cdot 626 = 85 \cdot 62626$$

$$13 \cdot 76432 = 13 \cdot 76432 = \underline{13 \cdot 76432}$$

$$\text{Difference,} = 71 \cdot 86193$$

(12)

(13)

$$\cdot 734 \text{ of a lb.} = 11 \cdot 744 \text{ oz.} \quad 2 \text{ ft. } 5\frac{1}{2} \text{ in.} = 29\frac{1}{2} \text{ in.} = \frac{59}{2} \text{ in.}$$

$$\cdot 198 \text{ of an oz.} = \cdot 198 \text{ oz.}$$

$$27 \cdot 3 \text{ ft.} = 27\frac{3}{4} \text{ ft.} = 328 \text{ in.}$$

$$\text{Difference,} = 11 \cdot 546 \text{ oz.}$$

$$20 \cdot 16 \text{ ft.} = 20\frac{1}{2} \text{ ft.} = 242 \text{ in.}$$

$$328 \times 242 \div \frac{59}{2} = 3\frac{7}{8} \times 2\frac{1}{2} \times \frac{3}{8} = 2706 \text{ in.} = 75\frac{1}{8} \text{ yds.}$$

(14)

$$3 \cdot 145 = 3\frac{145}{1000} = 3\frac{29}{200} = \frac{173}{88} \text{ and } 4 \cdot 297 = 4\frac{297}{1000} = 4\frac{11}{37} = \frac{159}{37}.$$

$$\frac{173}{88} \times \frac{159}{37} = \frac{27507}{2636} = 13 \cdot 5169533.$$

(15)

$\frac{3}{40}$ . Here  $40 = 2^3 \times 5$ . Therefore the equivalent decimal will contain 3 places.

|                               |   |                          |   |   |   |   |   |   |   |
|-------------------------------|---|--------------------------|---|---|---|---|---|---|---|
| $\frac{7}{24}$ .              | " | $24 = 2^3 \times 3$ .    | " | " | " | " | " | 3 | " |
| $\frac{8}{15}$ .              | " | $15 = 5 \times 3$ .      | " | " | " | " | " | 1 | " |
| $\frac{14}{44}$ .             | " | $144 = 2^4 \times 3^2$ . | " | " | " | " | " | 4 | " |
| $\frac{6}{90} = \frac{1}{15}$ | " | $15 = 5 \times 3$ .      | " | " | " | " | " | 1 | " |
| $\frac{119}{3584}$ .          | " | $3584 = 2^9 \times 7$ .  | " | " | " | " | " | 9 | " |

(16)

| Dissimilar.         |     | Similar                      |     | Similar and Coterminous                                      |
|---------------------|-----|------------------------------|-----|--|
| $81\frac{2}{3}$     | $=$ | $81\cdot\dot{6}$             | $=$ | $81\cdot\dot{6}\dot{6}\dot{6}\dot{6}\dot{6}\dot{6}$          |
| $61\cdot12\dot{6}$  | $=$ | $61\cdot12\dot{6}$           | $=$ | $61\cdot12\dot{6}\dot{6}\dot{6}\dot{6}\dot{6}\dot{6}$        |
| $328\cdot2\ddot{3}$ | $=$ | $328\cdot23\ddot{2}\ddot{3}$ | $=$ | $328\cdot23\ddot{2}\ddot{3}\ddot{2}\ddot{3}\ddot{2}\ddot{3}$ |
| $5\cdot62\dot{4}$   | $=$ | $5\cdot624\dot{6}\dot{2}$    | $=$ | $5\cdot624624\dot{6}\dot{2}$                                 |
|                     |     |                              |     | 2 carried  |
|                     |     |                              |     |  |
|                     |     | Sum,                         | $=$ | $476\cdot6502811\dot{9}$                                     |

(17)

$$\left( \frac{4\cdot4 - 2\cdot8\dot{3}}{1\cdot6 + 2\cdot62\dot{9}} \times \frac{6\cdot8 \times 3}{2\cdot25} \right) + \frac{2\cdot8 \times 2\cdot2\ddot{7}}{1\cdot13\ddot{6}}$$

$$= \left( \frac{1\cdot61}{4\cdot29\dot{6}} \times \frac{20\cdot4}{2\cdot25} \right) + \frac{2\frac{1}{2} \times 2\frac{2}{3}}{1\frac{1}{3}\frac{5}{6}}$$

$$= \left( \frac{1\frac{5}{6}}{4\frac{2}{3}\frac{9}{6}} \times \frac{20\frac{2}{3}}{2\frac{1}{4}} \right) + \frac{2\frac{1}{2} \times 2\frac{3}{4}}{1\frac{3}{2}\frac{5}{2}}$$

$$= \left( \frac{1\frac{1}{8}}{4\frac{2}{3}\frac{9}{6}} \times \frac{10\cdot2}{\frac{9}{4}} \right) + \frac{1\frac{1}{2} \times 2\frac{5}{4}}{2\frac{5}{2}}$$

$$= \left( \frac{1\frac{9}{8}}{4\frac{2}{3}\frac{9}{6}} \times \frac{34}{\frac{3}{4}} \right) + \frac{1\frac{1}{2} \times 1\frac{5}{4}}{2\frac{5}{2}}$$

$$= \left( \frac{\frac{1}{2}}{1\frac{1}{3}} \times \frac{34}{\frac{3}{4}} \right) + \frac{70}{2\frac{5}{2}} = \left( \frac{1\frac{1}{3}}{2\frac{5}{2}} \times \frac{136}{16} \right) + \frac{28}{5}$$

$$= \left( \frac{2}{5} \times \frac{136}{16} \right) + \frac{28}{5} = \frac{17}{1} + \frac{28}{5} = \frac{45}{5} = 9.$$



## EXERCISE 78—Page 198.

(1)

$$\begin{array}{r} \text{v} \\ 9)4312131 \\ \hline \end{array}$$

$$\begin{array}{r} 9)224322..8 \\ \hline \end{array}$$

$$\begin{array}{r} 9)12043..5 \\ \hline \end{array}$$

$$\begin{array}{r} 9)344..7 \\ \hline \end{array}$$

$$\begin{array}{r} 9)21..0 \\ \hline \end{array}$$

$$\begin{array}{r} 1..2 \\ \hline \end{array}$$

$$\begin{array}{r} \text{v} \\ 3)4312131 \\ \hline \end{array}$$

$$\begin{array}{r} 3)1234023..2 \\ \hline \end{array}$$

$$\begin{array}{r} 3)224322..2 \\ \hline \end{array}$$

$$\begin{array}{r} 3)41240..2 \\ \hline \end{array}$$

$$\begin{array}{r} 3)12043..1 \\ \hline \end{array}$$

$$\begin{array}{r} 3)2144..1 \\ \hline \end{array}$$

$$\begin{array}{r} 3)344..2 \\ \hline \end{array}$$

$$\begin{array}{r} 3)113..0 \\ \hline \end{array}$$

$$\begin{array}{r} 3)21..0 \\ \hline \end{array}$$

$$\begin{array}{r} 3)3..2 \\ \hline \end{array}$$

$$\begin{array}{r} 1..0 \\ \hline \end{array}$$

$$\begin{array}{r} \text{v} \\ 8)4312131 \\ \hline \end{array}$$

$$\begin{array}{r} 8)242343..7 \\ \hline \end{array}$$

$$\begin{array}{r} 8)14022..2 \\ \hline \end{array}$$

$$\begin{array}{r} 8)1032..1 \\ \hline \end{array}$$

$$\begin{array}{r} 8)32..6 \\ \hline \end{array}$$

$$\begin{array}{r} 2..1 \\ \hline \end{array}$$

v

ix

iii

viii

$$4312131 =$$

$$\begin{array}{r} 5 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \hline \end{array}$$

$$\begin{array}{r} 116 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \hline \end{array}$$

$$\begin{array}{r} 582 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \hline \end{array}$$

$$\begin{array}{r} 2911 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \hline \end{array}$$

$$\begin{array}{r} 14558 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \hline \end{array}$$

$$\begin{array}{r} 72791 \text{ decimal.} \\ \hline \end{array}$$

$$120758 =$$

$$\begin{array}{r} 9 \\ \hline \end{array}$$

$$\begin{array}{r} 11 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \hline \end{array}$$

$$\begin{array}{r} 99 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \hline \end{array}$$

$$\begin{array}{r} 898 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \hline \end{array}$$

$$\begin{array}{r} 8087 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \hline \end{array}$$

$$\begin{array}{r} 72791 \text{ dec.} \\ \hline \end{array}$$

$$10200211222 =$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$\begin{array}{r} 11 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$\begin{array}{r} 33 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$\begin{array}{r} 99 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$\begin{array}{r} 299 \\ \hline \end{array}$$

$$\begin{array}{r} 299 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$\begin{array}{r} 898 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$\begin{array}{r} 2695 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$\begin{array}{r} 8087 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$\begin{array}{r} 24263 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \hline \end{array}$$

$$\begin{array}{r} 72791 \text{ dec.} \\ \hline \end{array}$$

$$216127$$

$$\begin{array}{r} 8 \\ \hline \end{array}$$

$$\begin{array}{r} 17 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \hline \end{array}$$

$$\begin{array}{r} 142 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \hline \end{array}$$

$$\begin{array}{r} 1137 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \hline \end{array}$$

$$\begin{array}{r} 9098 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \hline \end{array}$$

$$\begin{array}{r} 72791 \text{ dec.} \\ \hline \end{array}$$

(3)

$$976.432 \div .00000096 = 97643200000 \div 96 \text{ and } 96 = 12 \times 8.$$

$$12)97643200000$$

$$\begin{array}{r} 8)8136933333.3 \\ \hline \end{array}$$

$$1017116666.6$$

(4)

$$(2\frac{7}{8} + .5625 - 1.5 + \frac{1}{16}) \div \frac{11}{8}$$

$$\frac{(1\frac{8}{11} \times \frac{1}{9} \times 296 \times \frac{1}{101} \div \frac{1}{8}) \div .9472947}{\frac{19}{6}} =$$

$$\frac{(2\frac{7}{8} + \frac{9}{16} - 1\frac{1}{2} + \frac{1}{16}) \times \frac{8}{11}}{\frac{1}{1} \times \frac{8}{11}} =$$

$$\frac{(\frac{19}{11} \times \frac{1}{9} \times \frac{296}{1} \times \frac{1}{101} \times \frac{8}{11}) \div \frac{9472}{9999}}{\frac{19}{11} \times \frac{1}{9} \times \frac{296}{1} \times \frac{1}{101} \times \frac{8}{11} \times \frac{9999}{9472}} =$$

$$\frac{\frac{\frac{19}{11}}{\frac{19}{11}}}{\frac{19}{6}} = \frac{\frac{16}{11}}{\frac{6}{11}} = \frac{16}{6} = 2\frac{2}{3}.$$

(5)

| lbs.  | oz. | dr. | scr. | lbs.   | oz.   | dr.             | scr.             | grs. |
|-------|-----|-----|------|--------|-------|-----------------|------------------|------|
| 9     | 7   | 7   | 2    | )      | 97    | 3               | 4                | 1    |
| 12    |     |     |      |        | 12    |                 |                  | 17   |
| <hr/> |     |     |      |        | <hr/> |                 |                  |      |
| 115   |     |     |      |        | 1167  |                 |                  |      |
| 8     |     |     |      |        | 8     |                 |                  |      |
| <hr/> |     |     |      |        | <hr/> |                 |                  |      |
| 927   |     |     |      |        | 9340  |                 |                  |      |
| 3     |     |     |      |        | 3     |                 |                  |      |
| <hr/> |     |     |      |        | <hr/> |                 |                  |      |
| 2783  |     |     |      |        | 28021 |                 |                  |      |
| 20    |     |     |      |        | 20    |                 |                  |      |
| <hr/> |     |     |      |        | <hr/> |                 |                  |      |
| 55660 | )   |     |      | 560437 | (10   | <sup>3837</sup> | <sub>55660</sub> |      |
|       |     |     |      | 55660  |       |                 |                  |      |
|       |     |     |      | <hr/>  |       |                 |                  |      |
|       |     |     |      | 3837   |       |                 |                  |      |

(6)

$$15 \text{ yds.} = 540 \text{ in. and } 7 \text{ ft.} = 84 \text{ in.}$$

$$6 \text{ ft.} = 72 \text{ in. and } 4 \text{ ft.} = 48 \text{ in.}$$

$$(540 \times 84 \times 13) - (72 \times 48 \times 13) = 589680 - 44928 = 544752.$$

$$544752 \div 108 = 5044.$$

(7)

$$\begin{array}{r} 9 \text{ ft. } 6' \quad 4'' \quad 7''' \\ 11 \quad 7 \quad 9 \quad 11 \\ \hline \quad \quad \quad 8 \quad 8 \quad 10'''' \quad 2'''' \quad 5'''' \\ \quad \quad 7 \quad 1 \quad 9 \quad 5 \quad 3 \\ \quad 5 \quad 6 \quad 8 \quad 8 \quad 1 \\ 104 \quad 10 \quad 2 \quad 5 \\ \hline 111 \quad 0 \quad 9 \quad 7 \quad 4 \quad 5 \quad 5 \end{array}$$

(8)

$$\frac{4\frac{2}{7} + \frac{8}{9} - \frac{7}{12}}{\frac{3}{4} \text{ of } \frac{8}{13} + \frac{1}{6} \text{ of } \frac{5}{9}} = \frac{4\frac{7}{13} - \frac{7}{12}}{\frac{6}{13} + \frac{5}{14}} = \frac{11\frac{5}{13}}{2\frac{5}{13}} = \frac{11\frac{5}{13}}{1\frac{1}{13}} = \frac{451\frac{2}{13}}{54\frac{6}{13}} = 8\frac{55\frac{5}{13}}{44\frac{6}{13}}.$$

(9)

$$\begin{array}{r} \text{pts.} \\ 2) 782436 \\ \hline 4) 391218 \dots 0 \text{ pt.} \\ \hline 2) 97804 \dots 2 \text{ qt.} \\ \hline 4) 48902 \dots 0 \text{ gal.} \\ \hline 12225 \dots 2 \text{ pks.} \\ 12225 \text{ bush. } 2 \text{ pks. } 0 \text{ gal. } 2 \text{ qts.} \end{array}$$

(10)

$$\begin{array}{r} 77 \times 27 \dots 42 \dots 27 \dots 21 \dots 33 \dots 14 \dots 7 \dots 11 \dots 63 \dots 30 \\ 27 \quad \quad \quad 6 \dots 27 \dots 3 \dots 3 \dots 2 \quad \quad \quad 3 \dots 30 \\ 10 \quad \quad \quad 2 \quad \quad \quad 2 \quad \quad \quad 10 \\ 77 \times 27 \times 10 = 20790 = 1 \text{ c. m.} \end{array}$$

(11)

XII

IX

28e4)36t87942(1375t·12

3762814

28e49

9e47

34

82t09

18679

312

172749

14054

2810

118889

23882

25298

235549

32t·0

227683

28e·49

5t·80

2049151

55·t8

4·94

(12)

$$150528 = 2^{10} \times 3 \times 7^2.$$

$$10+1=11$$

$$1+1=2$$

$$2+1=3$$

$$11 \times 3 \times 2 = 66.$$

(13)

(14)

2 wks. 2 dys. = 16 dys.

$$728\frac{1}{2} = 8\frac{1}{2} + 2 \times 10 + 7 \times 10 \times 10.$$

·1234625

lbs. oz. dr.

lbs. oz. dr.

16

$$27 \ 4 \ 3 \times 8\frac{1}{2} =$$

$$231 \ 11 \ 9\frac{1}{2}$$

10

7407750

1234625

$$272 \ 9 \ 14 \times 2 =$$

$$545 \ 3 \ 12$$

10

1·9754000 dys.

24

$$2726 \ 2 \ 12 \times 7 =$$

$$19083 \ 3 \ 4$$

39016000

$$19860 \ 2 \ 9\frac{1}{2}$$

19508000

23·4096000 hrs.

60

24·5760000 min.

60

$$34·5600000 = 34 \frac{1}{2} \text{ sec.}$$

(15)

$$\begin{aligned} £16 \text{ 3s. } 8\frac{1}{2}\text{d.} &= \$64.74\frac{7}{12} \text{ and } £67 \text{ 17s. } 7\frac{1}{2}\text{d.} = \$271.52\frac{1}{2}. \\ \$98.17 + \$42.29 + \$64.74\frac{7}{12} + \$97.19 + \$127.87\frac{1}{2} &= \$430.27\frac{1}{2} \\ \$430.27\frac{1}{2} - \$271.52\frac{1}{2} &= \$158.75. \end{aligned}$$

(16)

$$\begin{aligned} .8 &= \frac{8}{10}. & .76 &= \frac{76}{100}. & .9123 &= \frac{9123-91}{9900} = \frac{9032}{9900} = \frac{2258}{2475}. \\ .003327 &= \frac{3327-3}{999000} = \frac{3324}{999000} = \frac{277}{83250}. \end{aligned}$$

(18)

$$\begin{aligned} & \frac{[(2\frac{1}{3} \times .5 \text{ of } 1\frac{1}{2}) + 9\frac{1}{2} + .09 + \frac{23}{31}] - 11\frac{6}{7}}{(\frac{1}{5} \text{ of } .16)} \\ & \frac{[(.7632763 \times 11) \times \frac{1}{8} \text{ of } \frac{101}{106}] \times (\frac{1}{2} \text{ of } .2 \text{ of } .3 \text{ of } .25 \text{ of } 96) \div .2}{\frac{1}{4} \text{ of } .6732467 \div \frac{1}{9}} \\ & \frac{[(\frac{7}{3} \times \frac{1}{2} \times 1\frac{1}{2}) + 9\frac{1}{2} + 1\frac{1}{7} + \frac{23}{31}] - 11\frac{6}{7}}{(\frac{1}{5} \text{ of } \frac{1}{6})} \\ & \frac{(\frac{7632}{9999} \times \frac{1}{1} \times \frac{1}{8} \times \frac{101}{106} \times \frac{1}{2} \times \frac{1}{6} \times \frac{1}{3} \times \frac{1}{4} \times \frac{25}{1}) \div \frac{2}{9}}{\frac{1}{4} \times \frac{67324}{99999} \div \frac{1}{9}} \\ & \frac{\{(2 + 9\frac{1}{2} + 1\frac{1}{7} + \frac{23}{31}) - 11\frac{6}{7}\} \div (\frac{1}{5} \times \frac{1}{6})}{\frac{7632}{9999} \times \frac{1}{1} \times \frac{1}{8} \times \frac{101}{106} \times \frac{1}{2} \times \frac{1}{6} \times \frac{1}{3} \times \frac{1}{4} \times \frac{25}{1} \times \frac{9}{2}} \\ & \frac{1}{4} \times \frac{67324}{99999} \times \frac{9}{1} \\ & \frac{(12 - 11\frac{6}{7}) \div (\frac{1}{5} \times \frac{1}{6})}{\frac{18}{6}} = \frac{\frac{1}{7} \times \frac{5}{1} \times \frac{6}{1}}{\frac{18}{6}} = \frac{\frac{18}{6}}{\frac{18}{6}} = \frac{\frac{5}{1}}{\frac{18}{6}} = \\ & \frac{16831}{11111} = \frac{16831}{11111} = \frac{16831}{11111} = \frac{16831}{11111} = \\ & \frac{5555}{16831} = 3\frac{5062}{16831}. \end{aligned}$$

(19)

8 children will have 8 children's shares.

One woman will have 3 children's shares  $\therefore$  6 women will have

$$6 \times 3 = 18 \text{ children's shares.}$$

One man will have 6 children's shares  $\therefore$  4 men will have

$$4 \times 6 = 24 \text{ children's shares.}$$

4 men, 6 women, and 8 child. will therefore have 50 child. shares.

$$£550 \text{ 3s. } 1\frac{1}{2}\text{d.} \div 50 = £11 \text{ 0s. } 0\frac{3}{4}\text{d.} = \text{child's share.}$$

$$£11 \text{ 0s. } 0\frac{3}{4}\text{d.} \times 3 = £33 \text{ 0s. } 2\frac{1}{4}\text{d.} = \text{woman's share.}$$

$$£33 \text{ 0s. } 2\frac{1}{4}\text{d.} \times 2 = £66 \text{ 0s. } 4\frac{1}{2}\text{d.} = \text{man's share.}$$

(20)

$$16\frac{7}{11} + 19\frac{1}{2} + 23\frac{7}{8} + 129\frac{6}{7} = 16 + 19 + 23 + 129 + \\ (\frac{7}{11} + \frac{1}{2} + \frac{7}{8} + \frac{6}{7}) = 187 + 3\frac{519}{3080} = 190\frac{519}{3080}.$$

(21)

$$8100 = 2^3 \times 3^4 \times 5^2.$$

1..3..9..27..81

1..2..4

1..3..9..27..81..2..6..18..54..162..4..12..36..108..324

1..5..25

1..3..9..27..81..2..6..18..54..162..4..12..36..108..324..

5..15..45..135..405..10..30..90..270..810..20..60..180..

540..1620..25..75..225..675..2025..50..150..450..1350..

4050..100..300..900..2700..8100.

Therefore the divisors of 8100 are 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 25, 27, 30, 36, 45, 50, 54, 60, 75, 81, 90, 100, 108, 135, 150, 162, 180, 225, 270, 300, 324, 405, 450, 540, 675, 810, 900, 1350, 1620, 2025, 2700, 4050, 8100.

(22)

$$\begin{array}{r} 2691 \overline{)11817(4} \\ 10764 \\ \hline 1053 \end{array}$$

$$\begin{array}{r} 2691(2 \\ 2106 \\ \hline 585 \end{array}$$

$$\begin{array}{r} 1053(1 \\ 585 \\ \hline 468 \end{array}$$

$$\begin{array}{r} 585(1 \\ 468 \\ \hline 117 \end{array}$$

$$\begin{array}{r} 468(4 \\ 468 \end{array}$$

9828 is divisible by 117  $\therefore$  117 is the G. C. M.

(23)

$$\begin{array}{r} \text{sec.} \\ 60 \overline{)2551443} \end{array}$$

$$60 \overline{)42524.. 3}$$

$$24 \overline{)708..44}$$

$$29..12$$

$$29\text{d.}, 12\text{ h.}, 44\text{ m.}, 3\text{ sec.}$$

$$\begin{array}{r} \text{sec.} \\ 60 \overline{)31556928} \end{array}$$

$$60 \overline{)525948..48}$$

$$24 \overline{)8765..48}$$

$$365.. 5$$

$$365\text{ d.}, 5\text{ h.}, 48\text{ m.}, 48\text{ sec.}$$

(24)

$$14\text{ ft. } 11\text{ in.} = 179\text{ in.}$$

$$38\text{ miles} = 2407680\text{ in.}$$

$$2407680 \div 179 = 13450\frac{1}{2}$$

(25)

$$11\text{ ft.} \times 13\text{ ft.} \times 15\text{ ft.} = 2145\text{ cub. ft.}$$

$$\text{One cubic foot weighs } 62\frac{1}{2}\text{ lbs. } 2145 \times 62\frac{1}{2} = 134062\frac{1}{2} = \text{weight of } 2145\text{ cub. ft.}$$

$$\text{One gallon weighs } 10\text{ lbs. } 134062\frac{1}{2} \div 10 = 13406\frac{1}{4} = \text{gals. in } 134062\frac{1}{2}\text{ lbs.}$$

(26)

$$\pounds 73 \times 400 = \$292.00$$

$$17\text{s.} \times 20 = 3.40$$

$$11\frac{1}{4}\text{d.} = 47\text{ far.} \times 5 \div 12 = 19\frac{7}{12}$$

$$\pounds 73 \quad 17\text{s.} \quad 11\frac{1}{4}\text{d.} = \$295.59\frac{7}{12}$$

(27)

$$93\frac{1}{4} - 76\frac{1}{2} = 92\frac{1}{4} - 76\frac{1}{2} = 16\frac{1}{4}$$

$$\frac{4206}{253} \div \frac{17}{253} = \frac{4206}{253} \times \frac{253}{17} = \frac{4206}{17} = 247\frac{7}{17}$$

(28)

$$\frac{5\frac{5}{8} \div \frac{2}{3}}{1\frac{1}{5} \text{ of } \frac{5}{9} \div 10\frac{1}{3}} \times \frac{\frac{3}{8} \text{ of } \frac{1\frac{1}{2} \text{ of } 4\frac{1}{9}}{13\frac{7}{8} \text{ of } 5\frac{1}{3}}}{\frac{4\frac{5}{16} \times 3}{1 \times \frac{1}{31}}} = \frac{4\frac{5}{8} \times \frac{3}{2}}{\frac{6 \times 5 \times 31}{2 \times 3}} \times \frac{3}{8} \times \frac{\frac{3}{2} \times 3\frac{7}{9}}{1\frac{1}{8} \times 1\frac{1}{3}} =$$

$$\frac{2 \times \frac{1}{31}}{3\frac{7}{8} \times 2} =$$

$$\frac{5 \times 3 \times 31}{16 \times 2} \times \frac{3}{5} \times \frac{37}{2 \times 3 \times 37 \times 2} = \frac{3 \times 9 \times 31}{16 \times 2 \times 2 \times 2} =$$

$$\frac{837}{128} = 6\frac{69}{128}.$$

(29)

$$\begin{array}{r} \text{XI} \\ 5)91342 \\ \hline 5)19074..4 \\ \hline 5)4015..1 \\ \hline 5)891..0 \\ \hline 5)184..3 \\ \hline 5)39..3 \\ \hline 5)8..2 \\ \hline 1..3 \end{array}$$

$$\begin{array}{r} \text{XI} \\ 12)91342 \\ \hline 12)834..9 \\ \hline 12)773..1 \\ \hline 12)70..3 \\ \hline 6..5 \end{array}$$

$$\begin{array}{r} \text{XI} \\ 2)91342 \\ \hline 2)46176..1 \\ \hline 2)23093..0 \\ \hline 2)11541..1 \\ \hline 2)6246..0 \\ \hline 2)3153..0 \\ \hline 2)1627..0 \\ \hline 2)869..0 \\ \hline 2)434..1 \\ \hline 2)217..1 \\ \hline 2)109..0 \\ \hline 2)54..0 \\ \hline 2)27..1 \\ \hline 2)15..0 \\ \hline 2)8..0 \\ \hline 2)4..0 \\ \hline 2)2..0 \\ \hline 1..0 \end{array}$$



(29 continued.)

| XI          | V           | XII         | II                 |             |
|-------------|-------------|-------------|--------------------|-------------|
| 91342       | 13233014    | 65319       | 100000100110000101 |             |
| <u>11</u>   | <u>5</u>    | <u>12</u>   | <u>2</u>           |             |
| 100         | 8           | 77          | 2                  | 260         |
| <u>11</u>   | <u>5</u>    | <u>12</u>   | <u>2</u>           | <u>2</u>    |
| 1103        | 42          | 927         | 4                  | 521         |
| <u>11</u>   | <u>5</u>    | <u>12</u>   | <u>2</u>           | <u>2</u>    |
| 12137       | 213         | 11125       | 8                  | 1043        |
| <u>11</u>   | <u>5</u>    | <u>12</u>   | <u>2</u>           | <u>2</u>    |
| 133509 dec. | 1068        | 133509 dec. | 16                 | 2086        |
|             | <u>5</u>    |             | <u>2</u>           | <u>2</u>    |
|             | 5340        |             | 32                 | 4172        |
|             | <u>5</u>    |             | <u>2</u>           | <u>2</u>    |
|             | 26701       |             | 65                 | 8344        |
|             | <u>5</u>    |             | <u>2</u>           | <u>2</u>    |
|             | 133509 dec. |             | 130                | 16688       |
|             |             |             | <u>2</u>           | <u>2</u>    |
|             |             |             | 200                | 33377       |
|             |             |             |                    | <u>2</u>    |
|             |             |             |                    | 66754       |
|             |             |             |                    | <u>3</u>    |
|             |             |             |                    | 133509 dec. |

|                                  |                     |  |
|----------------------------------|---------------------|--|
| (30)                             | (31)                | (32)   |
| 2)7680 = $2^9 \times 3 \times 5$ | m. f. p. y. ft. in. |  |
| <u>2)3840</u>                    | 72 3 7 2 1 7        |  |
| <u>2)1920</u>                    | 8                   | $\$47 \times 97 = \$45.59.$                                    |
| <u>2)960</u>                     | 579 fur.            | (33)   |
| <u>2)480</u>                     | 40                  |  |
| <u>2)240</u>                     | 23167 per.          | $(73 \times 4 \times 11) \div 128 = 25\frac{3}{2}.$            |
| <u>2)120</u>                     | 5½                  | $\$3.62\frac{1}{2} \times 25\frac{3}{2} = \$90.96\frac{3}{4}.$ |
| <u>2)60</u>                      | 115837              |  |
| <u>2)30</u>                      | 11583½              |  |
| <u>3)15</u>                      | 127420½ yds.        |  |
| <u>5</u>                         | 3                   |  |
|                                  | 382262½ ft.         |  |
|                                  | 12                  |  |
|                                  | 4587157 in.         |  |
|                                  | 12                  |  |
|                                  | 55045884 lines      |  |

(34)

$$93\cdot723 = 93\frac{716}{990} = 93\frac{2786}{990} \text{ and } 29\cdot4173 = 29\frac{4169}{990} = 29\frac{3879}{990}.$$

$$\frac{93\frac{2786}{990}}{29\frac{3879}{990}} = \frac{92786}{990} \times \frac{111}{293879} = \frac{92786 \times 111}{11 \times 293879} =$$

$$\frac{10299246}{3232669} = 3.185988 +$$

(35)

One bushel of oats weighs 34 lbs. ∴ in 73429 lbs. there are  $73429 \div 34 = 2159\frac{3}{4}$  bushels.

(36)

In 719630 lbs. of wheat there are  $719630 \div 60 = 11993\frac{5}{6}$  bus.

$$\$1.80 \times 11993\frac{5}{6} = \$21588.90,$$

Or  $\$1.80$  per bushel = 3 cents per lb.

$$719630 \times 3 = 2158890 \text{ cents.} = \$21588.90.$$

(38)

21389)180781(8

171112

(37)

9669)21389(2

19338

$$\$72.14 + \$93.76 = \$165.90$$

$$\$165.90 \times 9.47 = \$1571.0730$$

$$\$1571.0730 \div 11 = \$142.8248+$$

2051)9669(4

8204

1465)2051(1

1465

586)1465(2

1172

293)586(2

Last divisor 293 = G. C. M.

(39)

 $\frac{7}{11}, \frac{1}{5}, \frac{2}{7}, \frac{3}{33}, \frac{11}{14}, \frac{7}{10}, \frac{1}{2}.$ 

The least common multiple of 11, 5, 7, 33, 14, 10 and 2 is 2310.

The multiplier for both terms of the first fraction is  $\frac{2310}{11} = 210$ ; for the second,  $\frac{2310}{5} = 462$ ; for the third,  $\frac{2310}{7} = 330$ ; for the fourth,  $\frac{2310}{33} = 70$ ; for the fifth,  $\frac{2310}{14} = 165$ ; for the sixth,  $\frac{2310}{10} = 231$ ; for the seventh,  $\frac{2310}{2} = 1155$ .

Multiplying by these numbers, we obtain  $\frac{1470}{2310}, \frac{1848}{2310}, \frac{2970}{2310}, \frac{560}{2310}, \frac{1815}{2310}, \frac{1617}{2310}$ , and  $\frac{1155}{2310}$  for the required fractions.

(40)

$$\begin{aligned} \$11 \times 17 &= \$1.87. \quad \$37\frac{1}{2} \times 19 = \$7.12\frac{1}{2}. \quad \$2.17 \times 14\frac{1}{2} = \\ & \$31.46\frac{1}{2}. \quad \$27 \times 67 = \$18.09. \quad \$1.37\frac{1}{2} \times 15 = \$20.62\frac{1}{2}. \\ \$1.87 + \$7.12\frac{1}{2} + \$31.46\frac{1}{2} + \$4.75 + \$11.50 + \$18.09 + \\ & \$20.62\frac{1}{2} + \$7.93 = \$103.35\frac{1}{2}. \end{aligned}$$

## EXERCISE 84—Page 210.

(1)

$$\begin{array}{r} \text{Baskets.} \quad 1 \cdot 22 \\ 11 : 87 :: \$13 \cdot 42 : \frac{\$13 \cdot 42 \times 87}{11} = \$106 \cdot 14. \end{array}$$

(2)

$$\begin{array}{r} \text{Cords.} \quad 19 \\ 28 : 25 :: \$266 : \frac{\$266 \times 25}{28} = \$237 \cdot 50. \end{array}$$

(3)

$$\begin{array}{r} \text{days} \quad 4 \\ \$29 \cdot 20 : \$83 \cdot 60 :: 16 : \frac{16 \times 83 \cdot 60}{7 \cdot 3} = 45\frac{2}{3} \text{ days.} \end{array}$$

(4)

$$\begin{array}{r} \text{Bags.} \quad \cdot 8 \\ 16 : 156 :: \$12 \cdot 80 : \frac{\$12 \cdot 80 \times 156}{16} = \$124 \cdot 80. \end{array}$$

(5)

$$\begin{array}{r} \text{Feet.} \quad \text{ft.} \quad 7 \times 112 \\ 5 : 112 :: 7 : \frac{7 \times 112}{5} = 156\frac{4}{5} \text{ ft.} \end{array}$$

(6)

$$\begin{array}{r} \text{Cows.} \quad \text{days.} \quad 9 \\ 55 : 27 :: 99 : \frac{99 \times 27}{55} = 48\frac{3}{5} \text{ days.} \end{array}$$

(7)

$$\begin{array}{r} \text{Acres.} \quad \text{bus.} \quad 9 \times 48 \\ 5 : 48 :: 9 : \frac{9 \times 48}{5} = 86\frac{2}{5} \text{ bush.} \end{array}$$

(8)

$$\begin{array}{r} \text{Perches. days. } 2 \times 803 \\ 73 : 803 :: 2 : \frac{11}{73} \times 803 = 22 \text{ days.} \end{array}$$

(9)

$$\begin{array}{r} \text{Pails. lbs. } 100 \times 1128 \\ 176 : 1128 :: 100 : \frac{141}{176} \times 1128 = 640 \frac{10}{11} \text{ lbs.} \end{array}$$

(10)

$$\begin{array}{r} 108 : 465 :: \$20.88 : \frac{.58 \quad 155}{\$20.88 \times 465} = \$ 89.90. \end{array}$$

(11)

$$\begin{array}{r} \$ \quad \$ \quad \text{brls. } 72 \times 1278 \\ 16 : 1278 :: 72 : \frac{9 \quad 639}{72 \times 1278} = 5751 \text{ barrels.} \end{array}$$

(12)

$$\begin{array}{r} \text{Men. Acres } 165 \times 3 \\ 11 : 3 :: 165 : \frac{15}{11} = 45 \text{ acres.} \end{array}$$

(13)

$$\begin{array}{r} \text{Barrels. loaves } 250 \times 67 \\ 4 : 67 :: 250 : \frac{125}{4} = 4187 \frac{1}{2} \text{ loaves.} \end{array}$$

(14)

$$\begin{array}{r} \text{Bushels. brls. } 16 \times 38 \\ 190 : 38 :: 16 : \frac{190}{5} = 3 \frac{1}{2} \text{ barrels.} \end{array}$$

(15)

$$\begin{array}{rcl} & 6 & \\ \text{Days.} & \text{men} & 90 \times 12 \\ 15 : 12 :: 90 : \frac{\quad}{15} & = & 72 \text{ men} \end{array}$$

(16)

$$\begin{array}{rcl} \text{D'. work. brls.} & 2 \times 279 & \\ 17 : 279 :: 2 : \frac{\quad}{17} & = & 32\frac{1}{17} \text{ barrels.} \end{array}$$

(17)

$$\begin{array}{rcl} \text{Hours.} & \text{miles.} & \\ 1 : 24 :: 27 : 27 \times 24 & = & 648 \text{ miles.} \end{array}$$

(18)

$$\begin{array}{rcl} \text{Cows.} & \text{lbs.} & 30 \times 23 \\ 7 : 23 :: 30 : \frac{\quad}{7} & = & 98\frac{4}{7} \text{ lbs.} \end{array}$$

## EXERCISE 85—Page 211.

(1)

$$\frac{3}{16} : \frac{21}{16} :: \$9750 : \frac{375}{1} \times \frac{7}{21} \times \frac{16}{8} = \$42000.$$

(2)

$$\begin{array}{rcl} \text{Yard.} & \text{s.} & 5 \\ \frac{7}{8} : \frac{1}{4} :: \frac{5}{6} : \frac{1}{6} \times \frac{1}{4} \times \frac{2}{7} & = & \frac{5}{21} = 2\frac{1}{2} \text{ d.} \end{array}$$

(3)

$$\begin{array}{rcl} \text{Tons.} & & \\ \frac{7}{9} : 8\frac{1}{3} :: \$7.49 : \frac{\$7.49 \times 8\frac{1}{3}}{7} = \frac{1.07}{1} \times \frac{25}{8} \times \frac{3}{7} = \$80.25. \end{array}$$

(4)

$$\begin{array}{rcl} \text{Yards.} & & \\ 5\frac{1}{2} : \frac{4}{7} :: \$28.42 : \frac{.14}{1} \times \frac{4}{7} \times \frac{5}{29} = \$2.80. \end{array}$$

(5)

$$\begin{array}{c} \text{Dollar.} \quad \text{bag} \quad \frac{4}{5} \quad 7 \quad \frac{25}{20} \\ \frac{1}{2} : \frac{7}{20} :: \frac{4}{5} : \frac{7}{20} \times \frac{25}{20} \times \frac{25}{12} = \frac{7}{12} \text{ of a bag.} \end{array}$$

(6)

$$\begin{array}{c} \$ \quad \$ \quad \$ \\ 100 : 472\frac{1}{2} :: 98\frac{7}{8} : \frac{98\frac{7}{8} \times 472\frac{1}{2}}{100} = \frac{98 \cdot 875 \times 472 \cdot 44}{100} = \$467 \cdot 12\frac{1}{2}. \end{array}$$

(7)

$$\begin{array}{c} \text{Tons.} \quad \text{days.} \quad 107\frac{3}{4} \times 11\frac{1}{4} \quad \frac{295}{1180} \quad \frac{9}{198} \quad \frac{5}{88} \\ 17\frac{3}{8} : 11\frac{1}{4} :: 107\frac{3}{4} : \frac{107\frac{3}{4} \times 11\frac{1}{4}}{17\frac{3}{8}} = \frac{1180}{11} \times \frac{198}{17} \times \frac{5}{88} = 70\frac{1}{8} \text{ dys.} \end{array}$$

(8)

$$\begin{array}{c} \text{Tons.} \quad \text{cords.} \quad 22\frac{1}{2} \times 11\frac{27}{26} \quad \frac{202}{9} \quad \frac{295}{26} \quad \frac{13}{202} \\ 15\frac{7}{8} : 11\frac{9}{26} :: 22\frac{1}{2} : \frac{22\frac{1}{2} \times 11\frac{27}{26}}{15\frac{7}{8}} = \frac{202}{9} \times \frac{295}{26} \times \frac{13}{202} = 16\frac{7}{8} \text{ cords.} \end{array}$$

(9)

$$\begin{array}{c} \text{Yds.} \quad \text{yds} \quad \$ \\ \frac{1}{2} \text{ of } \frac{3}{8} \text{ of } 3\frac{1}{2} : \frac{3}{8} \text{ of } \frac{1}{2} \text{ of } \frac{55}{66} :: \frac{7}{8} \text{ of } \frac{3}{11} \text{ of } 4\frac{2}{3} : \frac{\frac{7}{8} \text{ of } \frac{3}{11} \text{ of } 4\frac{2}{3} \times \frac{3}{8} \text{ of } \frac{1}{2} \text{ of } \frac{55}{66}}{\frac{1}{2} \text{ of } \frac{3}{8} \text{ of } 3\frac{1}{2}} = \end{array}$$

$$\begin{array}{c} \frac{4}{11} \times \frac{15}{165} = \$2\frac{1}{4}. \\ \frac{4}{11} \times \frac{165}{896} = \\ \frac{4}{11} \times \frac{224}{224} = \end{array}$$

## EXERCISE 86—Page 212.

(1)

$$37 \text{ sq. yds. } 4 \text{ ft. } 120 \text{ in.} = 48648 \text{ in., and } 9 \text{ sq. yds. } 2 \text{ ft.} = 11952 \text{ in.}$$

$$\begin{array}{c} \text{Inches.} \\ 11952 : 48648 :: \$3 \cdot 50 : \frac{3 \cdot 50 \times 48648}{11952} = \$14 \cdot 245 + \\ \frac{3 \cdot 50 \times 48648}{11952} \\ \frac{1494}{498} \end{array}$$

(2)

$$12 \text{ lbs. } 10 \text{ oz.} = 154 \text{ oz.}$$

Ounces.

$$1 : 154 :: \$1.25 : 1.25 \times 154 = \$192.50.$$

(3)

$$10 \text{ yds.} = 40 \text{ qrs., and } 3 \text{ yds. } 2 \text{ qrs.} = 14 \text{ qrs.}$$

$$\begin{array}{r} .17 \quad 7 \end{array}$$

Quarters.

$$\$ .40 \times 14$$

$$40 : 14 :: \$3.40 : \frac{\quad}{\quad} = \$1.19.$$

$$40$$

$$20$$

(4)

$$15 \text{ oz. } 12 \text{ dwt. } 16 \text{ grs.} = 7504 \text{ grs., and } 13 \text{ oz. } 14 \text{ grs.} = 6254 \text{ grs.}$$

$$\begin{array}{r} .95 \quad 3127 \end{array}$$

Grains.

$$\$ .80 \times 6254$$

$$7504 : 6254 :: \$3.80 : \frac{\quad}{\quad} = \$3.167 +$$

$$7504$$

$$1876$$

$$938$$

(5)

$$3 \text{ lbs. } 1 \text{ oz. } 11 \text{ dwt.} = 751 \text{ dwt. and } 12 \text{ lbs. } 6 \text{ oz. } 4 \text{ dwt.} = 3004 \text{ dwt.}$$

$$150$$

Dwt.

$$\$ \quad 600 \times 751$$

$$3004 : 751 :: 600 : \frac{\quad}{\quad} = \$150.$$

$$3004$$

$$4$$

(6)

$$\text{Barrels.} \quad \text{h. m. s.} \quad 2 \text{ h. } 46 \text{ m. } 39 \text{ s.} \times 24$$

$$54 : 24 :: 2 \text{ } 46 \text{ } 30 : \frac{\quad}{\quad} = 1 \text{ hr. } 14 \text{ min.}$$

$$54$$

$$9$$



(7)

73 yds. 3 qrs. 2 na. 1 in = 2660½ in. 3 Fl. e. 2 qrs. 1 na. = 101¼ in.  
 And £4 17s. 8¼d. = 1172¼d.

$$\begin{array}{r} \text{Inches.} \quad \text{d.} \quad 1172\frac{1}{4} \times 2660\frac{1}{2} \quad \overset{521}{\cancel{4689}} \quad 5321 \quad \times \quad \frac{4}{405} = \\ 101\frac{1}{4} : 2660\frac{1}{2} :: 1172\frac{1}{4} : \frac{1172\frac{1}{4} \times 2660\frac{1}{2}}{101\frac{1}{4}} = \frac{1172\frac{1}{4} \times 2660\frac{1}{2}}{101\frac{1}{4}} \times \frac{4}{405} = \\ 2772\frac{1}{2} \text{ d.} = £128 \text{ 6s. } 10\frac{1}{2}\text{d.} \end{array}$$

(8)

$$\begin{array}{r} 8\frac{1}{2} \text{ lbs.} = 136\frac{2}{3} \text{ oz.} \\ \text{Ounces.} \quad \text{s.} \quad 287 \quad 410 \quad 9 \quad \text{s.} \\ 49 : 136\frac{2}{3} :: 8\frac{31}{32} : \frac{287}{32} \times \frac{410}{8} \times \frac{9}{41} = \frac{287 \times 410 \times 9}{32 \times 8} = 13 \text{ 9s. } 0\frac{1}{2}\text{d.} \end{array}$$

(9)

$$\begin{array}{r} \text{Pages.} \quad \overset{52}{156} \times 400 \\ 327 : 400 :: 156 : \frac{156 \times 400}{327} = 190\frac{90}{109}, \text{ i. e. on the 191st p.} \end{array}$$

(10)

$$\begin{array}{r} 46 \text{ a., } 3 \text{ r., } 14 \text{ p.} = 7494 \text{ p., and } 35 \text{ a., } 2 \text{ r., } 10 \text{ p.} = 5690 \text{ p.} \\ \text{Perches.} \quad \text{£} \quad \overset{50}{100} \times 5690 \\ 7494 : 5690 :: 100 : \frac{100 \times 5690}{7494} = £75 \text{ 18s. } 6\frac{3}{4}\text{d.} \end{array}$$

(11)

$$\begin{array}{r} \text{Days.} \quad \text{miles.} \quad \overset{17}{12} \times 68 \\ 48 : 68 :: 12 : \frac{12 \times 68}{48} = 17 \text{ miles per day.} \end{array}$$

(12)

$$\begin{array}{rcl} \text{Shillings.} & \text{lbs.} & \\ 21\frac{1}{2} : 32\frac{3}{4} :: 16\frac{1}{2} : \frac{113}{7} \times \frac{113}{226} \times \frac{3}{84} = \frac{38307}{1568} = 24\frac{625}{1568} \text{ lbs} \end{array}$$

(13)

$$17493 \times 1000 \times 5 \text{ cub. ft.} = 87465000 \text{ cub. ft.}$$

$$192724 \times 1000 \times 4 \text{ cub. ft.} = 770896000 \text{ cub. ft.}$$

$$87465000 + 770896000 = 858361000 \text{ cub. ft.}$$

$$\begin{array}{rcl} \text{Cubic feet.} & \text{ton.} & \\ 9000 : 858361000 :: 1 : \frac{858361000}{9000} = 95373\frac{1}{3} \text{ tons.} \end{array}$$

(14)

$$50000 \times 9000 = 450000000 = \text{cub. ft. of gas in 50000 tons of coal}$$

$$\text{Cubic feet.} \quad \text{hour.}$$

$$4 : 450000000 :: 1 : \frac{450000000}{4} = 112500000 \text{ h.} = 12842 \text{ y. } 170 \text{ d.}$$

(15)

$$\text{lbs. lbs. lbs. lb. lb.}$$

$$4 + 3 + 2 + 1 + \frac{1}{2} = 10\frac{1}{2} \text{ lbs.}$$

$$\begin{array}{rcl} \text{lbs.} & & \\ 10\frac{1}{2} : 11270 :: 1 : \frac{11270}{10\frac{1}{2}} = 1073, \text{ and } 3\frac{1}{2} \text{ lbs. remaining.} \end{array}$$

(16)

$$180 \text{ miles} = 180 \times 1760 = 316800 \text{ yards.}$$

$$\text{Yards.} \quad \text{day.}$$

$$100 : 316800 :: 1 : \frac{316800 \times 1}{100} = 3168 \text{ dys. or about } 8\frac{2}{3} \text{ yrs.}$$

## EXERCISE 87—Page 216.

(1)

$$\left. \begin{array}{l} 120 : 90 \text{ bush.} \\ 6 : 14 \text{ horses.} \end{array} \right\} :: 56 \text{ days} : \frac{7 \ 15}{56 \times 90 \times 14} = 7 \times 14 = 98 \text{ days.}$$

$$\frac{120 \times 6}{8}$$

(2)

$$\left. \begin{array}{l} 28 : 32 \text{ ft. high.} \\ 8 : 15 \text{ days.} \end{array} \right\} :: 63 \text{ men} : \frac{9 \ 4}{63 \times 32 \times 15} = 9 \times 15 = 135 \text{ men.}$$

$$\frac{8 \times 28}{7}$$

(3)

$$\left. \begin{array}{l} 3 : 45 \text{ length.} \\ 1\frac{1}{4} : 1 \text{ width.} \end{array} \right\} :: 1 \text{ lb.} : \frac{45}{3 \times 1\frac{1}{4}} = \frac{45}{\frac{15}{4}} = \frac{3}{15} \times 4 = 3 \times 4 = 12 \text{ lbs.}$$

(4)

$$\left. \begin{array}{l} 10 : 100 \text{ length.} \\ 1\frac{1}{2} : 1\frac{1}{4} \text{ width.} \end{array} \right\} :: 3 \text{ lbs.} : \frac{3 \times 1\frac{1}{4} \times 100}{1\frac{1}{2} \times 10} = 2 \times 1\frac{1}{4} \times 10 = 25 \text{ lbs.}$$

(5)

$$\left. \begin{array}{l} 44 : 132 \text{ tons.} \\ 18 : 5 \text{ days.} \end{array} \right\} :: 12 \text{ horses} : \frac{2 \ 3}{12 \times 5 \times 182} = 2 \times 5 = 10 \text{ horses.}$$

$$\frac{44 \times 18}{6}$$

(6)

$$\left. \begin{array}{l} 4 : 14 \text{ men.} \\ 7 : 10 \text{ days.} \end{array} \right\} :: 27s. : \frac{2 \ 5}{27 \times 14 \times 10} = 27 \times 5 = 135s. = £6 \ 15s.$$

$$\frac{4 \times 7}{2}$$

(7)

$$\left. \begin{array}{l} 3:5 \text{ masters.} \\ 8:10 \text{ apprentices.} \\ 5:8 \text{ weeks} \\ 6:5\frac{1}{2} \text{ days per wk.} \end{array} \right\} :: \$144 : \frac{8}{24} \frac{144 \times 5\frac{1}{2} \times 8 \times 10 \times 5}{8 \times 8 \times 5 \times 6} = \$440.$$

(8)

$$\left. \begin{array}{l} 6 : 18 \text{ s.mak.} \\ 4 : 5 \text{ weeks.} \end{array} \right\} :: 36 \text{ pairs of men's shoes} : \frac{9}{88} \frac{3}{18} \times 18 \times 5 =$$

$$135 \text{ pairs men's and the women's} = \frac{2}{3} \frac{1}{6} = \frac{2}{3} \text{ of } 135 = 90 \text{ pairs.}$$

(9)

$$\left. \begin{array}{l} 9 : 18 \text{ feet high.} \\ 4 : 6 \text{ days.} \end{array} \right\} :: 12 \text{ men} : \frac{3}{12} \frac{2}{18} \times 18 \times 6 = 3 \times 2 \times 6 = 36 \text{ men.}$$

(10)

$$\left. \begin{array}{l} 130 : 390 \text{ miles.} \\ 7 : 14 \text{ hours.} \end{array} \right\} :: 3 \text{ days} : \frac{2}{3} \frac{3}{14} \times 14 \times 390 = 3 \times 2 \times 3 = 18 \text{ days.}$$

(11)

$$\left. \begin{array}{l} 10 : 60 \text{ oz.} \\ 22\frac{1}{2} : 30 \text{ d.} \end{array} \right\} :: 1 \text{ d.} : \frac{60 \times 30}{10 \times 22\frac{1}{2}} = \frac{80}{1} \times \frac{80}{1} \times \frac{1}{10} \times \frac{2}{45} = 4 \times 2 = 8 \text{ d.}$$

(12)

$$\left. \begin{array}{l} 10 : 5 \text{ composers} \\ 7 : 14 \text{ hours.} \\ 20 : 40 \text{ sheets.} \\ 24 : 16 \text{ pages.} \\ 50 : 60 \text{ lines.} \\ 40 : 50 \text{ letters.} \end{array} \right\} :: 16 \text{ days} : \frac{2}{16} \times \frac{2}{14} \times \frac{2}{40} \times \frac{1}{24} \times \frac{2}{60} \times \frac{2}{50} =$$

$$\frac{10 \times 7 \times 20 \times 24 \times 50 \times 40}{5 \times 8} = 2 \times 16 = 32 \text{ days.}$$

(13)

$$\begin{array}{l}
 336 : 240 \text{ men.} \\
 5 : 9 \text{ days.} \\
 10 : 12 \text{ hours.} \\
 6 : 5 \text{ degrees.} \\
 5 : 3 \text{ yards wide} \\
 3 : 2 \text{ yards deep}
 \end{array}
 \left. \vphantom{\begin{array}{l} 336 : 240 \text{ men.} \\ 5 : 9 \text{ days.} \\ 10 : 12 \text{ hours.} \\ 6 : 5 \text{ degrees.} \\ 5 : 3 \text{ yards wide} \\ 3 : 2 \text{ yards deep} \end{array}} \right\} :: 70 \text{ yards : } \frac{\overset{7}{70} \times \overset{5}{240} \times \overset{2}{9} \times 12 \times 5 \times 3 \times 2}{\underset{48}{336} \times 5 \times 10 \times 6 \times 5 \times 3} =$$

$$9 \times 2 \times 2 = 36 \text{ yards.}$$

(14)

$$\begin{array}{l}
 6 : 12 \text{ horses.} \\
 4 : 9 \text{ months.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 6 : 12 \text{ horses.} \\ 4 : 9 \text{ months.} \end{array}} \right\} :: 16 \text{ acres : } \frac{\overset{4}{16} \times \overset{2}{12} \times 9}{\underset{6 \times 4}{6} \times 4} = 4 \times 2 \times 9 = 72 \text{ acres.}$$

(15)

$$\begin{array}{l}
 25 : 139 \text{ persons} \\
 1 : 7 \text{ years.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 25 : 139 \text{ persons} \\ 1 : 7 \text{ years.} \end{array}} \right\} :: 300 \text{ bush. : } \frac{\overset{12}{300} \times 139 \times 7}{\underset{25}{25}} = 11676 \text{ bushels.}$$

(16)

$$\begin{array}{l}
 48 : 32 \text{ men.} \\
 36 : 864 \text{ feet long.} \\
 8 : 5 \text{ feet high.} \\
 4 : 3 \text{ feet wide.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 48 : 32 \text{ men.} \\ 36 : 864 \text{ feet long.} \\ 8 : 5 \text{ feet high.} \\ 4 : 3 \text{ feet wide.} \end{array}} \right\} :: 4 \text{ days : } \frac{\overset{3}{4} \times \overset{2}{864} \times \overset{108}{8} \times 5 \times 3}{\underset{16}{48} \times 36 \times 8 \times 4} = 30 \text{ days.}$$

(17)

$$\begin{array}{l}
 679 : 22407 \text{ sold's.} \\
 336 : 112 \text{ days.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 679 : 22407 \text{ sold's.} \\ 336 : 112 \text{ days.} \end{array}} \right\} :: 702 \text{ bushels : } \frac{\overset{234}{702} \times \overset{33}{22407} \times 112}{\underset{3}{679} \times 336} =$$

$$234 \times 33 = 7722 \text{ bushels.}$$

(18)

$$\begin{array}{l}
 13 : 494 \text{ suits.} \\
 19 : 27 \text{ days.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 13 : 494 \text{ suits.} \\ 19 : 27 \text{ days.} \end{array}} \right\} :: 12 \text{ tailors : } \frac{\overset{2}{12} \times \overset{26}{494} \times 27}{\underset{19 \times 19}{19} \times 19} = 648 \text{ tailors.}$$

(19)

$$\begin{array}{l}
 17:40 \text{ head of cattle} \\
 30:51 \text{ days.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 17:40 \\ 30:51 \end{array}} \right\} :: 5 \text{ a. } 2 \text{ r. } 10 \text{ p.} : \frac{5 \text{ a. } 2 \text{ r. } 10 \text{ p.} \times 4 \times 51}{17 \times 30} =$$

$$5 \text{ a. } 2 \text{ r. } 10 \text{ p.} \times 4 = 22 \text{ a. } 1 \text{ r.}$$

(20)

$$\begin{array}{l}
 20 : 100 \text{ ft. long} \\
 6 : 4 \text{ feet wide.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 20 \\ 6 \end{array}} \right\} :: 180 \text{ bricks} : \frac{180 \times 100 \times 4}{20 \times 6} =$$

$$30 \times 5 \times 4 = 600 \text{ bricks.}$$

## EXERCISE 88.—Page 21

(1)

$$\begin{array}{ll}
 7 \text{ cords} & = 116 \text{ lbs.} \\
 87 \text{ lbs.} & = 23 \text{ barrels} \\
 19 \text{ barrels} & = 34 \text{ days' work} \\
 92 \text{ days' work} & = 57 \text{ baskets peaches} \\
 31 \text{ baskets peaches} & = 24 \text{ dollars} \\
 12 \text{ dollars} & = 2 \text{ tons} \\
 35 \text{ tons} & = x \text{ cords}
 \end{array}
 \left. \vphantom{\begin{array}{l} 7 \\ 87 \\ 19 \\ 92 \\ 31 \\ 12 \\ 35 \end{array}} \right\} =$$

$$\frac{7 \times 87 \times 19 \times 92 \times 31 \times 12 \times 35}{116 \times 23 \times 34 \times 57 \times 24 \times 2} = \frac{31 \times 35}{2 \times 2 \times 2} = \frac{1085}{8} = 135\frac{5}{8}.$$

(2)

$$\begin{array}{ll}
 6 \text{ lbs. tea} & = 29 \text{ lbs. sugar} \\
 17 \text{ lbs. sugar} & = 1 \text{ bushel} \\
 27 \text{ bushels} & = 4 \text{ tons} \\
 34 \text{ tons} & = 15 \text{ cows} \\
 29 \text{ cows} & = 1160 \text{ dollars} \\
 20 \text{ dollars} & = x \text{ lbs. tea.}
 \end{array}
 \left. \vphantom{\begin{array}{l} 6 \\ 17 \\ 27 \\ 34 \\ 29 \\ 20 \end{array}} \right\} =$$

$$\frac{6 \times 17 \times 27 \times 34 \times 29 \times 20}{29 \times 1 \times 4 \times 15 \times 1160} = \frac{17 \times 17 \times 27}{5 \times 58} = \frac{7803}{290} = 26\frac{243}{290}$$

(3)

|                 |   |                   |   |   |
|-----------------|---|-------------------|---|---|
| 11 bush. barley | = | 21 bush. potatoes | } | = |
| 19 " potatoes   | = | 29 " oats         |   |   |
| 115 " oats      | = | 44 " wheat        |   |   |
| 14½ " wheat     | = | 38 " peas         |   |   |
| 60 " peas       | = | 55 " rye          |   |   |
| 75 " rye        | = | 11½ " clover sd.  |   |   |
| 36 " clover sd. | = | x " barley        | } |   |

$$\frac{11 \times 19 \times 115 \times 14\frac{1}{2} \times 60 \times 75 \times 36}{21 \times 29 \times 44 \times 38 \times 55 \times 11\frac{1}{2}} = \frac{5 \times 75 \times 18}{7 \times 11} = \frac{6750}{77} = 87\frac{1}{2}$$

(4)

|                  |   |                  |   |   |
|------------------|---|------------------|---|---|
| 16 baskets pears | = | 29 turkeys       | } | = |
| 17 turkeys       | = | 7 days' work     |   |   |
| 7½ days' work    | = | 187 loaves       |   |   |
| 3½ loaves        | = | 4 lbs. veal      |   |   |
| 1 lb. veal       | = | 11 cents         |   |   |
| 792 cents        | = | 63 lbs. sugar    |   |   |
| x lbs. sugar     | = | 21 baskets pears | } |   |

$$\frac{29 \times 7 \times 187 \times 4 \times 11 \times 63 \times 21}{16 \times 17 \times 7\frac{1}{2} \times 3\frac{1}{2} \times 1 \times 792} = \frac{11 \times 7 \times 21}{4} = \frac{1617}{4} = 404\frac{1}{4}$$

(5)

|             |   |   |   |
|-------------|---|---|---|
| 7 A = 11 B  | } | = | $\frac{7 \times 5 \times 15 \times 11 \times 42}{11 \times 8 \times 21 \times 5} = \frac{7 \times 15}{4} = \frac{105}{4} = 26\frac{1}{4}$ |
| 5 B = 8 C   |   |   |   |
| 15 C = 21 D |   |   |   |
| 11 D = 5 E  |   |   |   |
| 42 E = x A  |   |   |   |

(6)

|                 |                     |
|-----------------|---------------------|
| 7 barrels flour | = 23 cords          |
| 6 cords         | = 11 cwt.           |
| 46 cwt.         | = £28               |
| £77             | = 9 sheep           |
| 5 sheep         | = 8 tons            |
| 9 tons          | = $x$ barrels flour |

$$\left. \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right\} =$$

$$\frac{3 \quad 2 \quad 7}{7 \times 6 \times 46 \times 77 \times 5 \times 9} = \frac{3 \times 7 \times 5}{28 \times 11 \times 28 \times 9 \times 8} = \frac{105}{8} = 13\frac{1}{8}.$$

(7)

|               |                  |
|---------------|------------------|
| 15 N. England | = 20 New York    |
| 24 New York   | = 22½ N. Jersey  |
| 30 New Jersey | = 20 Canada      |
| 4807½ Canada  | = $x$ N. England |

$$\left. \begin{array}{l} \\ \\ \\ \end{array} \right\} =$$

$$\frac{2 \quad 6 \quad 8 \quad 961\frac{1}{2}}{15 \times 24 \times 30 \times 4807\frac{1}{2}} = 961\frac{1}{2} \times 6 = 5769 \text{ s.} = £288 \text{ 9s.}$$

## EXERCISE 89.—Page 222.

(1)

$$\frac{7}{8} \times \frac{17}{11} \times \frac{28}{29} \times \frac{319}{119} \times \frac{2}{69} = \frac{2}{3} = 2 : 3.$$

(2)

$$\begin{array}{rcl} £119 \times 400 & = & \$476.00 \\ 16\text{s.} \times 20 & = & 3.20 \\ 6\frac{1}{2}\text{d.} = 26 \text{ far.} \times 5 \div 12 & = & .10\frac{5}{6} \\ \hline £119 \text{ 16s. } 6\frac{1}{2}\text{d.} & = & \$479.30\frac{5}{6} \end{array}$$



(4)

$$\left. \begin{array}{l} 9 : 13 = 9 \div 13 = .692 \\ 21 : 27 = 21 \div 27 = .777 \\ 7 : 10 = 7 \div 10 = .7 \\ 11 : 15 = 11 \div 15 = .733 \end{array} \right\} \text{Hence } 21 : 27 \text{ is the greatest,} \\ \text{and } 9 : 13 \text{ the least.}$$

(5)

Dissimilar.                  Similar.                  Similar and Coterminous.

$$\begin{array}{rclcl} 76 \cdot 23478 & = & 76 \cdot 234784 & = & 76 \cdot 234784784784784 \\ 19 \cdot 1342291 & = & 19 \cdot 1342291 & = & \underline{19 \cdot 134229122912291} \\ & & \text{Difference,} & = & 57 \cdot 100555661872493 \end{array}$$

(6)

71324<sub>t</sub> undenary = 1146287 denary, 23421 quinary = 1736 denary, and 4e7 duodenary = 17995 denary.

$1146287 \times 1736 = 1989954232 \div 17995 = 110583\frac{13147}{17995}$ .  
 $110583\frac{13147}{17995}$  denary\* =  $53ee3\frac{7737}{447}$  duodenary,  $12014313\frac{410042}{1033440}$   
 quinary, and  $76070\frac{3872}{1256\frac{1}{2}}$  undenary.

(7)

$$\begin{array}{rcl} 5.63 : 7.9 \text{ cubic inches.} & \left. \vphantom{\begin{array}{l} 5.63 : 7.9 \text{ cubic inches.} \\ 1 : 1.220 \text{ spec. grav.} \end{array}} \right\} & \text{oz.} \\ 1 : 1.220 \text{ spec. grav.} & & \text{3.254} \\ \hline & & \text{3.254} : \frac{3.254 \times 7.9 \times 1.220}{5.63} = \\ & & \frac{31.362052}{5.63} = 5.57052 \text{ oz. Ans.} \end{array}$$

\* To reduce the fractional part, reduce both numerator and denominator separately.

$$\begin{array}{cccccc} & & & (8) & & \\ \text{yds. qrs. na. in.} & \text{yds. qrs. na. in.} & & & & \\ 17)63 & 3 & 2 & 1 & (3 & 3 & 0 & 0\frac{1}{4} & (9) \end{array}$$

51

—

12

4

—

51

51

(10)

—

0

4

—

2

2½

—

$$5\frac{1}{2} = \frac{1}{2} \div 17 = \frac{1}{34}.$$

(12)

$$\text{Whole amount of increase} = 2571437 - 1842265 = 729172.$$

$$1842265 : 100 :: 729172 : \frac{729172 \times 100}{1842265} = 39 \text{ per cent.}$$

(13)

$$\frac{1}{4} \text{ of } \frac{2}{3} \text{ of } \frac{18}{29} - \frac{1}{8} \text{ of } \frac{2}{3} \text{ of } \frac{5}{7} = \frac{6}{29} - \frac{5}{84} = \frac{359}{2436}.$$

(14)

$$100 : 7 :: 11 : \frac{11 \times 7}{100} = \frac{77}{100}. \quad 11 - \frac{77}{100} = 10\frac{23}{100}.$$

(15)

$$79 \times 16 \times £.00163 = £2.06032 = £2 \text{ 1s. } 2\frac{2}{3}\text{d.}$$

(16)

$$\left. \begin{array}{l} 4:3 \text{ men} \\ 10:12 \text{ hours} \\ 20:35 \text{ acres} \end{array} \right\} :: 2\frac{1}{2} \text{ days} : \frac{2\frac{1}{2} \times 3 \times 12 \times 35}{4 \times 10 \times 20} = \frac{3}{16} = 3\frac{1}{16} \text{ days.}$$

(17)

$$\left(\frac{1}{2} \text{ of } \frac{9}{11} \times .02 \times .456\right) \div \left(\frac{16}{17} \text{ of } \frac{2}{3} \text{ of } \frac{1}{4} \text{ of } 51\right) =$$

$$\frac{\frac{2}{4} \times \frac{9}{11} \times \frac{2}{50} \times \frac{456}{37}}{\frac{16}{51} \times \frac{2}{11} \times \frac{1}{25} \times \frac{1}{37}} = \frac{2 \times 38}{5 \times 11 \times 25 \times 37} = \frac{76}{50875}.$$

(18)

$$\frac{2}{1} \times \frac{4}{7} \times \frac{13}{5} \times \frac{7}{2} \times \frac{5}{1} = 4 \times 13 = 52.$$

(19)

$$\begin{aligned} 50 \text{ barrels} &= 125 \text{ yards} \\ 80 \text{ yards} &= 6 \text{ bales,} \\ 13 \text{ bales} &= 3\frac{1}{2} \text{ hogsheads} \\ x \text{ hogsheads} &= 1000 \text{ barrels} \end{aligned} =$$

$$\frac{5 \times 3 \times 125}{125 \times 6 \times 3\frac{1}{2} \times 1000} = \frac{125 \times 3 \times 3\frac{1}{2}}{50 \times 80 \times 13} = 50\frac{25}{2}.$$

(20)

$$\frac{73 \cdot 47 \times .0063 \div 17 \cdot 2345}{\frac{7347}{100} \times \frac{63}{10000} \times \frac{3330}{57391}} = \frac{7347}{100} \times \frac{63}{10000} \div \frac{57391}{3330} = \frac{154132713}{8739100000} = .026856599989+$$

(21)

$$2 \text{ roods } 7 \text{ per. } 4 \text{ yds. } 3 \text{ ft. } 117 \text{ in.} = 3416481 \text{ in. and } 7 \text{ acres} = 43908480 \text{ inches.}$$

$$3416481 \div 43908480 = .0778+$$

(22)

$\frac{7}{8}$  of  $\frac{1}{2}$  of  $\frac{1}{3}$  of 70 miles =  $\frac{1}{3}$  miles = 5.33333+ miles.

$\cdot 73$  of 11 fur. = 8.03 fur. = 1.00375 mile.

$5.33333 - 1.00375 = 4.32958$  miles.

(23)

274312 nonary = 167195 denary, 1101011010 = 858 denary, and  
 .5555 septenary = 2000 denary.

$167195 - 858 = 166337 \times 2000 = 332674000$ .

332674000 denary = 764876837 nonary.

= 10011110101000011001111010000 binary,

= 11146453021 septenary.

(24)

$$\begin{array}{r|l} 275 & 44..275..18..190..200..225 \\ 38 & 4 \quad 18.. 38.. 19.. 9 \\ 18 & 2 \quad 9 \quad 9 \end{array}$$

$$275 \times 38 \times 18 = 188100 = 1. \text{ c. m.}$$

(25)

|                     |   |  |
|---------------------|---|--|
| 10:6 weeks          | } |  |
| 6:5 days            |   |  |
| 11:10 hours         |   |  |
| 2400:8742 feet long |   |  |
| 18:20 feet wide     |   |  |
| 11:8 feet high      |   | $\begin{array}{r} \text{men } 60 \times 6 \times 5 \times 10 \times 8742 \times 20 \times 8 \\ \text{:: } 60: \frac{10 \times 6 \times 11 \times 2400 \times 18 \times 11}{240 \quad 3} = \end{array}$ |

$$\frac{5 \times 2914 \times 2}{11 \times 3 \times 11} = \frac{2914^2}{363} = 80\frac{109}{363}.$$

(26)

$172000 = 2^5 \times 5^3 \times 43$ . Increasing each exponent by 1 and multiplying them together we obtain  $6 \times 4 \times 2 = 48$ .

(27)

$$42 \frac{7}{9} = 42\frac{7}{9} = \frac{385}{9} \text{ and } 9 \cdot 7\dot{1}2\dot{3} = 9\frac{7123}{999} = 9\frac{1186}{1665} = \frac{16171}{1665}.$$

$$\frac{385}{9} \times \frac{16171}{1665} = \frac{6225835}{14985} = 415 \cdot 471137804.$$

(28)

$$100 : 27 :: \$73 \cdot 42 : \frac{73 \cdot 42 \times 27}{100} = \$19 \cdot 8234.$$

$$\$73 \cdot 42 - \$19 \cdot 8234 = \$53 \cdot 5966.$$

(29)

$$6300 = 2^2 \times 3^2 \times 5^2 \times 7.$$

1..5..25

1..2.. 4

1..5..25..2..10..50..4..20..100

1..3.. 9

1..5..25..2..10..50..4..20..100..3..15..75..6..30..150..

12..60..300..9..45..225..18..90..450..36..180..900

1..7

1..5..25..2..10..50..4..20..100..3..15..75..6..30..150..

12..60..300..9..45..225..18..90..450..36..180..900..7..

35..175..14..70..350..28..140..700..21..105..525..42..

210..1050..84..420..2100..63..315..1575..126..630..3150

..252..1260..6300.

Therefore the divisors of 6300 are 1, 2, 3, 4, 5, 6, 7, 9, 10, 12, 14, 15, 18, 20, 21, 25, 28, 30, 35, 36, 42, 45, 50, 60, 63, 70, 75, 84, 90, 100, 105, 126, 140, 150, 175, 180, 210, 225, 252, 300, 315, 350, 420, 450, 525, 630, 700, 900, 1050, 1260, 1575, 2100, 3150, 6300

(30)

$$\frac{2}{7} \text{ of } \frac{2}{3} \text{ of } 3\frac{1}{2} \text{ lbs.} = \frac{2}{3} \text{ lbs.}, \quad \frac{2}{7} \text{ of } \frac{2}{3} \text{ of } \frac{2}{7} \text{ of } \frac{1}{2} \text{ of } \$1 = \$\frac{2}{7},$$

$$\text{and } \frac{2}{3} \text{ of } \frac{7}{9} \text{ of } \frac{5}{10} \text{ of } \frac{2}{3} \text{ of } 90 \text{ lbs.} = 1\frac{22}{30} \text{ lbs.}$$

$$\begin{array}{c} \text{lbs.} \\ \frac{2}{3} : \frac{1323}{200} :: \frac{2}{7} : \frac{\$ \frac{2}{7} \times \frac{1323}{200}}{\frac{2}{7}} = \frac{2}{7} \times \frac{\frac{221}{1323}}{\frac{200}{25}} \times \frac{8}{8} = \frac{126}{25} = \$5 \cdot 04. \end{array}$$

(31)

7 men will have 7 men's shares.

One woman has  $\frac{3}{11}$  of a man's share;  $\therefore$  2 women will have  $2 \times \frac{3}{11} = \frac{6}{11}$  of a man's share.

One child has  $\frac{2}{7}$  of  $\frac{3}{11} = \frac{6}{77}$  of a man's share;  $\therefore$  11 children will have  $11 \times \frac{6}{77} = \frac{6}{7}$  of a man's share.

7 men, 2 women, and 11 children will have  $7 + \frac{6}{11} + \frac{6}{7} = 8\frac{3}{7}$  men's shares.

$\$2739.18 \div 8\frac{3}{7} = \$325.99\frac{133}{147} =$  a man's share.

$\frac{3}{11}$  of  $\$325.99\frac{133}{147} = \$88.90\frac{448}{147} =$  a woman's share.

$\frac{2}{7}$  of  $\$88.90\frac{448}{147} = \$25.40\frac{128}{147} =$  a child's share.

(33)

(34)

|  | yds.            | ft.  | in.            |                |                     |
|--|-----------------|------|----------------|----------------|---------------------|
| $\frac{1}{2}$ of $6\frac{1}{2}$ yds.                   | $= \frac{2}{9}$ | yds. | $= 2$          | 2              | 8                   |
| $\frac{3}{8}$ of $\frac{1}{2}$ of $8\frac{1}{2}$ ft.   | $= 1$           | 0    | 0              | 2              | 23 : 8              |
| $\frac{2}{7}$ of $\frac{3}{11}$ of $7\frac{7}{10}$ in. | $=$             |      | $\frac{3}{8}$  | 4              | 11                  |
|  |                 |      |                | 2              | 6 : 5               |
|  |                 |      |                | 13             | 11                  |
| Sum  | $= 3$           | 2    | $8\frac{3}{8}$ | $3\frac{1}{2}$ | $38\frac{1}{2} : 8$ |

} = 104 : 5.

(35)

23 bush. 2 pks. 1 gal. 1 qt. 1 pt. = 1515 pts.

$1515 \times 9000 \times \frac{1}{4} = 4545000$  in. = 71 miles 5 fur. 34 per. 3 yds.

(36)

$$\frac{4158}{10395} = \frac{462}{1155} = \frac{66}{165} = \frac{22}{55} = \frac{2}{5}.$$

(37)

VIII.

$\frac{1}{2}, \frac{2}{3}, \frac{4}{5}, \frac{7}{7}$ . Here the common denominator is  $2 \times 3 \times 5 \times 7 = 322$ . The numerators of the fractions are, for the first,  $1 \times 3 \times 5 \times 7 = 151$ ; for the second,  $2 \times 2 \times 5 \times 7 = 214$ ; for the third,  $4 \times 2 \times 3 \times 7 = 250$ ; for the fourth,  $2 \times 2 \times 3 \times 5 = 74$ ; and the equivalent fractions are,  $\frac{151}{322}, \frac{214}{322}, \frac{250}{322}$ , and  $\frac{74}{322}$ , which when added together  $= \frac{731}{322} = 2\frac{65}{322}$ , the numbers all through being in the octenary scale.

(38)

$$\begin{array}{l}
 17 \text{ sheep} = 6 \text{ cows} \\
 26 \text{ cows} = 27\frac{1}{2} \text{ acres} \\
 12 \text{ acres} = 13 \text{ horses} \\
 11 \text{ horses} = 28 \text{ goats} \\
 x \text{ goats} = 68 \text{ sheep}
 \end{array}
 \left\{
 \begin{array}{l}
 \frac{3}{6} \times \frac{2\frac{1}{2}}{27\frac{1}{2}} \times \frac{13}{12} \times \frac{28}{11} \times \frac{17}{68} = \\
 \frac{17}{17} \times \frac{26}{26} \times \frac{13}{13} \times \frac{28}{11} \times \frac{1}{1} =
 \end{array}
 \right.$$

$$2\frac{1}{2} \times 28 = 70 \text{ goats.}$$

(39)

$$\begin{array}{l}
 27 : 54 \text{ days} \\
 24 : 18 \text{ cel.} \\
 36 : 48 \text{ ft. l.} \\
 21 : 28 \text{ ft. w.} \\
 10 : 9 \text{ ft. d.} \\
 3 : 5 \text{ hrs.}
 \end{array}
 \left\{
 \begin{array}{l}
 \therefore 50 \text{ men} : \frac{50 \times 54 \times 18 \times 48 \times 28 \times 9 \times 5}{27 \times 24 \times 36 \times 21 \times 10 \times 3} = 200 \text{ men.}
 \end{array}
 \right.$$

## EXERCISE 90.—Page 226.

(1)

$$\$ \cdot 35 \times 92647 = \$32426 \cdot 45.$$

(2)

|     |               | £    | s  | d. |                              |
|-----|---------------|------|----|----|------------------------------|
| 4d. | $\frac{1}{4}$ | 4746 | 17 | 0  | = cost of 94937 pails at 1s. |
| 1d. | $\frac{1}{4}$ | 1582 | 5  | 8  | = " " " at 4d.               |
|     |               | 395  | 11 | 5  | = " " " at 1d.               |

$$£6724 \ 14 \ 1 = \text{cost of 94937 pails at 1s. 5d.}$$

(3)

$$\$ \cdot 07\frac{1}{2} \times 95972 = \$7197 \cdot 90$$

(4)

$$\$28 \cdot 80 \times 62 = \$1785 \cdot 60.$$

(5)

$$\$ \cdot 32\frac{1}{2} \times 2310 = \$750 \cdot 75.$$

(6)

$$\$ \cdot 37\frac{1}{2} \times 2117 = \$793 \cdot 87\frac{1}{2}.$$

(7)

|                  |               | £   | s  | d.              |                              |
|------------------|---------------|-----|----|-----------------|------------------------------|
| 6d.              | $\frac{1}{4}$ | 375 | 6  | 0               | = price of 7506 pairs at 1s. |
| 3d.              | $\frac{1}{2}$ | 187 | 13 | 0               | = " " at 6d.                 |
| $\frac{1}{2}$ d. | $\frac{1}{2}$ | 93  | 16 | 6               | = " " at 3d.                 |
|                  |               | 23  | 9  | 1 $\frac{1}{2}$ | = " " at $\frac{1}{2}$ d.    |

$$£680 \ 4 \ 7\frac{1}{2} = \text{price of 7506 pairs at 1s. 9 $\frac{1}{2}$ d.}$$

(8)

$$\$ \cdot 17\frac{1}{2} \times 1217 = \$212.97\frac{1}{2}.$$

(9)

$$\$3.07\frac{1}{2} \times 2103 = \$6466.72\frac{1}{2}$$

(10)

|                   |                |       |   |   |   |                         |   |                                |
|-------------------|----------------|-------|---|---|---|-------------------------|---|--------------------------------|
| 10s.              | $\frac{1}{2}$  | 2096  |   |   |   |                         |   |                                |
|                   |                | 3     |   |   |   |                         |   |                                |
|                   |                |       |   |   |   |                         |   |                                |
|                   |                | £6288 | 0 | 0 | = | cost of 2096 oz. at £3. |   |                                |
| 5s.               | $\frac{1}{2}$  | 1048  | 0 | 0 | = | "                       | " | at 0 10s.                      |
| 2s. 6d.           | $\frac{1}{2}$  | 524   | 0 | 0 | = | "                       | " | at 0 5s.                       |
| 1s. 3d.           | $\frac{1}{2}$  | 262   | 0 | 0 | = | "                       | " | at 0 2s. 6d.                   |
| $1\frac{1}{2}$ d. | $\frac{1}{10}$ | 131   | 0 | 0 | = | "                       | " | at 0 1s. 3d.                   |
|                   |                | 13    | 2 | 0 | = | "                       | " | at 0 0 $1\frac{1}{2}$ d.       |
|                   |                | £8266 | 2 | 0 | = | "                       | " | at £3 18s. 10 $\frac{1}{2}$ d. |

(11)

|                |                |                     |                    |
|----------------|----------------|---------------------|--------------------|
| 10 dwt.        | $\frac{1}{2}$  | \$1.55              |                    |
|                |                | 6                   |                    |
|                |                | <hr/>               |                    |
|                |                | \$9.30              | = cost of 6 oz.    |
| 5 dwt.         | $\frac{1}{2}$  | .77 $\frac{1}{2}$   | = " 10 dwt.        |
| 2 dwt. 12 grs. | $\frac{1}{2}$  | .38 $\frac{1}{2}$   | = " 5 dwt.         |
| 1 dwt. 6 grs.  | $\frac{1}{2}$  | .19 $\frac{1}{2}$   | = " 2 dwt. 12 grs. |
| 2 grs.         | $\frac{1}{15}$ | .09 $\frac{11}{16}$ | = " 1 dwt. 6 grs.  |
|                |                | .00 $\frac{31}{48}$ | = " 2 grs.         |

$$\$10.75\frac{3}{4} = \text{cost of 6 oz. 18 dwt. 20 grs.}$$

(12)

|        |               |   |
|--------|---------------|---|
| 10s.   | $\frac{1}{2}$ | £98 0 0 = cost of 98 yards at £1.       |
| 5s.    | $\frac{1}{2}$ | 49 0 0 = " " 0 10s.                     |
|        |               | 24 10 0 = " " 0 5s.                     |
|        |               | £171 10 0 = cost of 98 yards at £1 15s. |
| 2 qrs. | $\frac{1}{2}$ | £1 15                                   |
| 1 qr.  | $\frac{1}{2}$ | 17 6 = cost of 2 qrs.                   |
| 1 na.  | $\frac{1}{4}$ | 8 9 = " 1 qr.                           |
|        |               | 2 2½ = " 1 na.                          |
|        |               | £1 8 5½ = cost of 3 qrs. 1 na.          |

£1 8 5½ = cost of 3 qrs. 1 na.

Then £171 10 0 = cost of 98 yards at £1 15s.

1 8  $5\frac{1}{4}$  = cost of 3 qrs. 1 na. at £1 15s. per yard.

£172 18 5½ = cost of 98 yds. 3 qrs. 1 na. at £1 15s. per yd



(13)

|       |                |       |    |   |   |                          |     |      |       |
|-------|----------------|-------|----|---|---|--------------------------|-----|------|-------|
| 1s.   | $\frac{1}{20}$ | 344   |    |   |   |                          |     |      |       |
|       |                | 4     |    |   |   |                          |     |      |       |
| <hr/> |                |       |    |   |   |                          |     |      |       |
|       |                | £1376 | 0  | 0 | = | rent of 344 acres at £4. |     |      |       |
| 1d.   | $\frac{1}{12}$ | 17    | 4  | 0 | = | "                        | "   | at 0 | 1s.   |
|       |                | 1     | 8  | 8 | = | "                        | "   | at 0 | 0 1d. |
| <hr/> |                |       |    |   |   |                          |     |      |       |
|       |                | £1394 | 12 | 8 | = | rent of 344 acres at £4  | 1s. | 1d   |       |

|         |               |    |                |                |   |                       |  |  |  |
|---------|---------------|----|----------------|----------------|---|-----------------------|--|--|--|
| 2 r.    | $\frac{1}{2}$ | £4 | 1              | 1              |   |                       |  |  |  |
| <hr/>   |               |    |                |                |   |                       |  |  |  |
| 1 r.    | $\frac{1}{2}$ | 2  | 0              | $6\frac{1}{2}$ | = | rent of 2 roods.      |  |  |  |
| 10 per. | $\frac{1}{4}$ | 1  | 0              | $3\frac{1}{4}$ | = | " 1 rood.             |  |  |  |
| 5 per.  | $\frac{1}{2}$ | 5  | $0\frac{1}{2}$ | $\frac{3}{4}$  | = | " 10 perches.         |  |  |  |
|         |               | 2  | $6\frac{1}{2}$ | $\frac{3}{4}$  | = | " 5 perches.          |  |  |  |
| <hr/>   |               |    |                |                |   |                       |  |  |  |
|         |               | £3 | 8              | $4\frac{3}{4}$ | = | " 3 roods 15 perches. |  |  |  |

|       |    |                |   |                         |       |     |     |         |  |
|-------|----|----------------|---|-------------------------|-------|-----|-----|---------|--|
| £1394 | 12 | 8              | = | rent of 344 acres at £4 | 1s.   | 1d. |     |         |  |
| 3     | 8  | $4\frac{3}{4}$ | = | " 3 roods 15 per.       | at £4 | 1s. | 1d. | per ac. |  |
| <hr/> |    |                |   |                         |       |     |     |         |  |
| £1398 | 1  | $0\frac{3}{4}$ | = | " 344 a. 3 r. 15 per.   | at £4 | 1s. | 1d. |         |  |

(14)

|         |               |    |                |                |           |                                   |   |  |  |
|---------|---------------|----|----------------|----------------|-----------|-----------------------------------|---|--|--|
| 5 dwt.  | $\frac{1}{4}$ | 5  | 10             |                |           |                                   |   |  |  |
|         |               | 5  |                |                |           |                                   |   |  |  |
| <hr/>   |               |    |                |                |           |                                   |   |  |  |
|         |               | £1 | 9              | 2              | =         | price of 5 oz. at 5s. 10d. per oz |   |  |  |
| 1 dwt.  | $\frac{1}{8}$ | 1  | $5\frac{1}{2}$ | =              | " 5 dwt.  | "                                 | " |  |  |
| 12 grs. | $\frac{1}{2}$ |    | $3\frac{1}{2}$ | =              | " 1 dwt.  | "                                 | " |  |  |
| 4 grs.  | $\frac{1}{4}$ |    | $1\frac{3}{4}$ | =              | " 12 grs. | "                                 | " |  |  |
| 1 gr.   | $\frac{1}{8}$ |    | $0\frac{7}{8}$ | =              | " 4 grs.  | "                                 | " |  |  |
|         |               |    | $0\frac{7}{8}$ | =              | " 1 gr.   | "                                 | " |  |  |
| <hr/>   |               |    |                |                |           |                                   |   |  |  |
|         |               | £1 | 11             | $1\frac{3}{8}$ | =         | " 5 oz. 6 dwt. 17 grs. at         |   |  |  |
|         |               |    |                |                |           | 5s. 10d. per oz.                  |   |  |  |

(15)

|        |               |                      |                                       |              |
|--------|---------------|----------------------|---------------------------------------|--------------|
| 2 qrs. | $\frac{1}{2}$ | £1 2 4               |                                       |              |
|        |               | 4                    |                                       |              |
|        |               | <hr/>                |                                       |              |
|        |               | £4 9 4               | = price of 4 yards at £1 2 4 per yard |              |
| 2 na.  | $\frac{1}{4}$ | 11 2                 | =                                     | " 2 qrs. " " |
| 1 na.  | $\frac{1}{2}$ | 2 9 $\frac{1}{2}$    | =                                     | " 2 na. " "  |
|        |               | 1 4 $\frac{3}{4}$    | =                                     | " 1 na. " "  |
|        |               | <hr/>                |                                       |              |
|        |               | £5 4 8 $\frac{1}{4}$ | = price of 4 yds. 2 qrs. 3 na.        | " "          |

(16)

|         |               |                       |                                    |             |
|---------|---------------|-----------------------|------------------------------------|-------------|
| 1 rood. | $\frac{1}{4}$ | £1 16                 |                                    |             |
|         |               | 32                    |                                    |             |
|         |               | <hr/>                 |                                    |             |
|         |               | £57 12 0              | = price of 32 acres at £1 16s.     |             |
| 10 per. | $\frac{1}{4}$ | 9 0                   | =                                  | " 1 rood. " |
| 2 per.  | $\frac{1}{8}$ | 2 3                   | =                                  | " 10 per. " |
| 2 per.  | $\frac{1}{8}$ | 5 $\frac{2}{8}$       | =                                  | " 2 per. "  |
|         |               | 5 $\frac{3}{8}$       | =                                  | " 2 per. "  |
|         |               | <hr/>                 |                                    |             |
|         |               | £58 4 1 $\frac{1}{8}$ | = price of 32 acres 1 rood 14 per. |             |

(17)

|        |               |                      |  |            |
|--------|---------------|----------------------|--|------------|
| 4 pts. | $\frac{1}{2}$ | 7 6                  |  |            |
|        |               | 3                    |  |            |
|        |               | <hr/>                |  |            |
|        |               | £1 2 6               | = price of 3 gals. at 7s. 6d. per gal. |            |
| 1 pt.  | $\frac{1}{4}$ | 3 9                  | =                                      | " 4 pts. " |
|        |               | 11 $\frac{1}{4}$     | =                                      | " 1 pt. "  |
|        |               | <hr/>                |  |            |
|        |               | £1 7 2 $\frac{1}{4}$ | = price of 3 gals. 5 pts.              |            |

(18)

$$\$1.67\frac{1}{2} \times 724 = \$1212.70.$$

(19)

$$\$1.93\frac{3}{4} \times 721 = \$1396.93\frac{3}{4}.$$

(20)

|                  |                |        |    |   |                            |   |   |    |                       |
|------------------|----------------|--------|----|---|----------------------------|---|---|----|-----------------------|
| 10s.             | $\frac{1}{2}$  | 4514   |    |   |                            |   |   |    |                       |
|                  |                | 2      |    |   |                            |   |   |    |                       |
| <hr/>            |                |        |    |   |                            |   |   |    |                       |
|                  |                | £9028  | 0  | 0 | = cost of 4514 rods at £2. |   |   |    |                       |
| 6s. 8d.          | $\frac{1}{3}$  | 2257   | 0  | 0 | =                          | " | " | at | 0 10                  |
| 10d.             | $\frac{1}{8}$  | 1504   | 13 | 4 | =                          | " | " | at | 0 6 8                 |
| 1d.              | $\frac{1}{16}$ | 188    | 1  | 8 | =                          | " | " | at | 0 0 10                |
| $\frac{1}{2}$ d. | $\frac{1}{2}$  | 18     | 16 | 2 | =                          | " | " | at | 0 0 1                 |
|                  |                | 9      | 8  | 1 | =                          | " | " | at | 0 0 0 $\frac{1}{2}$   |
| <hr/>            |                |        |    |   |                            |   |   |    |                       |
|                  |                | £13005 | 19 | 3 | =                          | " | " | at | £2 17 7 $\frac{1}{2}$ |

(21)

|       |                |        |    |                  |  |   |   |    |       |
|-------|----------------|--------|----|------------------|--|---|---|----|-------|
| 10s.  | $\frac{1}{2}$  | £3749  | 7  | 6                |  |   |   |    |       |
|       |                | 3      |    |                  |  |   |   |    |       |
| <hr/> |                |        |    |                  |  |   |   |    |       |
|       |                | £11248 | 2  | 6                | = price of 3749 $\frac{3}{8}$ acres at £3    |   |   |    |       |
| 5s.   | $\frac{1}{2}$  | 1874   | 13 | 9                | =  | " | " | at | 0 10  |
| 6d.   | $\frac{1}{16}$ | 937    | 6  | 10 $\frac{1}{2}$ | =  | " | " | at | 0 5   |
|       |                | 93     | 14 | 8 $\frac{1}{4}$  | =  | " | " | at | 0 0 6 |
| <hr/> |                |        |    |                  |  |   |   |    |       |
|       |                | £14153 | 17 | 9 $\frac{3}{4}$  | = price of 3749 $\frac{3}{8}$ acres at £3 15 |   |   |    | 6     |

(22)

|         |                |                    |                     |   |                             |    |                       |    |       |
|---------|----------------|--------------------|---------------------|---|-----------------------------|----|-----------------------|----|-------|
| 4s.     | $\frac{1}{3}$  | £17                | 0                   | 0 | = cost of 17 cwt. at £1     |    |                       |    |       |
| 8d.     | $\frac{1}{6}$  | 3                  | 8                   | 0 | =                           | "  | "                     | at | 0 4   |
| 1d.     | $\frac{1}{8}$  | 11                 | 4                   |   | =                           | "  | "                     | at | 0 0 8 |
|         |                | 1                  | 5                   |   | =                           | "  | "                     | at | 0 0 1 |
| <hr/>   |                |                    |                     |   |                             |    |                       |    |       |
|         |                | £21                | 0                   | 9 | = cost of 17 cwt. at £1 4 9 |    |                       |    |       |
| 1 qr.   | $\frac{1}{4}$  | £1                 | 4                   | 9 |                             |    |                       |    |       |
| <hr/>   |                |                    |                     |   |                             |    |                       |    |       |
| 16 lbs. | $\frac{1}{7}$  | 6                  | 2 $\frac{1}{4}$     |   | = cost of 1 qr.             |    |                       |    |       |
| 1 lb.   | $\frac{1}{16}$ | 3                  | 6 $\frac{3}{4}$     |   | =                           | "  | 16 lbs.               |    |       |
|         |                | 0                  | 2 $\frac{7}{11}$    |   | =                           | "  | 1 lb.                 |    |       |
| <hr/>   |                |                    |                     |   |                             |    |                       |    |       |
|         |                | 9                  | 11 $\frac{37}{112}$ |   | =                           | "  | 1 qr. 17 lbs.         |    |       |
| £21     | 0              | 9                  |                     |   | = cost of 17 cwt.           | at | £1 4s. 9d. per cwt.   |    |       |
|         |                | 9                  | 11 $\frac{37}{112}$ |   | =                           | "  | 1 qr. 17 lbs.         | "  | "     |
| <hr/>   |                |                    |                     |   |                             |    |                       |    |       |
| £21     | 10             | 8 $\frac{37}{112}$ |                     |   | =                           | "  | 17 cwt. 1 qr. 17 lbs. | "  | "     |

(23)

|        |               |                    |                                       |          |     |
|--------|---------------|--------------------|---------------------------------------|----------|-----|
| 2 qrs. | $\frac{1}{2}$ | \$11.55            |                                       |          |     |
|        |               | 78                 |                                       |          |     |
|        |               | <hr/>              |                                       |          |     |
|        |               | 9240               |                                       |          |     |
|        |               | 8085               |                                       |          |     |
|        |               | <hr/>              |                                       |          |     |
|        |               | \$900.90           | = cost of 78 cwt. at \$11.55 per cwt. |          |     |
| 1 qr.  | $\frac{1}{2}$ | 5.77 $\frac{1}{2}$ | =                                     | " 2 qrs. | " " |
| 7 lbs. | $\frac{1}{4}$ | 2.88 $\frac{3}{4}$ | =                                     | " 1 qr.  | " " |
| 4 lbs. | $\frac{1}{2}$ | .72 $\frac{3}{16}$ | =                                     | " 7 lbs. | " " |
| 1 lb.  | $\frac{1}{4}$ | .41 $\frac{1}{4}$  | =                                     | " 4 lbs. | " " |
|        |               | .10 $\frac{5}{16}$ | =                                     | " 1 lb.  | " " |
|        |               | <hr/>              |                                       |          |     |
|        |               | \$910.80           | = cost of 78 cwt. 3 qrs. 12 lbs.      |          |     |

(24)

£10 10

20

£210 0 = price of 20 tons at £10 10s.

19 cwt. 3 qrs. 27 $\frac{1}{2}$  lbs. = 1 ton. —  $\frac{1}{2}$  lb. The price of 1 ton is £10 10s., and the price of  $\frac{1}{2}$  lb. =  $\frac{1}{4480}$  of £10 10s. =  $\frac{63}{112}$  d. ∴ the price of 19 cwt. 3 qrs. 27 $\frac{1}{2}$  lbs. = £10 10s. —  $\frac{63}{112}$  d. = £10 9s. 11 $\frac{49}{112}$  d.

£210 0 0 = price of 20 tons at £10 10s.

10 9 11 $\frac{49}{112}$  = " 19 cwt. 3 qrs. 27 $\frac{1}{2}$  lbs.

£220 9 11 $\frac{49}{112}$  = " 20 tons 19 cwt. 3 qrs. 27 $\frac{1}{2}$  lbs. at £10 10s. per ton.

(25)

|         |               |                          |   |                                       |             |
|---------|---------------|--------------------------|---|---------------------------------------|-------------|
| 10 cwt. | $\frac{1}{2}$ | \$45.50                  |   |                                       |             |
|         |               | 219                      |   |                                       |             |
|         |               | <hr/>                    |   |                                       |             |
|         |               | 40950                    |   |                                       |             |
|         |               | 4550                     |   |                                       |             |
|         |               | 9100                     |   |                                       |             |
|         |               | <hr/>                    |   |                                       |             |
|         |               | \$9964.50                | = | price of 219 tons at \$45.50 per ton. |             |
| 5 cwt.  | $\frac{1}{2}$ | 22.75                    | = | "                                     | 10 cwt. " " |
| 1 cwt.  | $\frac{1}{5}$ | 11.37 $\frac{1}{2}$      | = | "                                     | 5 cwt. " "  |
| 2 qrs.  | $\frac{1}{2}$ | 2.27 $\frac{1}{2}$       | = | "                                     | 1 cwt. " "  |
| 1 qr.   | $\frac{1}{4}$ | 1.13 $\frac{3}{4}$       | = | "                                     | 2 qrs. " "  |
|         |               | 56 $\frac{7}{8}$         | = | "                                     | 1 qr. " "   |
|         |               | <hr/>                    |   |                                       |             |
|         |               | \$10002.60 $\frac{1}{2}$ | = | price of 219 tons 16 cwt. 3 qrs.      |             |

## EXERCISE 91—Page 228.

## BILLS OF PARCELS.

(No. 2.)

|                                       | s. | d.                       | £ | s. | d.               |
|---------------------------------------|----|--------------------------|---|----|------------------|
| 9 pair of worsted stockings, at.....  | 4  | 6 per pair               | 2 | 0  | 6                |
| 6 pair of silk ditto, at.....         | 15 | 9 "                      | 4 | 14 | 6                |
| 17 pair of thread ditto, at.....      | 5  | 4 "                      | 4 | 10 | 8                |
| 23 pair of cotton ditto, at.....      | 4  | 10 "                     | 5 | 11 | 2                |
| 14 pair of yarn ditto, at.....        | 2  | 4 "                      | 1 | 12 | 8                |
| 18 pair of women's silk gloves, at... | 4  | 2 "                      | 3 | 15 | 0                |
| 19 yards of flannel, at .....         | 1  | 7 $\frac{1}{2}$ per yard | 1 | 10 | 10 $\frac{1}{2}$ |

Ans. £23 15 4 $\frac{1}{2}$ 

(No. 3.)

|   |                               |                      |
|---|-------------------------------|----------------------|
| 75 $\frac{1}{2}$ lbs. of sugar, at.....   | 7 $\frac{1}{2}$ cents per lb. | \$5.85 $\frac{1}{2}$ |
| 63 lbs. of tea, at.....                   | 93 "                          | 58.59                |
| 126 lbs. of butter, at.....               | 13 "                          | 16.38                |
| 35 $\frac{1}{2}$ lbs. of raisins, at..... | 18 $\frac{1}{2}$ "            | 6.71 $\frac{1}{2}$   |
| 17 lbs. of sago, at.....                  | 15 "                          | 2.55                 |
| 23 lbs. of rice, at.....                  | 9 "                           | 2.07                 |
| 58 $\frac{1}{2}$ lbs. of starch, at.....  | 22 "                          | 12.87                |

Ans. \$105.02 $\frac{1}{2}$

## (No. 4.)

|   |        |          |
|---|--------|----------|
| 198 Sangster's National Arithmetic, at.....     | \$0.60 | \$118.80 |
| 197 Robertson's Philosophy of Grammar, at...    | 0.50   | 98.50    |
| 83 Hodgins' Geography, at.....                  | 1.00   | 83.00    |
| 57 Sangster's Algebraic Formula, at.....        | 0.12½  | 7.12½    |
| 217 Strachan's Canadian Penmanship, at.....     | 0.37½  | 81.37½   |
| 143 Hodgins' Geography of British Provinces, at | 0.45   | 64.35    |
| 227 Sangster's First Arithmetic, at.....        | 0.30   | 68.10    |

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*Ans.* \$521.25

## (No. 5.)

|                                   | s. | d.         | £  | s. | d. |
|-----------------------------------|----|------------|----|----|----|
| 9½ yards of silk, at.....         | 12 | 9 per yard | 6  | 1  | 1½ |
| 13 yards of flowered ditto, at... | 15 | 6 "        | 10 | 1  | 6  |
| 11½ yards of lustring, at.....    | 6  | 10 "       | 4  | 0  | 3½ |
| 14 yards of brocade, at.....      | 11 | 3 "        | 7  | 17 | 6  |
| 12½ yards of satin, at.....       | 10 | 8 "        | 6  | 10 | 8  |
| 11½ yards of velvet, at.....      | 18 | 0 "        | 10 | 4  | 9  |

---

*Ans.* £44 15 10

## (No. 6.)

|                             |        |        |
|-----------------------------|--------|--------|
| 14 oz. ipecacuanha, at..... | \$0.67 | \$9.38 |
| 23 " laudanum, at.....      | 0.89   | 20.47  |
| 17 " emetic tartar, at..... | 1.25   | 21.25  |
| 25 " cantharides, at.....   | 2.17   | 54.25  |
| 27 " gum mastic, at.....    | 0.61   | 16.47  |
| 56 " gum camphor, at.....   | 0.27   | 15.12  |

---

*Ans.* \$136.94

## (No. 7.)

|  | s. | d.        | £  | s.  | d. |
|--|----|-----------|----|-----|----|
| 15½ lbs. of currants, at.....          | 0  | 4 per lb. | 5  | 2   |    |
| 17½ lbs. of Malaga raisins, at.....    | 0  | 5½ "      | 7  | 10½ |    |
| 19½ lbs. of sun raisins, at.....       | 0  | 6 "       | 9  | 10½ |    |
| 17 lbs. of rice, at.....               | 0  | 3½ "      | 4  | 11½ |    |
| 8½ lbs. of pepper, at.....             | 1  | 6 "       | 12 | 9   |    |
| 3 loaves of sugar, weight 32½ lbs. at. | 0  | 8½ "      | 1  | 3   | 0½ |
| 13 oz. of cloves, at.....              | 0  | 9 per oz. | 9  | 9   |    |

---

*Ans.* £3 13 5½



(4)

| Dissimilar.             |   | Similar.                   |   | Similar and Coterminous.    |
|-------------------------|---|----------------------------|---|-----------------------------|
| $73.\dot{7}2\dot{3}$    | = | $73.723\dot{7}2\dot{3}$    | = | $73.723\dot{7}2372\dot{3}$  |
| $11.\dot{3}4\dot{2}$    | = | $11.342\dot{2}$            | = | $11.342\dot{2}2222\dot{2}$  |
| $16.\dot{7}1\dot{3}$    | = | $16.713\dot{0}$            | = | $16.71300000\dot{0}$        |
| $19.\dot{0}3\dot{4}$    | = | $19.034\dot{0}3\dot{4}$    | = | $19.034\dot{0}3403\dot{4}$  |
| $713.2134\dot{3}7$      | = | $713.2134\dot{3}7$         | = | $713.2134\dot{3}743\dot{7}$ |
| $12.\dot{3}456\dot{7}8$ | = | $12.3456\dot{7}834\dot{5}$ | = | $12.3456\dot{7}834\dot{5}$  |
|                         |   |                            |   | 2 carried.                  |
|                         |   | Sum                        | = | $846.37209576\dot{3}$       |

(5)

$$\begin{array}{l}
 5 : 7 = 5 \div 7 = .714+ \\
 9 : 13 = 9 \div 13 = .692+ \\
 12 : 17 = 12 \div 17 = .705+ \\
 7 : 10 = 7 \div 10 = .7
 \end{array}
 \left. \vphantom{\begin{array}{l} 5 : 7 \\ 9 : 13 \\ 12 : 17 \\ 7 : 10 \end{array}} \right\} \begin{array}{l} \text{Hence } 5 : 7 \text{ is the greatest,} \\ \text{and } 9 : 13 \text{ least.} \end{array}$$

$$\frac{5}{7} \times \frac{9}{13} \times \frac{12}{17} \times \frac{7}{10} = \frac{54}{221} = 54 : 221.$$

(6)

1 acre = 160 rods, and 25 acres 2 roods 35 rods = 4115 rods.

$$\begin{array}{r}
 \text{rods.} \\
 160 : 4115 :: \$80.50 : \frac{40.25 \quad 823}{80.50 \times 4115} = \$2070.3593. \\
 \frac{160}{80} \\
 \frac{80}{16}
 \end{array}$$

(8)

$$\$3.68\frac{1}{2} \times 7439 = \$27431.31\frac{1}{2}.$$



(9)

$\frac{135795}{22210}$ . The G. C. M. of 135795 and 22210 is 12345; when both terms of the fraction are divided by 12345, it becomes  $\frac{11}{18}$ .

$\frac{714235}{999999}$ . Here 714235 and 999999 have no G. C. M.;  $\therefore$  the fraction cannot be reduced.

$\frac{109375}{100000}$ . The G. C. M. of 109375 and 100000 is 3125; when both terms of the fraction are divided by 3125, it becomes reduced to  $\frac{35}{32}$ .

$\frac{20301}{33633}$ . The G. C. M. of 20301 and 33633 is 303; when both terms of the fraction are divided by 303, it is reduced to its lowest terms, viz.,  $\frac{67}{111}$ .

(10)

$$\left. \begin{array}{ll} 34\frac{1}{2} \text{ bushels turnips} & = 17 \text{ bushels potatoes} \\ 9 \text{ " potatoes} & = 59\frac{1}{2} \text{ lbs. tea} \\ 6 \text{ lbs. tea} & = 11\frac{1}{2} \text{ stone flour} \\ 13 \text{ stone flour} & = 360 \text{ cents} \\ 38 \text{ cents} & = 12 \text{ lbs. bread} \\ 119 \text{ lbs bread} & = x \text{ bushels turnips} \end{array} \right\} =$$

$$\frac{3}{84\frac{1}{2}} \times \frac{9}{59\frac{1}{2}} \times \frac{6}{11\frac{1}{2}} \times \frac{13}{360} \times \frac{19}{12} \times \frac{7}{1} = \frac{3 \times 13 \times 19}{8\frac{1}{2} \times 40} = 2\frac{61}{340}.$$

(11)

$$\left. \begin{array}{l} 54 : 27 \text{ men} \\ 11 : 8 \text{ hours} \\ 42 : 77 \text{ floors} \\ 20 : 24 \text{ feet long} \\ 16 : 22 \text{ feet wide} \\ 3 : 5 \text{ coats paint} \end{array} \right\} :: 7 \text{ days} : \frac{7 \times 27 \times 8 \times 77 \times 24 \times 22 \times 5}{54 \times 11 \times 42 \times 20 \times 16 \times 3} = \frac{7 \times 11}{2 \times 3} = 12\frac{5}{6} \text{ days.}$$

(13)

IX.  
12)72342

12)5403..2

12)407..0

12)30..7

2..3

IX.  
6)72342

6)11806..2

6)1731..0

6)264..4

6)40..4

6)6..0

1..0

IX.  
3)72342

3)23713..2

3)7234..0

3)2371..1

3)723..1

3)237..0

3)72..1

3)23..2

3)7..0

2..1

| IX.   |   | XII.  |   | VI.     |   | III        |       |
|-------|---|-------|---|---------|---|------------|-------|
| 72342 | = | 23702 | = | 1004402 | = | 2102101102 |       |
| 9     |   | 12    |   | 6       |   | 3          |       |
| ---   |   | ---   |   | ---     |   | ---        |       |
| 65    |   | 27    |   | 6       |   | 7          |       |
| 9     |   | 12    |   | 6       |   | 3          |       |
| ---   |   | ---   |   | ---     |   | ---        |       |
| 588   |   | 331   |   | 36      |   | 21         |       |
| 9     |   | 12    |   | 6       |   | 3          |       |
| ---   |   | ---   |   | ---     |   | ---        |       |
| 5296  |   | 3972  |   | 220     |   | 65         | 1765  |
| 9     |   | 12    |   | 6       |   | 3          | 3     |
| ---   |   | ---   |   | ---     |   | ---        | ---   |
| 47666 |   | 47666 |   | 1324    |   | 196        | 5296  |
|       |   |       |   | 6       |   | 3          | 3     |
|       |   |       |   | ---     |   | ---        | ---   |
|       |   |       |   | 7944    |   | 588        | 15888 |
|       |   |       |   | 6       |   | 3          | 3     |
|       |   |       |   | ---     |   | ---        | ---   |
|       |   |       |   | 47666   |   | 1765       | 47666 |

(14)

| II.          |           | IV.            | IV.         |
|--------------|-----------|----------------|-------------|
| 111111       | 100000    | 333333         | 100000      |
| 2            | 2         | 4              | 4           |
| --           | --        | —              | —           |
| 3            | 2         | 15             | 4           |
| 2            | 2         | 4              | 4           |
| --           | --        | —              | —           |
| 7            | 4         | 63             | 16          |
| 2            | 2         | 4              | 4           |
| —            | —         | —              | —           |
| 15           | 8         | 255            | 64          |
| 2            | 2         | 4              | 4           |
| —            | —         | —              | —           |
| 31           | 16        | 1023           | 256         |
| 2            | 2         | 4              | 4           |
| —            | —         | —              | —           |
| 63 Greatest. | 32 Least. | 4095 Greatest. | 1024 Least. |

| VI.             | VI.         | VIII.            | VIII.  |
|-----------------|-------------|------------------|--------|
| 555555          | 100000      | 777777           | 100000 |
| 6               | 6           | 8                | 8      |
| —               | --          | —                | —      |
| 35              | 6           | 63               | 8      |
| 6               | 6           | 8                | 8      |
| ---             | ---         | ---              | ---    |
| 215             | 36          | 511              | 64     |
| 6               | 6           | 8                | 8      |
| —               | —           | —                | —      |
| 1295            | 216         | 4095             | 512    |
| 6               | 6           | 8                | 8      |
| —               | —           | —                | —      |
| 7775            | 1296        | 32767            | 4096   |
| 6               | 6           | 8                | 8      |
| —               | —           | —                | —      |
| 46655 Greatest. | 7776 Least. | 262143 Greatest. | 32768  |

(Continued on next page.)

(14 continued.)

| XII.              | XII.          |
|-------------------|---------------|
| <i>eeeeee</i>     | 100000        |
| 12                | 12            |
| ----              | ----          |
| 143               | 12            |
| 12                | 12            |
| ----              | ----          |
| 1727              | 144           |
| .12               | 12            |
| ----              | ----          |
| 20735             | 1728          |
| 12                | 12            |
| ----              | ----          |
| 248831            | 20736         |
| 12                | 12            |
| ----              | ----          |
| 2985983 Greatest. | 248832 Least. |

(15)

$$1728 = 2^6 \times 3^3.$$

1..2..4..8..16..32..64

1 .3..9..27

1..2..4..8..16..32..64..3..6..12..24..48..96..192..9..

18..36..72..144..288..576..27..54..108..216..432..864..

1728.

Therefore the divisors of 1728 are 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 27, 32, 36, 48, 54, 64, 72, 96, 108, 144, 192, 216, 288, 432, 576, 864, 1728.

(16)

|     |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |
|-----|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|
| 30  | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 |
| 14  | 2 |   | 4 |   |    | 2  | 7  | 8  | 3  | 2  | 11 |    | 4  | 13 | 14 |
| 12  |   |   | 2 |   |    |    | 4  | 3  |    | 11 |    | 2  | 13 |    |    |
| 143 |   |   |   |   |    |    |    |    |    | 11 |    | 13 |    |    |    |

$$30 \times 14 \times 12 \times 143 = 720720 = 1. \text{ c. m.}$$

(17)

Dissimilar.

Similar.

Similar and Coterminous.

$$7 \cdot 91342 = 97 \cdot 913423 = 97 \cdot 913423423423423$$

$$8 \cdot 1234567 = 19 \cdot 1234567 = 18 \cdot 123456745674567$$

$$\text{Difference} = 79 \cdot 789966677748855$$

(18)

20 ft. 7'

19 ft. 5 7"

$$\begin{array}{r} 1 \quad 0 \quad 0 \quad 1''' \\ 8 \quad 6 \quad 11 \\ 91 \quad 1 \end{array}$$

$$00 \quad 7 \quad 11 \quad 1 = 44 \text{ sq. yds.} + \frac{4}{9} + \frac{7}{108} + \frac{11}{1296} + \frac{1}{15552} = 44 \frac{8053}{15552} \text{ sq. yds.} = 44 \cdot 517 + \text{sq. yds.}$$

$$\$2 \cdot 87\frac{1}{2} \times 44 \cdot 517 = \$127 \cdot 98 +.$$

(19)

$$916 \text{ acres } 3 \text{ roods } 17 \text{ per. } 7 \text{ yds.} = 4437591\frac{1}{4} \text{ sq. yds., and } 43 \text{ acres } 1 \text{ rood } 2 \text{ per. } 17 \text{ yds.} = 209407\frac{1}{2} \text{ sq. yds.}$$

$$4437591\frac{1}{4} \div 209407\frac{1}{2} = 4437591 \cdot 25 \div 207407 \cdot 5 = 21 \cdot 19117 +.$$

## EXERCISE 94---Page 233

(1)

(2)

$$\$742 \cdot 10 \times \cdot 05 = \$37 \cdot 10\frac{1}{2}. \quad \$1000 \times \cdot 11 = \$110.$$

(3)

$$\$734 \cdot 19 \times \cdot 10 = \$73 \cdot 419.$$

(4)

$$\$1624 \cdot 50 \times \cdot 875 = \$1421 \cdot 4375.$$

5)

$$\$994.70 \times .125 = \$124.3375.$$

(6)

$$\$777.50 \times .0875 = \$68.03125, \text{ or } \$68.03\frac{1}{2}.$$

(7)

(8)

$$\$7135.80 \times .0225 = \$160.5555.$$

$$2740 \times .20 = 548.$$

(9)

(10)

$$\$7490 \times .10 = \$749$$

$$\$740 \times .045 = \$33.30$$

$$\$7490 \times .17 = \$1273.30$$

$$\$1680 \times .025 = \$42.00$$

$$\$7490 \times .27 = \$2022.30$$

$$\$42.00 - \$33.30 = \$8.70$$

$$\$7490 \times .46 = \$3445.40$$

(11)

(12)

$$729 \times .11 = 80.19$$

$$\$763.22 \times .25 = \$190.8050$$

$$729 - 80.19 = 648.81 = 648\frac{81}{100}$$

$$\$847.16 \times .16 = 135.5456$$

$$\$1234.17 \times .0625 = 77.135625$$

---


$$\text{Sum} = \$403.486225$$

(13)

(14)

$$\$17429.40 \times .43 = \$7494.64\frac{1}{2}$$

$$68978 \times .36 = 24832.08.$$

$$\$17429.40 \times .37 = 6448.87\frac{1}{2}$$

(15)

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$$\$13943.52$$

$$29800 \times .17 = 5066$$

$$\$17429.40 - \$13943.52 = \$3485.88.$$

$$29800 - 5066 = 24734$$

---

EXERCISE 95—Page 235.

(1)

(2)

$$\$1000 \times .045 = \$45. \quad \$1678.30 \times .0225 = \$37.76175.$$

(3)

(4)

$$\$7531.19 \times .0375 = \$282.419625. \quad \$508.60 \times .0125 = \$6.3575$$

(5)

(6)

$$\$7863.50 \times .0175 = \$137.61125. \quad \$878.30 \times .025 = \$21.9575$$

(7)

(8)

$$\$7193.16 \times .03125 = \$224.78625. \quad \$6734.10 \times .17 = \$1144.797.$$

(9)

$$\$7.13 \times 718 \times .0425 = \$217.57195.$$

(10)

$$\$1.85 \times 8243 \times .05625 = \$857.7871875.$$

---

EXERCISE 96—Page 236.

(1)

(2)

$$\$7893.87 \times .02 = \$157.8774.$$

$$\$8000 \times .00875 = \$70.$$

(3)

$$\$8643.22 \times .0125 = \$108.04025.$$

(4)

$$\$78963.80 \times .00875 = \$690.93325.$$

(5)

$$\$1987.27 \times .0375 = \$74.522625.$$

## EXERCISE 97—Page 237.

(1)

$$\$4000 \div 1.0125 = \$3950.61728 + = \text{sum to be invested.}$$

$$\$4000 - 3950.61728 = \$49.38271 = \text{commission.}$$

(2)

$$\$7500 \div 1.045 = \$7177.03349 = \text{sum to be expended in laces.}$$

$$\$7500 - \$7177.03349 = \$322.96651 = \text{commission.}$$

(3)

$$\$8470 \div 1.05 = \$8066.66\frac{2}{3} = \text{sum to be invested.}$$

$$\$8066.66\frac{2}{3} \div \$6.40 = 1260\frac{5}{8} \text{ barrels.}$$

(4)

$$\$11000 \div 1.00875 = \$10904.584882 = \text{sum to be invested.}$$

(5)

$$\$13000 \div 1.045 = \$12440.1913 + = \text{sum to be invested.}$$

$$\$13000 - \$12440.1913 = \$559.8086 + = \text{commission.}$$

$$\$12440.1913 + \div \$3.63 = 3427.0499 \text{ yds.}$$

## EXERCISE 98—Page 238.

(1)

(2)

$$\$9000 \div 0.83 = \$10843.373. \quad \$8500 \div 1.11 = \$7657.6576$$

(3)

$$\$17500 \div 1.0125 = \$17283.951 = \text{amount to be invested.}$$

$$\$17283.951 \div 1.07 = \$16153.22 = \text{stock.}$$



(4)

 $\$20000 \div 1.0175 = \$19656.01965 = \text{amount to be invested.}$ 
 $\$19656.01965 \div 0.97 = \$20263.937 = \text{stock remitted.}$ 

(5)

 $\$200 \times 100 = \$20000 = \text{par value of 200 shares.}$ 
 $\$1 \text{ stock costs } \$1.055. \quad \$1.055 \times 20000 = \$21100 = \text{cost of stock.}$ 
 $\$21100 \times .00875 = \$184.625 = \text{brokerage.}$ 
 $\$21100 + \$184.625 = \$21284.625 = \text{whole cost.}$ 

## EXERCISE 99—Page 240..

(1)

(2)

 $\$7500 \times .0175 = \$131.25.$ 
 $\$8375 \times .0075 = \$62.8125.$ 

(3)

(4)

 $\$6000 \times .01875 = \$112.50$ 
 $\$5000 \times .0117 = \$58.50.$ 

(5)

(6)

 $\$6400 \times .0090 = \$57.60$ 
 $\$4500 \times .0035 = \$15.75.$ 

(7)

(8)

 $\$36000 \times .03 = \$1080.$ 
 $\$27000 \times .0482 \times 4 = \$5205.60.$ 

(9)

(10)

 $\$39000 \times .022 = \$858.$ 
 $\$17800 \times .005 = \$89.$ 

(11)

 $\$12350 \times .017 \times 7 = \$1235.$

## EXERCISE 100—Page 241.

(1)

$$\$17000 \div 965 = \$17616.58.$$

(2)

$$\$22750 \div .94 = \$24202.127.$$

(3)

$$\$15000 \div .9775 = \$15345.2685.$$

(4)

$$\$33000 \div .9425 = \$35013.2625.$$

## EXERCISE 101—Page 243.

(1)

$$1347 \times 5 = 6735 \text{ lbs.} = \text{gross weight.}$$

$$6735 \times .06 = 404.1 \text{ lbs. tare.}$$

$$\begin{array}{r} 6330.9 \text{ lbs.} = \text{net at } 3\frac{1}{2} \text{ cents per lb.} = 6330.9 \\ \times .035 = \$221.58. \end{array}$$

(2)

$$127 \times 11 = 1397 \text{ lbs.} = \text{gross weight.}$$

$$1397 \times .03 = 41.91 \text{ lbs.} = \text{tare.}$$

$$\begin{array}{r} 1355.09 \text{ lbs.} = \text{net at } \$ .012 \text{ per lb.} = 1355.09 \\ \times .012 = \$16.26. \end{array}$$

(3)

$$.29 \times .13 = \$16.77.$$

(4)

$$31 \times 207 = 6417 \text{ lbs.} = \text{gross weight.}$$

$$207 \times 2\frac{1}{4} = 465\frac{3}{4} \text{ lbs.} = \text{tare.}$$

$$\begin{array}{r} 5951\frac{1}{4} \text{ lbs.} = \text{net at } 5\frac{1}{2} \text{ cents per lb.} = 5951\frac{1}{4} \times \\ .0575 = \$342.1968. \end{array}$$

(5)

$$214 \times .47 = \$100.58.$$

---

## EXERCISE 102—Page 243.

(1)

(2)

$$\$17429.80 \times .21 = \$3660.2580. \quad \$2920.16 \times .075 = \$219.012.$$

(3)

(4)

$$\$71342.90 \times .25 = \$17835.725. \quad \$913.73 \times .2 = \$182.746.$$

(5)

$$\$14713.19 \times .33 = \$4855.3527.$$

---

## EXERCISE 103—Page 244.

(1)

$$\$23900 \div 7142300 = \$0.0033462 = \text{rate per dollar.}$$

$$\$0.0033462 \times 14729.50 = \$49.2878 +.$$

(2)

$$\$100000 \div 5793000 = \$0.017262 = \text{rate per dollar.}$$

$$\$0.017262 \times 18600 = \$321.0732.$$

(3)

$$\$100000 \div 5793000 = \$0.017262 = \text{rate per dollar.}$$

$$\$0.017262 \times 7500 = \$129.465.$$

(4)

$$\$100000 \div 5793000 = \$0.017262 = \text{rate per dollar.}$$

$$\$0.017262 \times 11400 = \$196.7868.$$

## EXERCISE 104—Page 252.

(1)

Here  $P = \$723.19$ ,  $r = .067$ , and  $t = 7.32$ .Then  $I = Prt = 723.19 \times .067 \times 7.32 = \$354.6813036$ .

(2)

Here  $P = 857.19$ ,  $r = .065$ , and  $t = 6\frac{1}{2}$  or  $6.5$ .Then  $A = P(1 + rt) = \$857.19 \times 1.4225 = \$1219.352775$ .

(3)

Here  $t = 11$ , and  $r = .725$ .Then  $n = tr + 1 = 11 \times .725 + 1 = 8.975$ .

(4)

Here  $P = \$654.32$ ,  $I = \$234.56$ , and  $r = .07$ .Then  $t = \frac{I}{Pr} = \frac{234.56}{654.32 \times .07} = 5.12112$  or 5 years 1 m. 13 d.

(5)

Here  $A = \$1200$ ,  $P = \$700$ , and  $t = 5$ .Then  $r = \frac{A-P}{Pt} = \frac{1200-700}{700 \times 5} = \frac{1}{7} = \text{rate per unit} \therefore 14\frac{2}{7} = \text{rate per cent.}$ 

(6)

Here  $n = 4$ , and  $r = .23$ .Then  $t = \frac{n-1}{r} = \frac{4-1}{.23} = 13$  years 15 days.

(7)

Here  $P = \$270$ ,  $I = \$87$  and  $r = .07$ .Then  $t = \frac{I}{Pr} = \frac{87}{270 \times .07} = 4$  years  $7\frac{5}{7}$  months.

(8)

Here  $P = \$680$ ,  $t = 11\frac{1}{2}$ , and  $r = .11$ .Then  $A = P(1 + rt) = 680 \times 2.265 = \$1540.20$ 

(9)

Here  $A = \$2000$ ,  $t = 20$ , and  $r = .08$ .Then  $P = \frac{A}{1 + rt} = \frac{2000}{2.6} = \$769.23\frac{1}{3}$ .

(10)

Here  $n = 21$ , and  $t = 24$ .Then  $r = \frac{n-1}{t} = \frac{21-1}{24} = .83\frac{1}{3} = \text{rate per unit. } \therefore 83\frac{1}{3} = \text{rate per cent.}$ 

(11)

Here  $n = 23$ , and  $r = .16$ .Then  $t = \frac{n-1}{r} = \frac{23-1}{.16} = 137\frac{1}{2} \text{ years}$ 

(12)

Here  $P = \$679.18$ ,  $r = .0775$ , and  $t = 11.73$ .Then  $I = Prt = 679.18 \times .0775 \times 11.73 = \$617.4255$ .

(13)

Here  $P = \$950$ ,  $A = \$1763.42$ , and  $t = 10$ .Then  $r = \frac{A - P}{Pt} = \frac{1763.42 - 950}{950 \times 10} = .08562 = \text{rate per unit}$   
 $\therefore 8.562 = \text{rate per cent.}$ 

K

(14)

Here  $P = \$666$ ,  $A = \$1347.50$ , and  $r = .06$ .

$$\text{Then } t = \frac{A - P}{Pr} = \frac{1347.50 - 666}{666 \times .06} = 17.054\frac{1}{2} \text{ years, or 17 years 19 days.}$$

(15)

Here  $P = \$273$ ,  $I = \$100$ , and  $r = .09$ 

$$\text{Then } t = \frac{I}{Pr} = \frac{100}{273 \times .09} = 4.07 \text{ years} = 4 \text{ years 25 days.}$$

(16)

Here  $P = \$476.30$ ,  $A = \$500$ , and  $t = 2$ .

$$\text{Then } r = \frac{A - P}{Pt} = \frac{500 - 476.30}{476.30 \times 2} = .0248 = \text{rate per unit.}$$

$$\therefore 2\frac{1}{2}\% = \text{rate per cent.}$$

(17)

Here  $P = \$749.49$ ,  $I = \$257$ , and  $t = 7$ .

$$\text{Then } r = \frac{I}{Pt} = \frac{257}{749.49 \times 7} = .04898 = \text{rate per unit.}$$

$$\therefore 4.898 = \text{rate per cent.}$$

(18)

Here  $A = \$1111.11$ ,  $t = 11$ , and  $r = .11$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1111.11}{2.21} = \$502.7647.$$

(19)

 $P = £167.47$ ,  $r = .11$ , and  $t = 9$ .

$$I = Prt = 167.47 \times .11 \times 9 = £165.7953 = £165 \text{ 15s. } 10\frac{1}{2}\text{d.}$$

## EXERCISE 105—Page 253.

(1)

$$11 \div 2 = 5\frac{1}{2} \text{ cents.}$$

(2)

$$16 \div 2 = 8 \text{ cents} = \$0.08.$$

(3)

$$9 \text{ years and } 8 \text{ months} = 116 \text{ months, and } 116 \div 2 = 58 \text{ cents} \\ = \$0.58.$$

(4)

$$16 \text{ years and } 3 \text{ months} = 195 \text{ months, and } 195 \div 2 = 97\frac{1}{2} \text{ cents} \\ = \$0.97\frac{1}{2}.$$

(5)

$$11 \text{ years and } 7 \text{ months} = 139 \text{ months, and } 139 \div 2 = 69\frac{1}{2} \text{ cents} \\ = \$0.695.$$

(6)

$$12 \text{ years and } 5 \text{ months} = 149 \text{ months, and } 149 \div 2 = 74\frac{1}{2} \text{ cents} \\ = \$0.745.$$

(7)

$$3 \text{ years and } 2 \text{ months} = 38 \text{ months, and } 38 \div 2 = 19 \text{ cents} = \\ \text{interest of } \$1 \text{ for given rate and time.} \\ \$0.19 \times 279.40 = \$53.086.$$

(8)

$$6 \text{ years and } 7 \text{ months} = 79 \text{ months, and } 79 \div 2 = 39\frac{1}{2} \text{ cents} = \\ \text{interest of } \$1 \text{ for given rate and time.} \\ \$0.395 \times 189.70 = \$74.9315.$$

(9)

3 years and 11 months = 47 months, and  $47 \div 2 = 23\frac{1}{2}$  cents = interest of \$1 for given rate and time.

$$\$0.235 \times 1463 = \$343.805.$$

(10)

11 years and 1 month = 133 months, and  $133 \div 2 = 66\frac{1}{2}$  cents = interest of \$1 for given rate and time.

$$\$0.665 \times 28967.50 = \$19263.3875.$$

## EXERCISE 106—Page 254.

(1)

(2)

$$2 \div 6 = \frac{1}{3} \text{ mill} = \$0.0003.$$

$$7 \div 6 = 1\frac{1}{6} \text{ mills} = \$0.001\frac{1}{6}.$$

(3)

(4)

$$11 \div 6 = 1\frac{5}{6} \text{ mills} = \$0.001\frac{5}{6}. \quad 27 \div 6 = 4\frac{1}{2} \text{ mills} = \$0.004\frac{1}{2}.$$

(5)

$$47 \div 6 = 7\frac{5}{6} \text{ mills} = \$0.007\frac{5}{6}.$$

(6)

$$8 \div 2 = 4 \text{ cents} = \$0.04.$$

$$12 \div 6 = 2 \text{ mills} = \$0.002 \text{ and } \$0.04 + \$0.002 = \$0.042$$

(7)

$$66 \div 6 = 11 \text{ mills} = \$0.011.$$

(8)

$$2 \text{ years } 2 \text{ m'ths} = 26 \text{ months, and } 26 \div 2 = 13 \text{ cents} = \$0.13.$$

$$19 \div 6 = 3\frac{1}{6} \text{ mills} = \$0.003\frac{1}{6} \text{ and } \$0.13 + \$0.003\frac{1}{6} = \$0.133\frac{1}{6}.$$



(9)

7 years 8 m'ths = 92 months, and  $92 \div 2 = 46$  cents = \$0.46.  
 $9 \div 6 = 1\frac{1}{2}$  mills = \$0.001 $\frac{1}{2}$  and  $\$0.46 + \$0.001\frac{1}{2} = \$0.461\frac{1}{2}$ .

(10)

17 years 11 months = 215 months, and  $215 \div 2 = 107\frac{1}{2}$  cents =  
 \$1.075.

$23 \div 6 = 3\frac{5}{6}$  mills = \$0.003 $\frac{5}{6}$ , and  $\$1.075 + \$0.003\frac{5}{6} = \$1.078\frac{5}{6}$ .

(11)

12 years 7 months = 151 months, and  $151 \div 2 = 75\frac{1}{2}$  cents =  
 \$0.755.

$17 \div 6 = 2\frac{5}{6}$  mills = \$0.002 $\frac{5}{6}$ , and  $\$0.755 + \$0.002\frac{5}{6} = \$0.757\frac{5}{6}$ .

## EXERCISE 107—Page 255.

(1)

Interest on \$1 for 7 months = \$0.035

Interest on \$1 for 17 days = 2 $\frac{5}{6}$

Therefore interest on \$1 for 7 months 17 days, = \$0.037 $\frac{5}{6}$

Then  $\$0.037\frac{5}{6} \times 917.30 = \$34.704516$ .

(2)

Interest on \$1 for 3 months = \$0.015

Interest on \$1 for 13 days = 2 $\frac{1}{6}$

Therefore interest on \$1 for 3 months 13 days = \$0.017 $\frac{1}{6}$

Then  $\$0.017\frac{1}{6} \times 842.50 = \$14.462916$ .

(3)

Interest on \$1 for 2 years 11 months = \$0.175

Interest on \$1 for 10 days = 1 $\frac{2}{3}$

Therefore interest on \$1 for 2 years 11 m'ths 10 days = \$0.176 $\frac{2}{3}$

Then  $\$0.176\frac{2}{3} \times 573.83 = \$101.3736$ .

(4)

$$\text{Interest on \$1 for 6 years 9 months} = \$0.405$$

$$\text{Interest on \$1 for 19 days} = 3\frac{1}{6}$$

$$\text{Therefore interest on \$1 for 6 years 9 m'ths 19 days} = \$0.408\frac{1}{6}$$

$$\text{Then } \$0.408\frac{1}{6} \times 642.30 = \$262.16545.$$

(5)

$$\text{Interest on \$1 for 5 years 5 months} = \$0.325$$

$$\text{Interest on \$1 for 7 days} = 1\frac{1}{6}$$

$$\text{Therefore interest on \$1 for 5 years 5 months 7 days} = \$0.326\frac{1}{6}$$

$$\text{Then } \$0.326\frac{1}{6} \times 1427.875 = \$465.7252.$$

(6)

$$\text{Interest on \$1 for 4 years 7 months} = \$0.275$$

$$\text{Interest on \$1 for 16 days} = 2\frac{2}{3}$$

$$\text{Therefore interest on \$1 for 4 years 7 m'ths 16 days} = \$0.277\frac{2}{3}$$

$$\text{Then } \$0.277\frac{2}{3} \times 709.63 = 197.040596.$$

(7)

$$\text{Interest on \$1 for 7 years 7 months} = \$0.455$$

$$\text{Interest on \$1 for 22 days} = 3\frac{2}{3}$$

$$\text{Therefore interest on \$1 for 7 years 7 m'ths 22 days} = \$0.458\frac{2}{3}$$

$$\text{Then } \$0.458\frac{2}{3} \times 2463.20 = \$1129.7877 + \$2463.20 = \$3592.9877.$$

(8)

$$\text{Interest on \$1 for 9 years 9 months} = \$0.585$$

$$\text{Interest on \$1 for 9 days} = 1\frac{1}{2}$$

$$\text{Therefore interest on \$1 for 9 years 9 months 9 days} = \$0.586\frac{1}{2}$$

$$\text{Then } \$0.586\frac{1}{2} \times 999.99 = \$586.494135.$$

(9)

$$\begin{array}{rcl} \text{Interest on \$1 for 3 years 4 months} & = & \$0.20 \\ \text{Interest on \$1 for 27 days} & = & \underline{4\frac{1}{2}} \end{array}$$

$$\text{Therefore interest on \$1 for 3 years 4 m'ths 27 days} = \$0.204\frac{1}{2}$$

$$\text{Then } \$0.2045 \times 68.70 = \$14.04915.$$

(10)

$$\begin{array}{rcl} \text{Interest on \$1 for 3 years} & = & \$0.18 \\ \text{Interest on \$1 for 28 days} & = & \underline{4\frac{2}{3}} \end{array}$$

$$\text{Therefore interest on \$1 for 3 years 28 days} = \$0.184\frac{2}{3}$$

$$\text{Then } \$0.184\frac{2}{3} \times 742.63 = \$137.139.$$

(11)

$$\begin{array}{rcl} \text{Interest on \$1 for 7 years 4 months} & = & \$0.44 \\ \text{Interest on \$1 for 11 days} & = & \underline{1\frac{5}{6}} \end{array}$$

$$\text{Therefore interest on \$1 for 7 years 4 m'ths 11 days} = \$0.441\frac{5}{6}$$

$$\text{Then } \$0.441\frac{5}{6} \times 200 = \$88.366 + \$200 = \$288.366.$$

(12)

$$\begin{array}{rcl} \text{Interest on \$1 for 9 years 3 months} & = & \$0.555 \\ \text{Interest on \$1 for 9 days} & = & \underline{1\frac{1}{2}} \end{array}$$

$$\text{Therefore interest on \$1 for 9 years 3 months 9 days} = \$0.556\frac{1}{2}$$

$$\text{Then } \$0.5565 \times 743.63 = \$413.830095 + \$743.63 = \$1157.460095.$$

## EXERCISE 108—Page 256.

(1)

$$\text{Interest on \$1 at 6 per cent. for given time} = \$0.526\frac{2}{3}.$$

$$\begin{array}{l} \text{Interest on \$1234.56 at 6 per cent. for given time} = \$0.526\frac{2}{3} \times \\ 1234.56 = \$650.2016. \end{array}$$

$$\begin{array}{l} \text{Hence interest on \$1234.56 at 7 per cent. for given time} = \\ \$650.2016 + \text{one sixth of } \$650.2016 = \$758.5685. \end{array}$$

(2)

Interest on \$1 at 6 per cent. for given time =  $\$0.126\frac{2}{3}$ .

Interest on \$9876.54 at 6 per cent. for given time =  $\$0.126\frac{2}{3} \times 9876.54 = \$1252.67449$ .

Hence interest on \$9876.54 at 3 per cent. for given time =  $\$1252.67449 \div 2 = \$626.337245$ .

(3)

Interest on \$1 at 6 per cent. for given time =  $\$0.216\frac{2}{3}$ .

Interest on \$715.30 at 6 per cent. for given time =  $\$0.216\frac{2}{3} \times 715.30 = \$154.98166$ .

Hence interest on \$715.30 at 8 per cent. for given time =  $\$154.98166 + \text{one third of } \$154.98166 = \$206.6422$ .

(4)

Interest on \$1 at 6 per cent. for given time =  $\$0.141\frac{1}{3}$ .

Interest on \$555.55 at 6 per cent. for given time =  $\$0.141\frac{1}{3} \times 555.55 = \$78.51773$ .

Hence interest on \$555.55 at 12 per cent. for given time =  $\$78.51773 \times 2 = \$157.03546$ ; amount =  $\$157.03546 + \$555.55 = \$712.58546$ .

(5)

Interest on \$1 at 6 per cent. for given time =  $\$0.016\frac{2}{3}$ .

Interest on \$7766.55 at 6 per cent. for given time =  $\$0.016\frac{2}{3} \times 7766.55 = \$129.4425$ .

Hence interest on \$7766.55 at 5 per cent. for given time =  $\$129.4425 - \text{one sixth of } \$129.4425 = \$107.86875$ .

Amount =  $\$107.86875 + \$7766.55 = \$7874.41875$ .

(6)

Interest on \$1 at 6 per cent. for given time =  $\$0.521\frac{1}{3}$ .

Interest on \$500 at 6 per cent. for given time =  $\$0.521\frac{1}{3} \times 500 = \$260.666\frac{2}{3}$ .

Hence interest on \$500 at 16 per cent. for given time =  $\$260.666\frac{2}{3} \times 2\frac{2}{3} = \$695.111$ ; amount =  $\$695.111 + \$500 = \$1195.111$ .

(7)

Interest on \$1 at 6 per cent. for given time =  $\$0.206\frac{1}{6}$ .

Interest on \$576 at 6 per cent. for given time =  $\$0.206\frac{1}{6} \times 576$   
 = \$118.752.

Hence interest on \$576 at 5 per cent. for given time = \$118.752  
 —one sixth of \$118.752 = \$98.96.

(8)

Interest on \$1 at 6 per cent. for given time =  $\$0.151\frac{5}{6}$ .

Interest on \$2478.91 at 6 per cent. for given time =  $\$0.151\frac{5}{6} \times$   
 2478.91 = \$376.38116.

Hence interest on \$2478.91 at  $4\frac{1}{2}$  per cent. for given time =  
 \$376.38116 — one fourth of \$376.38116 = \$282.285.

(9)

From May 9th to December 11th = 216 days. Interest on \$1 at  
 6 per cent. for 216 days = \$0.036.

Interest on \$780 at 6 per cent. for 216 days =  $\$0.036 \times 780 =$   
 \$28.08.

(10)

From August 16th 1851 to June 19th 1852 = 308 days.

Interest on \$1 at 6 per cent. for given time =  $\$0.051\frac{1}{3}$ .

Interest on \$1830.63 at 6 per cent. for given time =  $\$0.051\frac{1}{3} \times$   
 1830.63 = \$93.97234.

Hence interest on \$1830.63 at 7 per cent. for given time =  
 \$93.97234 + one sixth of \$93.97234 = \$109.63439.

(11)

From September 3rd 1858 to January 9th 1859 = 128 days.

Interest on \$1 at 6 per cent. for given time =  $\$0.021\frac{1}{3}$ .

Interest on \$6200 at 6 per cent. for given time =  $\$0.021\frac{1}{3} \times 6200$   
 = \$132.266.

Amount = \$132.266 + \$6200 = \$6332.266.

## EXERCISE 109.—Page 258.

(1)

|            |                 |                         |
|------------|-----------------|-------------------------|
| From June  | 2nd to July     | 17th there are 45 days. |
| " July     | 17th to October | 6th " 81 "              |
| " October  | 6th to December | 11th " 66 "             |
| " December | 11th to March   | 29th " 109 "            |
| " March    | 29th to October | 7th " 192 "             |

Whole sum \$1217·30 for 45 days = \$54778·50 for 1 day.  
 1st endorsement 207·80

---

Balance \$1009·50 for 81 days = \$81769·50 for 1 day.  
 2nd endorsement 209·60

---

Balance \$799·90 for 66 days = \$52793·40 for 1 day.  
 3rd endorsement 320·90

---

Balance \$479·00 for 109 days = \$52211·00 for 1 day.  
 4th endorsement 421·83

---

Balance \$57·17 for 192 days = \$10976·64 for 1 day.

---

Whole interest = that of \$252529·04 for 1 day.

Interest on \$252529·04 at 6 per cent. for 1 year = \$15151·7424.

Hence interest for 1 day =  $\$15151\cdot7424 \div 365 = \$41\cdot5116$ .

Then interest due = \$41·5116

Balance on Note = \$57·17

---

Principal and interest due = \$98·6816

(2)

From 17th June to 5th September there are 80 days.

|   |                               |   |     |   |
|---|-------------------------------|---|-----|---|
| " | 5th September to 7th December | " | 93  | " |
| " | 7th December to 11th June     | " | 186 | " |
| " | 11th June to 7th February     | " | 241 | " |
| " | 7th February to 19th December | " | 315 | " |
| " | 19th December to 1st May      | " | 133 | " |

Whole sum \$7348.25 for 80 days = \$587860.00 for 1 day.  
1st endorsement 2463.80

Balance \$4884.45 for 93 days = \$454253.85 for 1 day.  
2nd endorsement 392.20

Balance \$4492.25 for 186 days = \$835558.50 for 1 day.  
3rd endorsement 982.20

Balance \$3540.05 for 241 days = \$845922.05 for 1 day.  
4th endorsement 2842.90

Balance \$667.15 for 315 days = \$210152.25 for 1 day.  
5th endorsement 317.23

Balance \$349.92 for 133 days = \$46539.36 for 1 day.

Whole interest = that of \$2980286.01 for 1 day.

Interest on \$2980286.01 at 8 per cent. for 1 year = \$238422.8808.

Hence interest for 1 day =  $\$238422.8808 \div 365 = \$653.2133$ .

Then interest due = \$653.2133

Balance on Note = \$349.92

Principal and interest due = \$1003.1333

## EXERCISE 110.—Page 259.

(1)

\$1800  
108  

---

Principal.  
Interest for 1st year.

\$1908  
114.48  

---

Amount for 1 year = principal for 2nd year.  
Interest for 2nd year.

\$2022.48  
121.3488  

---

Amount for 2 years = principal for 3rd year.  
Interest for 3rd year.

\$2143.8288  
128.629728  

---

Amount for 3 years = principal for 4th year.  
Interest for 4th year.

\$2272.458528  
136.347511  

---

Amount for 4 years = principal for 5th year.  
Interest for 5th year.

\$2408.806039  
1800  

---

Amount for 5 years.  
Given Principal.

\$608.806 =

Compound interest required.

(2)

\$700  
49  

---

Principal.  
Interest for 1st half year.

\$749  
52.43  

---

Amount for 1 half y. = principal for 2nd half y.  
Interest for 2nd half year.

\$801.43  
56.1001  

---

Amount for 1 year = principal for 3rd half y.  
Interest for 3rd half year.

\$857.5301  
60.027107  

---

Amount for  $1\frac{1}{2}$  years = principal for 4th half y  
Interest for 4th half year.

\$917.557207  
64.229004  

---

Amount for 2 years = principal for 5th half y  
Interest for 5th half year.

\$981.786211  
68.725034  

---

Amount for  $2\frac{1}{2}$  years = principal for 6th half y.  
Interest for 6th half year.

\$1050.511245  
73.535787  

---

Amount for 3 years = principal for 7th half y  
Interest for 7th half year.

\$1124.047032  
700  

---

Amount for  $3\frac{1}{2}$  years.  
Given Principal.

\$424.047 =

Compound interest required.



(3)

|               |   |
|---------------|---|
| \$673.40      | Principal.  |
| 20.202        | Interest for 1st quarter.                         |
| <hr/>         |   |
| \$693.602     | Amount for 1 quar. = principal for 2nd quarter.   |
| 20.80806      | Interest for 2nd quarter.                         |
| <hr/>         |   |
| \$714.41006   | Amount for 1 half y. = principal for 3rd quart.   |
| 21.4323018    | Interest for 3rd quarter.                         |
| <hr/>         |   |
| \$735.8423618 | Amount for 3 quarters = principal for 4th quar.   |
| 22.0752708    | Interest for 4th quarter.                         |
| <hr/>         |   |
| \$757.9176326 | Amount for 1 year = principal for 5th quarter.    |
| 22.7375289    | Interest for 5th quarter.                         |
| <hr/>         |   |
| \$780.6551615 | Amount for 5 quarters = principal for 6th quar.   |
| 23.4196548    | Interest for 6th quarter.                         |
| <hr/>         |   |
| \$804.0748163 | Amount for 3 half y. = principal for 7th quarter. |
| 24.1222444    | Interest for 7th quarter.                         |
| <hr/>         |   |
| \$828.1970607 | Amount for 7 quarters = principal for 8th qr.     |
| 24.8459118    | Interest for 8th quarter.                         |
| <hr/>         |   |
| \$853.0429 =  | Amount for 2 years required.                      |
| 673.40        | Given Principal.                                  |
| <hr/>         |   |
| \$179.6429 =  | Compound Interest required.                       |

(4)

|                |   |
|----------------|---|
| \$860          | Principal.  |
| 34.4           | Interest for 1st half year.                         |
| <hr/>          |   |
| \$894.4        | Amount for 1 half year = principal for 2nd half y.  |
| 35.776         | Interest for 2nd half year.                         |
| <hr/>          |   |
| \$930.176      | Amount for 1 year = principal for 3rd half year.    |
| 37.20704       | Interest for 3rd half year.                         |
| <hr/>          |   |
| \$967.38304    | Amount for 3 half years = principal for 4th half y. |
| 38.69532       | Interest for 4th half year.                         |
| <hr/>          |   |
| \$1006.07836   | Amount for 2 years = principal for 5th half year.   |
| 40.24313       | Interest for 5th half year.                         |
| <hr/>          |   |
| \$1046.32149   | Amount for 5 half years = principal for 6th half y. |
| 41.85285       | Interest for 6th half year.                         |
| <hr/>          |   |
| \$1088.17434 = | Amount for 3 years required.                        |
| 860            | Given Principal.                                    |
| <hr/>          |   |
| \$228.1743 =   | Compound Interest required.                         |

## EXERCISE 111—Page 261.

(1)

By the table the am't of \$1 at 6 per cent. for 11 years = \$1.8983.

Then  $\$1.8983 \times 875 = \$1661.0125 = \text{Amount.}$ 

|     |                   |            |
|-----|-------------------|------------|
| 875 | <u>          </u> | Principal. |
|-----|-------------------|------------|

 $\$786.0125 = \text{Interest.}$ 

(2)

By the table the am't of \$1 for the given time and rate = \$2.77247.

Then  $\$2.77247 \times 643.98 = \$1785.41523 = \text{Amount.}$ 

|        |                   |            |
|--------|-------------------|------------|
| 643.98 | <u>          </u> | Principal. |
|--------|-------------------|------------|

 $\$1141.43523 = \text{Interest.}$ 

(3)

By the table the am't of \$1 at 6 per cent. for 45 years = \$13.76461.

Then  $\$13.76461 \times .01 = \$1.37646 = \text{Amount.}$ 

|     |                   |            |
|-----|-------------------|------------|
| .01 | <u>          </u> | Principal. |
|-----|-------------------|------------|

 $\$.127646 = \text{Interest.}$ 

(4)

By the table the am't of \$1 for the given time and rate = \$2.28793.

Then  $\$2.28793 \times 78.2 = \$178.916 = \text{Amount.}$ 

|      |                   |            |
|------|-------------------|------------|
| 78.2 | <u>          </u> | Principal. |
|------|-------------------|------------|

 $\$100.716 = \text{Interest.}$ 

(5)

By the table the am't of \$1 for the given rate and time = \$2.40662

Then  $\$2.40662 \times 777.77 = \$1871.7968 = \text{Amount.}$ 

|        |                   |            |
|--------|-------------------|------------|
| 777.77 | <u>          </u> | Principal. |
|--------|-------------------|------------|

 $\$1094.0268 = \text{Interest.}$

(6)

$$£44 \text{ 5s. 9d.} = £44.2875.$$

By the table the am't of £1 at 6 per cent. for 11 years = £1.8983

Then  $£1.8983 \times 44.2875 = £84.07096 = £84 \text{ 1 5} = \text{Amount.}$

44 5 9    Principal

                      
£39 15 8 = Interest.

(7)

$$£32 \text{ 4s. } 9\frac{3}{4}\text{d.} = £32.240625.$$

By the table the amount of £1 for the given time and rate =

£1.26532. Then  $£1.26532 \times 32.240625 =$

$£40.7947076 = £40 \text{ 15 } 10\frac{3}{4} \text{ nearly} = \text{Amount.}$

32 4 9 $\frac{3}{4}$     Principal.

                      
£8 11 1 = Interest.

EXERCISE 112—Page 262.

(1)

Amount of \$1 for 7 years at 4 per cent. = \$1.31593.

$\$7439.87 \div 1.31593 = \$5653.697.$

(2)

Amount of \$1 at 5 per cent. for 20 years = \$2.6533.

$\$9193.90 \div 2.6533 = \$3465.081.$

(3)

$$£595 \text{ 10s. } 2\frac{1}{2}\text{d.} = £595.51$$

Amount of £1 at 6 per cent. for 3 years = £1.19102.

$£595.51 \div 1.19102 = £500.$

(4)

Amount of \$1 at 6 per cent. for 7 years = \$1.50363.

$\$7111.11 \div 1.50363 = \$4729.295.$

(5)

$$£268 \text{ 0s. } 4\frac{1}{2}\text{d.} = £268.02.$$

$$\text{Amount of } £1 \text{ at 5 per cent for 6 years} = £1.3401.$$

$$£268.02 \div 1.3401 = £200.$$

## EXERCISE 113—Page 263.

(1)

$$\text{Here } A = \$962, r = .04, \text{ and } t = 1. \text{ Whence } 1 + rt = 1.04.$$

$$\text{Then } P = \frac{A}{1 + rt} = \frac{962}{1.04} = \$925.$$

(2)

$$\text{Here } A = \$2202, r = .06, \text{ and } t = 5.75. \text{ Whence } 1 + rt = 1.345.$$

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2202}{1.345} = \$1637.174.$$

(3)

$$\text{Here } A = \$1003.50, r = .06, \text{ and } t = \frac{2}{3} \text{ year. Whence } 1 + rt = 1.04.$$

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1003.50}{1.04} = \$964.9038.$$

(4)

$$\text{Here } A = \$716, r = .08, \text{ and } t = \frac{7}{12} \text{ year. Whence } 1 + rt = 1.04\frac{2}{3}.$$

$$\text{Then } P = \frac{A}{1 + rt} = \frac{716}{1.04\frac{2}{3}} = \$684.0764.$$

(5)

Here  $A = \$1342.50$ ,  $r = .065$ , and  $t = \frac{25}{12}$  year. Whence  $1 + rt = 1.022\frac{19}{24}$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1342.50}{1.022\frac{19}{24}} = \$1313.266.$$

(6)

Here  $A = \$2400$ ,  $r = .05$ , and  $t = \frac{336}{12}$  year. Whence  $1 + rt = 1.03\frac{17}{24}$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2400}{1.03\frac{17}{24}} = \$2324.84.$$

(7)

Here  $A = \$2202$ ,  $r = .05$ , and  $t = .75$  year. Whence  $1 + rt = 1.0375$ .

$$\$2202 \div 1.0375 = \$2122.40963+ = \text{Present worth.}$$

$$\$2202 - \$2122.40963+ = \$79.59036 = \text{Discount.}$$

(8)

Here  $A = \$4360$ ,  $r = .06$ , and  $t = 1\frac{5}{8}$ . Whence  $1 + rt = 1.085$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{4360}{1.085} = \$4018.43317.$$

(9)

Here  $A = \$1647$ ,  $r = .06$ , and  $t = 1\frac{1}{2}$  year. Whence  $1 + rt = 1.055$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1647}{1.055} = \$1561.13744.$$

(10)

Here  $A = \$2000$ ,  $r = .06$ , and  $t = 3\frac{7}{8}$ . Whence  $1 + rt = 1.215$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2000}{1.215} = \$1646.09053.$$

(11)

Here  $A = \$2070.90$ ,  $r = .05$ , and  $t = 1\frac{1}{2}$ . Whence  $1 + rt = 1.07\frac{1}{2}$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{2070.90}{1.07\frac{1}{2}} = \$1918.9806.$$

$\$2070 - \$1918.9806 = \$151.019 = \text{Discount required.}$

(12)

Here  $A = \$970.63$ ,  $r = .08$ , and  $t = \frac{1}{2}$  year. Whence  $1 + rt = 1.07\frac{1}{2}$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{970.63}{1.07\frac{1}{2}} = \$904.313.$$

(13)

Here in first case  $A = \$1512$ ,  $r = .07$ , and  $t = .5$  year. Whence  $1 + rt = 1.035$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1512}{1.035} = \$1460.8695.$$

Also  $A = 1512$ ,  $r = .07$ , and  $t = 1$ . Whence  $1 + rt = 1.07$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{1512}{1.07} = \$1413.0841.$$

$\$1460.8695 + \$1413.0841 = \$2873.9536 = \text{Present worth of whole amount.}$

$\$3024 - \$2873.9536 = \$150.0464 = \text{Discount required.}$

(14)

Here in first case  $A = \$440$ ,  $r = .08$ , and  $t = 1.25$ . Whence  $1 + rt = 1.1$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{440}{1.1} = \$400.$$

In second case  $A = \$896$ ,  $r = .08$ , and  $t = 1.5$ . Whence  $1 + rt = 1.12$ .

$$\text{Then } P = \frac{A}{1 + rt} = \frac{896}{1.12} = \$800.$$

$\$400 + \$800 = \$1200.$

## EXERCISE 114—Page 265.

(1)

Here the time the note has to run is 2 years 3 months 3 days.

Interest of \$1 at 7 per cent. for 2 yrs., 3 m., 3 days =  $\$0.1580\frac{5}{6}$ .

Interest of \$986 at 7 per cent. for 2 years, 3 months, 3 days =  
 $\$0.1580\frac{5}{6} \times 986 = \$155.8701$ .

(2)

Here the time the note has to run is 103 days = 3 months 13 days.

Interest of \$1 at 8 per cent. for 3 months 13 days =  $\$0.022\frac{3}{4}$ .

Interest of \$640 at 8 per cent. for 3 months, 13 days =  
 $\$0.022\frac{3}{4} \times 640 = \$14.6488$ .

(3)

Here the time the note has to run is 94 days = 3 months 4 days.

Interest of \$1 at 6 per cent. for 3 months 4 days =  $\$0.015\frac{2}{3}$ .

Interest of \$563.80 at 6 per cent. for 3 months 4 days =  
 $\$0.015\frac{2}{3} \times 563.80 = \$8.8328$  and  $\$563.80 - \$8.8328 = \$554.967$ .

## EXERCISE 115—Page 266.

(1)

Interest on \$1 for 93 days at 7 p. c. =  $\$0.0180\frac{5}{6}$ , and this taken from \$1 gives a remainder of  $\$0.9819\frac{1}{6}$  = present worth of \$1.

Then  $\$3755 \div 0.9819\frac{1}{6} = \$3824.15$ .

(2)

Interest on \$1 for 6 months 3 days at 5 per cent. =  $\$0.0254\frac{1}{2}$   
 and this taken from \$1 gives a remainder  $\$0.9745\frac{1}{2}$  = present worth of \$1.

Then  $\$1147.80 \div 0.9745\frac{1}{2} = \$1177.734$ .

(3)

Interest on \$1 for 48 days at  $3\frac{1}{2}$  per cent. =  $\$0.004\frac{2}{3}$ , and this taken from \$1 gives a remainder  $\$0.9953\frac{1}{3}$  = present worth of \$1.

$$\text{Then } \$713.90 \div 0.995\frac{1}{3} = \$717.2471.$$

## EXERCISE 116—Page 268.

(1)

$$\begin{array}{r} \$200 \times 3 = 600 \\ 150 \times 4 = 600 \\ 250 \times 6 = 1500 \\ \hline 600 \quad 600)2700(4\frac{1}{2} \text{ months.} \\ \underline{2400} \\ 300 \\ \hline 600 \end{array} \left. \vphantom{\begin{array}{r} 2700 \\ 2400 \\ 300 \\ 600 \end{array}} \right\} = \frac{1}{2}$$

(2)

$$\begin{array}{r} \frac{1}{4} \times 0 = 0 \\ \frac{1}{4} \times 3 = \frac{3}{4} \\ \frac{1}{4} \times 6 = 1\frac{1}{2} \\ \frac{1}{4} \times 9 = 2\frac{1}{4} \\ \hline 1 \quad 1)4\frac{1}{2} \\ \hline 4\frac{1}{2} \text{ months.} \end{array}$$

(3)

$$\begin{array}{r} \$50 \times 2 = 100 \\ 40 \times 5 = 200 \\ 30 \times 7 = 210 \\ \hline 120 \quad 120)510(4\frac{1}{4} \text{ months.} \\ \underline{480} \\ 30 \\ \hline 120 \end{array} \left. \vphantom{\begin{array}{r} 510 \\ 480 \\ 30 \\ 120 \end{array}} \right\} = \frac{1}{4}$$

(4)

$$\begin{array}{r} \$1000 \times 0 = 0 \\ 1500 \times 1 = 1500 \\ 600 \times 3 = 1800 \\ 700 \times 5 = 3500 \\ 1400 \times 7 = 9800 \\ \hline 5200 \quad 5200)16600(3\frac{5}{26} \text{ months.} \\ \underline{15600} \\ 1000 \\ \hline 5200 \end{array} \left. \vphantom{\begin{array}{r} 16600 \\ 15600 \\ 1000 \\ 5200 \end{array}} \right\} = \frac{5}{26}$$



(5)

Six months from 15th January = 15th July, and from 1st July to 15th July there are 14 days.

Six months from 10th February = 10th August, and from 1st July to 10th August there are 40 days.

Six months from 6th March = 6th September, and from 1st July to 6th September there are 67 days.

Six months from 8th June = 8th December, and from 1st July to 8th December there are 160 days.

$$\begin{array}{r}
 \$3750 \times 14 = 52500 \\
 3000 \times 40 = 120000 \\
 2400 \times 67 = 160800 \\
 2250 \times 160 = 360000 \\
 \hline
 11400 \quad 11400) 693300 (60\frac{3}{8} \text{ days.} \\
 \underline{684000} \\
 9300 \\
 \underline{11400} \quad \left. \vphantom{\begin{array}{r} 9300 \\ 11400 \end{array}} \right\} = \frac{3}{8}
 \end{array}$$

Therefore the note must be made payable on the 61st day from the 1st of July, which is the 31st of August.

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EXERCISE 117.—Page 269.

(1)

Whole stock : A's stock :: whole profit : A's profit.

$$1117 \times 3000$$

That is, \$4300:\$3000::\$1117:  $\frac{1117 \times 3000}{4300} = \$779.302 = \text{A's sh.}$

\$1117 - \$779.302 = \$337.697 = B's share.

(2)

Whole stock = \$6470 + \$3780 + \$9860 = \$20110.

Whole stock : A's stock :: whole profit ; A's profit,

$$7890 \times 6470$$

That is, \$20110 : \$6470 :: \$7890 :  $\frac{7890 \times 6470}{20110} = \$2538.453+ = A's \text{ sh.}$

Again, whole stock : B's stock :: whole profit ; B's profit.

$$7890 \times 3780$$

That is, \$20110 : \$3780 :: \$7890 :  $\frac{7890 \times 3780}{20110} = \$1483.053+ = B's \text{ sh.}$

Lastly, whole stock : C's stock :: whole profit : C's profit.

$$7890 \times 9860$$

That is, \$20110 : \$9860 :: \$7890 :  $\frac{7890 \times 9860}{20110} = \$3868.493+ = C's \text{ sh.}$

(3)

Whole stock : B's stock :: whole gain : B's gain.

$$80 \times 120$$

That is, \$320 : \$120 :: \$80 :  $\frac{80 \times 120}{320} = \$30 = B's \text{ gain.}$

Again, whole stock : C's stock :: whole gain : C's gain.

$$80 \times 200$$

That is, \$320 : \$200 :: \$80 :  $\frac{80 \times 200}{320} = \$50 = C's \text{ share.}$

(4)

Whole stock : B's stock :: whole gain : B's gain.

$$728 \times 1200$$

That is, \$2800 : \$1200 :: \$728 :  $\frac{728 \times 1200}{2800} = \$312 = B's \text{ gain.}$

Again, whole stock : C's stock :: whole gain : C's gain.

$$728 \times 1600$$

That is, \$2800 : \$1600 :: \$728 :  $\frac{728 \times 1600}{2800} = \$416 = C's \text{ gain,}$

(5)

Whole stock : B's stock :: whole amount to be divided : B's share

$$\text{That is, } \$3 : \$2 :: \$100 : \frac{100 \times 2}{3} = \$66.66\frac{2}{3} = \text{B's share.}$$

Again, whole st'k : C's st'k :: whole amo't to be divided : C's sh'e

$$\text{That is, } \$3 : \$1 :: \$100 : \frac{100 \times 1}{3} = \$33.33\frac{1}{3} = \text{C's share.}$$

(6)

$$£1400 : £500 :: £1100 : \frac{1100 \times 500}{1400} = £392\frac{5}{7} = \text{B's share.}$$

$$£1100 - £392\frac{5}{7} = £707\frac{2}{7} = \text{C's share.}$$

(7)

$$\begin{array}{l} \text{casks. casks. } 180 \times 200 \\ 900 : 200 :: 180 : \frac{\quad}{900} = 40 \text{ casks} = \text{B's loss.} \end{array}$$

$$900 : 300 :: 180 : \frac{180 \times 300}{900} = 60 \text{ casks} = \text{C's loss.}$$

$$180 - (40 + 60) = 80 \text{ casks} = \text{D's loss.}$$

(8)

$$\$1800 : \$800 :: \$100 : \frac{100 \times 800}{1800} = \$44.44\frac{4}{9} = \text{B's share.}$$

$$\$1800 : \$600 :: \$100 : \frac{100 \times 600}{1800} = \$33.33\frac{1}{3} = \text{C's share.}$$

$$\$44.44\frac{4}{9} + \$33.33\frac{1}{3} = \$77.77\frac{7}{9}, \text{ and } \$100 - \$77.77\frac{7}{9} = \$22.22\frac{2}{9} = \text{D's share.}$$

(9)

$$6 : 1 :: 120 : \frac{120 \times 1}{6} = 20$$

$$6 : 2 :: 120 : \frac{120 \times 2}{6} = 40$$

$$6 : 3 :: 120 : \frac{120 \times 3}{6} = 60$$

(10)

$$\text{Whole loss} = \$900 - \$540 = \$360.$$

$$8 : 1 :: \$360 : \frac{360}{8} = \$45 = \text{B's loss.}$$

$$8 : 2 :: \$360 : \frac{360 \times 2}{8} = \$90 = \text{C's loss.}$$

$$\$45 + 90 = \$135, \text{ and } \$360 - 135 = \$225 = \text{D's loss.}$$

(11)

$$\$12 : \$6 :: \$1320 : \frac{1320 \times 6}{12} = \$660 = \text{B's gain.}$$

$$\$12 : \$4 :: \$1320 : \frac{1320 \times 4}{12} = \$440 = \text{C's gain.}$$

$$\$12 : \$2 :: \$1320 : \frac{1320 \times 2}{12} = \$220 = \text{D's gain.}$$

(12)

$$£35 + £29 = £64, \text{ and } £110 - £64 = £46 = \text{D's profit.}$$

$$\text{D's profit} : \text{B's profit} :: \text{D's stock} : \text{B's stock.}$$

$$\text{That is, } £46 : £35 :: £1090 : \frac{1090 \times 35}{46} = £829 \text{ 6s. } 11\frac{1}{2}\text{d.} = \text{B's st.}$$

$$\text{Again, D's profit} : \text{C's profit} :: \text{D's stock} : \text{C's stock.}$$

$$\text{That is, } £46 : £29 :: £1090 : \frac{1090 \times 29}{46} = £687 \text{ 3s. } 5\frac{1}{2}\text{d.} = \text{C's st.}$$

## EXERCISE 118.—Page 271.

(1)

$$\left. \begin{array}{l} \$357 \times 5 = \$1785 \text{ for one month} \\ 371 \times 7 = 2597 \text{ for one month} \\ 154 \times 11 = 1694 \text{ for one month} \end{array} \right\} = \$6076 \text{ for one month.}$$

$$\$6076 : \$1785 :: \$347 \cdot 20 : \frac{347 \cdot 20 \times 1785}{6076} = \$102$$

$$\$6076 : \$2597 :: \$347 \cdot 20 : \frac{347 \cdot 20 \times 2597}{6076} = \$148 \cdot 40.$$

$$\$6076 : \$1694 :: \$347 \cdot 20 : \frac{347 \cdot 20 \times 1694}{6076} = \$96 \cdot 80$$

(2)

$$\left. \begin{array}{l} 40 \times 6 = 240 \text{ for one month} \\ 30 \times 5 = 150 \text{ for one month} \\ 50 \times 1 = 50 \text{ for one month} \end{array} \right\} = 440 \text{ for one month.}$$

$$440 : 240 :: \$160 : \frac{160 \times 240}{440} = \$87 \cdot 27_{11}^3; \text{ B's share.}$$

$$440 : 150 :: \$160 : \frac{160 \times 150}{440} = \$54 \cdot 54_{11}^6; \text{ C's share.}$$

$$440 : 50 :: \$160 : \frac{160 \times 50}{440} = \$18 \cdot 18_{11}^2; \text{ D's share.}$$

(3)

$$\left. \begin{array}{l} £150 \times 6 = £900 \text{ for one month} \\ 200 \times 3 = 600 \text{ for one month} \\ 125 \times 16 = 2000 \text{ for one month} \end{array} \right\} = £3500 \text{ for one month.}$$

$$£3500 : £900 :: £291 \text{ 13s. 4d.} : \frac{£291 \text{ 13s. 4d.} \times 900}{3500} = £75.$$

$$£3500 : £600 :: £291 \text{ 13s. 4d.} : \frac{£291 \text{ 13s. 4d.} \times 600}{3500} = £50.$$

$$£3500 : £2000 :: £291 \text{ 13s. 4d.} : \frac{£291 \text{ 13s. 4d.} \times 2000}{3500} = £166 \text{ 13s. 4d.}$$

(4)

$$\left. \begin{array}{l} \$4000 \times 12 = \$48000 \text{ for one month} \\ 3000 \times 15 = 45000 \text{ for one month} \\ 5000 \times 8 = 40000 \text{ for one month} \end{array} \right\} = \$133000 \text{ for one month}$$

$$\$133000 : \$48000 :: \$665 : \frac{665 \times 48000}{133000} = \$240 ; \text{B's share.}$$

$$\$133000 : \$45000 :: \$665 : \frac{665 \times 45000}{133000} = \$225 ; \text{C's share.}$$

$$\$133000 : \$40000 :: \$665 : \frac{665 \times 40000}{133000} = \$200 ; \text{D's share.}$$

(5)

$$\left. \begin{array}{l} 56 \times 12 = 672 \text{ for one day} \\ 64 \times 15 = 960 \text{ for one day} \\ 80 \times 18 = 1440 \text{ for one day} \end{array} \right\} = 3072 \text{ for one day.}$$

$$3072 : 672 :: \$320 : \frac{320 \times 672}{3072} = \$70 = \text{rent to be paid by 1st troop.}$$

$$3072 : 960 :: \$320 : \frac{320 \times 960}{3072} = \$100 = \text{ " " " 2nd "}$$

$$3072 : 1440 :: \$320 : \frac{320 \times 1440}{3072} = \$150 = \text{ " " " 3rd "}$$

(7)

$$\text{Sum of profits} = 240 + 800 + 400 = \$1440.$$

$$\text{Whole profit} : \text{A's profit} :: \text{Whole stock for 1 m.} : \text{A's st. for 1 m.}$$

$$\text{That is, } 1440 : 240 :: 34560 : \frac{34560 \times 240}{1440} = 5760 = \text{A's stock}$$

for one month. Hence, since A's stock was in for 6 months, it will be  $\$5760 \div 6 = \$960$ .

(Continued on next page.)

(7 Continued.)

Whole profit : B's profit :: Whole stock for 1 m. : B's st. for 1 m.

$$1440 : 800 :: 34560 : \frac{34560 \times 800}{1440} = 19200 = \text{B's stock for one}$$

month. And, since B's stock was in for 12 months,  $19200 \div 12 = \$1600$  will be his stock.

Whole profit : C's profit :: whole stock for 1 m. : C's st. for 1 m.

$$1440 : 400 :: 34560 : \frac{34560 \times 400}{1440} = \$9600 = \text{C's stock for one}$$

month, and hence his stock will be  $\$9600 \div 15 = \$640$ .

(8)

A's profit was \$240 for 6 months = \$40 for 1 month.

B's profit was \$800 for 12 months = \$66 $\frac{2}{3}$  for 1 month.C's profit was \$400 for 15 months = \$26 $\frac{2}{3}$  for 1 month.

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Sum of profits for 1 month = \$133 $\frac{1}{3}$

Whole profit for 1 m. : A's profit for 1 m. :: whole stock : A's st.

$$133\frac{1}{3} : 40 :: 3200 : \frac{3200 \times 40}{133\frac{1}{3}} = \$960 = \text{A's stock.}$$

$$133\frac{1}{3} : 66\frac{2}{3} :: 3200 : \frac{3200 \times 66\frac{2}{3}}{133\frac{1}{3}} = \$1600 = \text{B's stock.}$$

$$133\frac{1}{3} : 26\frac{2}{3} :: 3200 : \frac{3200 \times 26\frac{2}{3}}{133\frac{1}{3}} = \$640 = \text{C's stock.}$$


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EXERCISE 119—Page 275.

(1)

\$0.12 $\frac{1}{2}$  = selling price.

\$0.09 = buying price.

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\$0.03 $\frac{1}{2}$  = gain per lb..

\$0.03 $\frac{1}{2}$   $\times$  317 = \$11.095.

(2)

\$1.20 = selling price

\$0.87 $\frac{1}{2}$  = buying price.

---

\$0.32 $\frac{1}{2}$  = gain per bushel

\$0.32 $\frac{1}{2}$   $\times$  2138 = \$694.85.

(3)

$\$0.15 \times 317 \times 13 = \$618.15 = \text{cost of 13 barrels at } \$0.15 \text{ per lb.}$   
 $\$735 - 618.15 = \$116.85 \text{ gain.}$

(4)

$\$3.15 \times 22 \times 17 = \$1178.10 = \text{price of 17 kegs at } \$3.15 \text{ per gal.}$   
 $\$0.37\frac{1}{2} \times 1178.1 = \$441.7875 = \text{ad valorem duty.}$   
 $\$1178.10 + \$441.7875 + \$26.33 = \$1646.2175 = \text{whole cost.}$   
 $\$1646.2175 - \$1625 = \$21.2175 \text{ loss.}$

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EXERCISE 120—Page 276.

(1)

Here for every \$1 I expend I wish to receive \$1.30, and hence the selling price will be  $\$3.25 \times 1.30 = \$4.22\frac{1}{2}$ .

(2)

Here for every \$1 I expend I wish to receive \$1.05, and hence the selling price will be  $\$1.05 \times 13420 = \$14091$ .

(3)

Here for every \$1 I expend I desire to receive \$1.15, and hence the selling price will be  $\$1.15 \times .11 = \$0.1265 = 12\frac{1}{2}\frac{3}{4} \text{ cents.}$

(4)

Here for every \$1 I expend I wish to receive \$1.23, and hence the selling price will be  $\$1.23 \times 15.25 = \$18.75\frac{3}{4}$ .

(5)

Here for every \$1 I expend I am willing to receive \$0.89, and hence the selling price will be  $\$0.89 \times 7890 = \$7022.10$ .



## EXERCISE 121—Page 277.

(1)

Here the whole gain is  $\$0.87\frac{1}{2} - \$0.60 = \$0.27\frac{1}{2}$ .

That is,  $\$0.60$  gains  $\$0.27\frac{1}{2}$ , and therefore 1 cent gains  $\frac{27\frac{1}{2}}{60} =$

$$\frac{55}{120} = \frac{11}{24} \text{ of a cent.}$$

And hence, the gain per cent  $= \frac{11}{24} \times 100 = \frac{1100}{24} = 45\frac{5}{6}$  per cent.

(2)

Here the loss on each lb. is 2 cents.

That is, every 13 cents invested gives a loss of 2 cents.

Therefore every cent invested loses  $\frac{1}{13}$  of 2  $= \frac{2}{13}$  cents.

And hence, the loss per cent  $= \frac{2}{13} \times 100 = \frac{200}{13} = 15\frac{5}{13}$  per c.

(3)

Here the gain on each barrel is  $\$1.60$ .

That is, every  $\$6.20$  invested gives a gain of  $\$1.60$ .

Therefore every  $\$1$  invested gains  $\frac{160}{620}$  of 160  $= \frac{8}{31}$  of a \$.

And hence, the gain per cent  $= \frac{8}{31} \times 100 = \frac{800}{31} = 25.8 = 25\frac{8}{31}$  p. c.

(4)

Here the gain on each yard is 35 cents.

That is, every  $\$2.75$  invested gives a gain of 35 cents.

Therefore every  $\$1$  invested gains  $\frac{35}{275}$  of 35  $= \frac{35}{275} = \frac{7}{55}$  of a dollar.

And hence the gain per cent  $= \frac{7}{55} \times 100 = \frac{700}{55} = 12\frac{8}{11}$  p. c.

(5)

Here the gain on every bushel is 9 cents.

That is, every 47 cents invested gives a gain of 9 cents.

Therefore every cent invested gains  $\frac{1}{47}$  of 9  $= \frac{9}{47}$  cents.

And hence the gain per cent  $= \frac{9}{47} \times 100 = \frac{900}{47} = 19\frac{7}{47}$  p. c.

(6)

Here the loss on each lb. is  $1\frac{1}{2}$  cents.

That is every 12 cents invested gives a loss of  $1\frac{1}{2}$  cents.

Therefore every cent invested loses  $\frac{1}{12}$  of  $1\frac{1}{2} = \frac{1}{8}$  of a cent.

And hence, the loss per cent  $= \frac{1}{8} \times 100 = \frac{100}{8} = 12\frac{1}{2}$  p. c.

(7)

Here the whole gain is  $\$127 - \$93 = \$34$ .

That is,  $\$93$  gains  $\$34$ , and therefore  $\$1$  gains  $\frac{34}{93}$  of a dollar.

Hence, gain per cent  $= \frac{34}{93} \times 100 = \frac{3400}{93} = 36\frac{52}{93}$  per cent.

(8)

Here the loss is  $\$6742.50 - \$6000 = \$742.50$ .

That is,  $\$6742.50$  loses  $\$742.50$ , and therefore  $\$1$  loses  $\frac{742.50}{6742.50}$  of  $742.50 = \frac{99}{899}$  of a dollar.

Hence loss per cent  $= \frac{99}{899} \times 100 = \frac{9900}{899} = 11\frac{11}{899}$  per cent.

(9)

Here  $\$5700 + \$275 + \$1987.32 = \$7962.32 =$  whole sum expended.

Whole gain  $= \$8750 - \$7962.32 = \$787.68$ .

That is,  $\$7962.32$  gains  $\$787.68$ , and therefore  $\$1$  gains  $\frac{787.68}{7962.32}$  of  $787.68 = \frac{9846}{99629}$  of a  $\$$ .

Hence gain per cent  $= \frac{9846}{99629} \times 100 = \frac{984600}{99629} = 9.89$  or nearly 10 per cent.

(10)

$\$4.25 \times 723 = \$3072.75 =$  price of 723 yds. @  $\$4.25$ .

$\$3072.75 \times .07 = \$215.0925 =$  amount for Insurance.

$\$3072.75 \times .22 = \$676.005 =$  amount for ad valorem duty.

Then whole cost  $= \$3072.75 + \$215.0925 + \$23.70 + \$2.70 + \$3.16 + \$676.005 = \$3993.4075$ .

Whole gain  $= \$5270 - \$3993.4075 = \$1276.5925$ .

That is,  $\$3993.4075$  gains  $\$1276.5925 \therefore \$1$  gains  $\frac{1276.5925}{3993.4075}$  of  $\$1276.5925 = \frac{510637}{1597363}$  of a  $\$$ .

Hence gain per cent  $= \frac{510637}{1597363} \times 100 = 31.96749$  or nearly 32 per cent.

## EXERCISE 122—Page 278.

(1)

Loss on \$1 is 4 cents, or for every \$1 paid I receive \$0.96.

Hence cost =  $\$24.60 \div 0.96 = \$25.625$ .

(2)

Loss on \$1 is 10 cents, or for every \$1 paid he receives \$0.90.

Hence cost =  $\$2360 \div .90 = \$2622.22$ .

(3)

Gain on \$1 is 11 cents, or for every \$1 paid he receives \$1.11.

Hence cost =  $\$7400 \div 1.11 = \$6666.666$ .

(4)

Gain on \$1 is 17 cents, or for every \$1 paid he receives \$1.17.

$$\$117 : \$100 :: \$3789.40 : \frac{3789.40 \times 100}{117} = \$3238.803.$$

(5)

Loss on \$1 is 13 cents, or for every \$1 paid I receive \$0.87.

$$\$87 : \$100 :: \$2740 : \frac{2740 \times 100}{87} = \$3149.425.$$

## EXERCISE 123—Page 279.

(1)

\$2 gains 50 cents.

$$\text{Hence } \$0.50 : \$0.10 :: \$2.00 : \frac{2.00 \times 10}{50} = 40 \text{ cents.}$$

(2)

$$\$2.00 : \$2.80 :: \$2.50 : \frac{2.50 \times 2.80}{2.00} = \$3.50.$$

(3)

8 cents gain 5 cents in 9 months.

$$\text{Hence } 9 \text{ mo's} : 6 \text{ mo's} :: 5 \text{ cents} : \frac{5 \times 6}{9} = 3\frac{1}{3} = \text{gain for } 6 \text{ mo's}.$$

$$8 \text{ cts.} : 12 \text{ cts.} :: 3\frac{1}{3} : \frac{3\frac{1}{3} \times 12}{8} = 5 \text{ cts. gain on } 12 \text{ cts. for } 6 \text{ mo's}.$$

Therefore  $12 + 5 = 17$  cents = his selling price.

(4)

$$\$1.60 : \$1.85 :: \$0.55 : \frac{1.85 \times .55}{1.60} = \$0.6359375 = \text{what L}$$

ought to get in order to sell at the same profit as K.

But L only gets 60 cents, therefore K has the advantage.

$$70 \text{ yds. of cloth at } \$1.85 = \$1.85 \times 70 = \$129.50.$$

$$\$129.50 \div \$ .60 = 215\frac{5}{6}.$$

(5)

$$5 \text{ tons of butter at } \$102 = \$102 \times 5 = \$510$$

$$10\frac{1}{2} \text{ tons of tallow at } \$135 = \$135 \times 10\frac{1}{2} = \$1417.50$$

$$\text{Total value} = \$1927.50$$

$$\text{Deduct ready money, } \$600.30$$

$$\underline{\$1327.20}$$

$$\$1327.20 \div \$4.20 = 316 \text{ barrels.}$$

## EXERCISE 124—Page 281.

(1)

$$7 \text{ oz.} \times 22 = 154 \text{ carats.}$$

$$12\frac{1}{2} \text{ " } \times 21 = 262\frac{1}{2} \text{ "}$$

$$17 \text{ " } \times 9 = 153 \text{ "}$$

$$\begin{array}{r} 36\frac{1}{2} \end{array} \quad \begin{array}{r} 36\frac{1}{2} \end{array} 569\frac{1}{2} \text{ "}$$

$$\begin{array}{r} 2 \quad 2 \end{array} \text{ "}$$

$$73) 1139 (15\frac{4}{7} \text{ carats.}$$

$$\begin{array}{r} 73 \\ \hline \end{array}$$

$$\begin{array}{r} 409 \\ \hline \end{array}$$

$$\begin{array}{r} 365 \\ \hline \end{array}$$

$$\begin{array}{r} 44 \end{array}$$

(2)

$$2 \text{ gallons @ } 14\text{s.} = 28\text{s.}$$

$$1 \text{ " @ } 12\text{s.} = 12\text{s.}$$

$$2 \text{ " @ } 9\text{s.} = 18\text{s.}$$

$$4 \text{ " @ } 8\text{s.} = 32\text{s.}$$

$$\begin{array}{r} - \end{array}$$

$$9 \quad 90\text{s.}$$

$$\begin{array}{r} 10\text{s.} \end{array}$$

(3)

$$15 \text{ bushels @ } \$1.20 = \$18.00$$

$$30 \text{ " @ } \$1.50 = \$45.00$$

$$60 \text{ " @ } \$1.10 = \$66.00$$

$$83 \text{ " @ } \$1.75 = \$145.25$$

$$\begin{array}{r} 188 \end{array}$$

$$188) \$274.25 (\$1.458$$

$$\begin{array}{r} 188 \\ \hline \end{array}$$

$$\begin{array}{r} 86.2 \\ \hline \end{array}$$

$$\begin{array}{r} 75.2 \\ \hline \end{array}$$

$$\begin{array}{r} 11.05 \\ \hline \end{array}$$

$$\begin{array}{r} 9.40 \\ \hline \end{array}$$

$$\begin{array}{r} 1.650 \\ \hline \end{array}$$

$$\begin{array}{r} 1.504 \\ \hline \end{array}$$

$$\begin{array}{r} .146 \end{array}$$

(4)

$$\begin{array}{rcl}
 12 \text{ lbs. @ } 50 \text{ cents} & = & 600 \text{ cents.} \\
 16 \text{ " @ } 72 \text{ " } & = & 1152 \text{ " } \\
 22 \text{ " @ } 65 \text{ " } & = & 1430 \text{ " } \\
 18 \text{ " @ } 85 \text{ " } & = & 1530 \text{ " } \\
 100 \text{ " @ } 42 \text{ " } & = & 4200 \text{ " } \\
 \hline
 168 & & 8912 \text{ cents (53}\frac{1}{2}\text{ cents.} \\
 & & 840 \\
 & & \hline
 & & 512 \\
 & & 504 \\
 & & \hline
 & & 8 \\
 & & \hline
 & & 168 \} = 31.
 \end{array}$$

EXERCISE 125—Page 283.

(1)

Prices. Differences. Prices.

$$125 = \left\{ \begin{array}{l} 160 - 35 \quad \text{---} \quad 15 + 110 \\ 140 - 15 \quad \text{---} \quad 25 + 100 \end{array} \right\} = 125$$

Prices. Differences. Prices.

$$125 = \left\{ \begin{array}{l} 160 - 35 \quad \text{---} \quad 15 + 110 \\ 140 - 15 \quad \text{---} \quad 25 + 100 \end{array} \right\} = 125.$$

*Ans.* 35 bush. @ \$1.10, 15 @ \$1.60, 15 @ \$1, and 25 @ \$1.40.  
 35 bush. @ \$1.00, 15 @ \$1.40, 15 @ \$1.10, and 25 @ \$1.60.

(2)

Prices. Differences. Prices.

$$45 = \left\{ \begin{array}{l} 60 - 15 \text{ — } 3 + 42 \\ 50 - 5 \text{ — } 7 + 38 \end{array} \right\} = 45.$$

$\swarrow$   
 $15 + 30$

Prices. Differences. Prices.

$$45 = \left\{ \begin{array}{l} 60 - 15 \text{ — } 3 + 42 \\ 50 - 5 \text{ — } 7 + 38 \end{array} \right\} = 45.$$

$\swarrow$   
 $15 + 30$

*Ans.* 15 quarts @ 42 cents, 3 @ 60 cents, 5 @ 38 cents, 5 @ 30 cents, and  $7 + 15 = 22$  @ 50 cents.

15 quarts @ 28 cents, 3 @ 50 cents, 5 @ 42 cents, 15 @ 30 cents, and  $7 + 15 = 22$  @ 60 cents.

(3)

Prices. Differences. Prices.

$$12\frac{1}{2} = \left\{ \begin{array}{l} 18 - 5\frac{1}{2} \\ 17 - 4\frac{1}{2} \\ 16 - 3\frac{1}{2} \\ 15 - 2\frac{1}{2} \\ 14 - 1\frac{1}{2} \end{array} \right\} = 12\frac{1}{2}.$$

$\swarrow$   
 $\frac{1}{2} + 12$   
 $2\frac{1}{2} + 10$

*Ans.*  $\frac{1}{2}$  lb. @ 18 cents,  $\frac{1}{2}$  @ 17 cents,  $\frac{1}{2}$  @ 16 cents,  $2\frac{1}{2}$  @ 15 cents,  $2\frac{1}{2}$  @ 14 cents,  $5\frac{1}{2} + 4\frac{1}{2} + 3\frac{1}{2} = 13\frac{1}{2}$  @ 12 cents, and  $2\frac{1}{2} + 1\frac{1}{2} = 4$  @ 10 cents.

(4)

Prices. Differences. Prices.

$$10 = \left\{ \begin{array}{l} 13 - 3 \text{ — } 3 + 7 \\ 12 - 2 \text{ — } 5 + 5 \end{array} \right\} = 10.$$

*Ans.* 3 lbs. @ 7d., 3 @ 13d., 2 @ 5d., and 5 @ 12d.

## EXERCISE 126—Page 284.

(1)

By Case I we find that 17 quarts @ 31 cents, 6 @ 16 cents, 6 @ 19 cents, and 6 @ 23 cents will make a mixture worth 25 cents per quart.

Therefore 17 qts. : 87 qts. :: 6 qts. :  $\frac{6 \times 87}{17} = 30\frac{12}{17}$  quarts @

16 cents, and as there are 6 lbs. at each of the other prices, the same statement may be used, and the answer is therefore  $30\frac{12}{17}$  quarts @ each price.

(2)

To produce a mixture worth 75 cents per bushel, we require 45 bushels @ 80 cents, 5 @ 37 cents, and 5 @ 68 cents.

Therefore 45 bush. : 70 bush. :: 5 bush. :  $\frac{5 \times 70}{45} = 7\frac{7}{9}$  bush.

oats @ 37 cents.

45 bush. : 70 bush. :: 5 bush. :  $\frac{5 \times 70}{45} = 7\frac{7}{9}$  bush.

barley @ 68 cents.

(3)

To produce a mixture worth 1s. per lb., we require  $1\frac{1}{2}$  lbs. @ 16d.,  $1\frac{1}{2}$  @ 14d., and 6 @ 10½d.

Then  $1\frac{1}{2}$  lbs. : 50 lbs. ::  $1\frac{1}{2}$  lbs. : 50 lbs. brass @ 14d.

$1\frac{1}{2}$  lbs. : 50 lbs. :: 6 lbs. : 200 lbs. pewter @ 10½d.

(4)

By Case I we find that 1 oz. of 20 carats fine, 1 of 21 carats fine and 3 of 23 carats fine, will make a mixture 22 carats fine.

Then 1 oz. : 30 oz. :: 1 oz. : 30 oz. of 21 carats fine.

1 oz. : 30 oz. :: 3 oz. : 90 oz. of 23 carats fine.



## EXERCISE 127—Page 285.

(1)

To produce a mixture worth \$1.40 per lb., we require 20 lbs. @ \$1.00, 40 @ \$1.20, 40 @ \$1.60, and 20 @ \$1.80. But all of these added together, will make 120 lbs.

lbs. lbs. lbs. lbs.  
 Therefore 120 : 20 :: 168 :  $\frac{168 \times 20}{120} = 28$  lbs., the required quantity @ \$1.00.

120 : 40 :: 168 :  $\frac{168 \times 40}{120} = 56$  lbs., the required quantity @ \$1.20.

120 : 40 :: 168 :  $\frac{168 \times 40}{120} = 56$  lbs., the required quantity @ \$1.60.

120 : 20 :: 168 :  $\frac{168 \times 20}{120} = 28$  lbs., the required quantity @ \$1.80.

(2)

To produce a mixture worth 4s. 4d. per lb., we require 10 lbs. @ 5s. and 8 @ 3s. 6d. But these added together make 18 lbs.

lbs. lbs. lbs. lbs.  
 Therefore 18 : 10 :: 27 :  $\frac{27 \times 10}{18} = 15$  lbs., the required quantity of tea @ 5s.

18 : 8 :: 27 :  $\frac{27 \times 8}{18} = 12$  lbs., the required quantity of tea @ 3s. 6d.

(3)

To produce a mixture worth \$2.70 per gallon, we require 20 gallons @ \$2.40, 10 @ \$2.60, 10 @ \$2.80, and 30 @ \$2.90. But all of these added together will make 70 gallons. Therefore

$$70 : 20 :: 63 : \frac{63 \times 20}{70} = 18 \text{ gallons, the required quantity of brandy @ \$2.40.}$$

$$70 : 10 :: 63 : \frac{63 \times 10}{70} = 9 \text{ gallons, the required quantity of brandy @ \$2.60.}$$

$$70 : 10 :: 63 : \frac{63 \times 10}{70} = 9 \text{ gallons, the required quantity of brandy @ \$2.80.}$$

$$70 : 30 :: 63 : \frac{63 \times 30}{70} = 27 \text{ gallons, the required quantity of brandy @ \$2.90.}$$

## EXERCISE 128—Page 289.

(1)

$$1974.80 \times \frac{3}{8} = £740.55 = £740 \text{ 11s.}$$

(2)

$$765.43 \times \frac{2}{5} = £306.172 = £306 \text{ 3s. } 5\frac{7}{5}\text{d.}$$

(3)

$$8172.19 \times \frac{1}{4} = £2043.0475 = £2043 \text{ 0s. } 11\frac{3}{5}\text{d.}$$

## EXERCISE 129—Page 289.

(1)

$$£743 \text{ 18s. 11d.} = £743.94583 \text{ and } 743.94583 \div \frac{3}{10} = \$2479.8194.$$

(2)

$$£119 \text{ 9s. } 8\frac{1}{4}\text{d.} = £119.484375 \text{ and } 119.484375 \div \frac{3}{8} = \$318.625.$$

(3)

$$£473 \text{ 17s. } 1\frac{1}{4}\text{d.} = £473.8572916, \text{ and } 473.8572916 \div \frac{7}{30} = \$2030.816964.$$

## EXERCISE 130—Page 290.

(1)

$$1006.90 \div 4.867 = £206.88309 = £206 \text{ 17s. } 7\frac{1}{4}\text{d.}$$

(2)

$$916.87 \div 4.867 = £188.38504 = £188 \text{ 7s. } 8\frac{1}{4}\text{d.}$$

(3)

$$2114.81 \div 4.867 = £434.52023 = £434 \text{ 10s. } 4\frac{1}{4}\text{d.}$$

## EXERCISE 131—Page 290.

(1)

$$£2043 \text{ 11s. } 3\text{d.} = £2043.5625 \text{ and } 2043.5625 \times 4.867 = \$9946.01868.$$

(2)

$$£777 \text{ 7s. } 7\text{d.} = £777.37916 \text{ and } 777.37916 \times 4.867 = \$3783.50437.$$

(3)

$$£557 \text{ 19s. } 5\frac{1}{4}\text{d.} = £557.972916 \text{ and } 557.972916 \times 4.867 = \$2715.65418.$$

## EXERCISE 132—Page 294.

(1)

$$\$16785.25 \times 5.04 = 84597 \text{ francs } 66 \text{ centimes.}$$

(2)

Commercial value of the marc banco = 35 cents.

Add 1 per cent 35

---

3535

$$\text{Then } \$0.3535 \times 4000 = \$1414.$$

(3)

$$\$35678 \times 1.0225 = \$36480.755.$$

(4)

The par value of 1 ruble = 75 cents.

Deduct 2 per cent 15

---

735

$$\text{Then } \$0.735 \times 2560 = \$1881.60.$$

(5)

Old commercial par of £1 sterling = \$4.444 = \$4.44444

Add 8 per cent .35555

---

\$4.79999

$$\text{Then } \$4.79999 \times 800 = \$3839.999 = \$3840.00.$$

## EXERCISE 133—Page 295.

(1)

$$£1 = 420d.$$

$$19\frac{1}{3}d. = 1 \text{ franc.}$$

$$300 \text{ francs} = 60 \text{ ducats.}$$

$$1 \text{ ducat} = 360 \text{ maravedis.}$$

$$x = £1000.$$

$$x = \frac{84 \times 420 \times 1 \times 60 \times 360 \times 1000}{19\frac{1}{3} \times 300 \times 5} = 1564138 \text{ maravedis by cir. ex.}$$

$$42\frac{1}{2}d. : £1000 :: 272 \text{ maravedis} : \frac{272 \times 1000 \times 20 \times 12}{42\frac{1}{2}} =$$

$$\frac{16 \times 272 \times 1000 \times 8 \times 12}{17} = 1536000 \text{ maravedis by direct exchange.}$$

$$\text{Difference} = 1564138 - 1536000 = 28138 \text{ maravedis.}$$

$$34)28138$$

$$8)827 \text{ reals } 20 \text{ maravedis}$$

$$103 \text{ piastres } 3 \text{ reals } 20 \text{ maravedis.}$$

(2)

$$\text{Old commercial par of } £1 \text{ sterling} = \$4.444$$

$$\text{To which add 10 per cent. of itself} = .4444$$

$$\text{Gives price of } £1 \text{ sterling} = \$4.8884$$

$$\$4888.40 \div \$4.8884 = £1000 = \text{amount of bill he receives if he remits direct to London.}$$

$$\left. \begin{array}{l} \$1 = 515 \text{ centimes.} \\ 2580 \text{ cen.} = £1 \text{ sterling.} \\ x = \$4888.40 \end{array} \right\} x = \frac{515 \times 4888.40}{2580} = £975.78526.$$

$$= £975 \text{ } 15s. \text{ } 8\frac{1}{4}d. + = \text{amount of bill he receives if he remits through Paris.}$$

$$35 \text{ cents} = 1 \text{ marc.}$$

$$33\frac{3}{4} \text{ marcs} = £1 \text{ sterling.}$$

$$x = \$4888.40.$$

$$x = \frac{4888.40}{.35 \times 13\frac{3}{4}} = \frac{391072}{385} = £1015.77142 = £1015 \text{ } 15s. \text{ } 5d. + =$$

$$\text{amount of bill he receives by remitting through Hamburg.}$$

(3)

$$\left. \begin{array}{l} 18 \text{ cents.} = 1 \text{ franc,} \\ 25 \text{ francs.} = 240 \text{d.} \\ 180 \text{d.} = 3 \text{ milrees,} \\ 5 \text{ milrees.} = 18 \text{ marcs ban.} \\ 1200 \text{ marcs ban.} = x \end{array} \right\} x = \frac{18 \times 25 \times 180 \times 1200 \times 5}{240 \times 3 \times 18}$$

$= \$375 =$  circuitous exchange or sum he pays for 1200 marks.

$1200 \times .35 = \$420 =$  direct exchange or sum paid for 1200 marks,  $\$420 - \$375 = \$45 =$  gain by circuitous exchange.

## EXERCISE 134—Page 298.

(1)

$$(3)^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243.$$

(2)

$$(20)^{10} = 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 \times 20 = 10240000000000.$$

(3)

$$(1.05)^6 = 1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 = 1.340095640625.$$

(4)

$$\left(\frac{3}{8}\right)^7 = \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} = \frac{3^{187}}{8^{128}}.$$

(5)

$$\left(\frac{5}{9}\right)^5 = \frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} \times \frac{5}{9} = \frac{5^{125}}{9^{49}}.$$

(6)

$$11\frac{2}{5} = \frac{57}{5}, \quad \left(\frac{57}{5}\right)^3 = \frac{57}{5} \times \frac{57}{5} \times \frac{57}{5} = \frac{185193}{125} = 1481\frac{68}{125}.$$

## EXERCISE 135—Page 299.

(1)

$$4^2 \times 4^4 \times 4^5 \times 4^7 = 4^{2+4+5+7} = 4^{18}.$$

(2)

(3)

$$13^{11} \div 13^2 = 13^{11-2} = 13^9. \quad (3^3)^5 = 3^{3 \times 5} = 3^{15}.$$

(4)

$$\begin{aligned} \{(7^4 \times 7^3) \div (7^2 \times 7^2)\}^6 &= \{(7^4 + 3) \div (7^2 + 2)\}^6 = \\ \{7^7 \div 7^4\}^6 &= (7^{7-4})^6 = (7^3)^6 = 7^{3 \times 6} = 7^{18}. \end{aligned}$$

(5)

$$\begin{aligned} \{(5^3 \times 5^4 \times 5^{11} \times 5^9) \div (5^3 \times 5^2 \times 5^7 \times 5^5)\}^3 &= \\ \{(5^{3+4+11+9}) \div (5^{3+2+7+5})\}^3 &= \{5^{27} \div 5^{17}\}^3 \\ (5^{27-17})^3 &= (5^{10})^3 = 5^{10 \times 3} = 5^{30}. \end{aligned}$$

## EXERCISE 137—Page 304.

(1)

(2)

(3)

$$\begin{array}{r} \dots \\ 195364(442 \\ \underline{16} \\ 84)353 \\ \underline{336} \\ 882)1764 \\ \underline{1764} \end{array}$$

$$\begin{array}{r} \dots \\ .0676(.26 \\ \underline{4} \\ 46)276 \\ \underline{276} \end{array}$$

$$\begin{array}{r} \dots \\ 984064(992 \\ \underline{81} \\ 189)1740 \\ \underline{1701} \\ 1982)3964 \\ \underline{3964} \end{array}$$

(4)

$$\begin{array}{r}
 5\cdot0000000000(2\cdot23606 \\
 4 \\
 \hline
 42)1\cdot00 \\
 \cdot84 \\
 \hline
 443)1\cdot600 \\
 \cdot1329 \\
 \hline
 4466)27100 \\
 26796 \\
 \hline
 447206)3040000\ 1 \\
 2683236 \\
 \hline
 356764
 \end{array}$$

(6)

$$\begin{array}{r}
 60\cdot487129(7\cdot777 \\
 49 \\
 \hline
 147)1148 \\
 1029 \\
 \hline
 1547)11971 \\
 10829 \\
 \hline
 15547)114229 \\
 108829 \\
 \hline
 5400
 \end{array}$$

(8)

$$\begin{array}{r}
 0\cdot0000012321(0\cdot00111 \\
 1 \\
 \hline
 21)23 \\
 21 \\
 \hline
 221)221 \\
 221 \\
 \hline
 \end{array}$$

(5)

$$\begin{array}{r}
 5\cdot0000000000(7\cdot07106 \\
 49 \\
 \hline
 1407)10000 \\
 9849 \\
 \hline
 14141)15100 \\
 14141 \\
 \hline
 1414206)9590000 \\
 8485236 \\
 \hline
 1104764
 \end{array}$$

(7)

$$\begin{array}{r}
 79792266297612001(282475249 \\
 4 \\
 \hline
 48)397 \\
 384 \\
 \hline
 562)1392 \\
 1124 \\
 \hline
 5644)26826 \\
 22576 \\
 \hline
 56487)425062 \\
 395409 \\
 \hline
 564945)2965397 \\
 2824725 \\
 \hline
 5649502)14067261 \\
 11299004 \\
 \hline
 56495044)276825720 \\
 225980176 \\
 \hline
 564950489)5084554401 \\
 5084554401 \\
 \hline
 \end{array}$$



## EXERCISE 138—Page 304.

(1)

$$\dot{1} = \frac{1}{9} \text{ and } \sqrt{\frac{1}{9}} = \frac{1}{3}.$$

(3)

$$5\frac{1}{2} = 5.142857142857 \text{ and } \sqrt{5.142857142857} = 2.267786.$$

(4)

$$2\frac{17}{38} = .4033457249 \text{ and } \sqrt{.4033457249} = .63509.$$

(5)

$$13\frac{1}{2} = 13.2 \text{ and } \sqrt{13.2} = 3.63318$$

## EXERCISE 139—Page 305.

(1)

$$\begin{array}{r} \dot{1} \dot{1} \dot{3} \dot{3} \dot{3} \dot{1} \dot{1} (2626 \\ 4 \\ - \\ 46) 433 \\ 411 \\ \hline 552) 2233 \\ 1434 \\ \hline 5546) 46611 \\ 46611 \end{array}$$

(2)

$$\begin{array}{r} \dot{3} \dot{3} \dot{2} \dot{3} \dot{3} \dot{3} \dot{4} \dot{4} (4344 \\ 24 \\ - \\ 123) 523 \\ 413 \\ \hline 1304) 11033 \\ 10024 \\ \hline 13124) 100544 \\ 100544 \end{array}$$

(3)

$$\begin{array}{r}
 \cdot \cdot \cdot \cdot \cdot \\
 4234 \cdot 101230 (43 \cdot 412 \\
 \underline{31} \\
 133) 1134 \\
 \underline{1004} \\
 1414) 130 \cdot 10 \\
 \underline{122 \cdot 21} \\
 14231) 2 \cdot 3412 \\
 \underline{1 \cdot 4231} \\
 142322) \cdot 413130 \\
 \underline{\cdot 340144} \\
 22431
 \end{array}$$

(4)

$$\begin{array}{r}
 \cdot \cdot \cdot \cdot \cdot \\
 888888 \cdot 88880 (888 \cdot 88 \\
 \underline{71} \\
 178) 1788 \\
 \underline{1601} \\
 1878) 18788 \\
 \underline{16801} \\
 18878) 1887 \cdot 88 \\
 \underline{1688 \cdot 01} \\
 188878) 188 \cdot 8780 \\
 \underline{168 \cdot 8801} \\
 18 \cdot 8878
 \end{array}$$

(5)

$$\begin{array}{r}
 \cdot \cdot \cdot \cdot \cdot \\
 248664e \cdot 69 (54373 \\
 \underline{21} \\
 t4) 386 \\
 \underline{354} \\
 t83) 3264 \\
 \underline{2809} \\
 t867) 657e \cdot t \\
 \underline{62et \cdot 1} \\
 t8723) 281969 \\
 \underline{281969}
 \end{array}$$

EXERCISE 140—Page 307.

(1)

$$\begin{array}{r}
 100^2 = 10000 \\
 60^2 = 3600 \\
 \hline
 \end{array}$$

$$\text{Difference} = 6400 \text{ and } \sqrt{6400} = 80.$$

(2)

$$50^2 = 2500$$

$$80^2 = 6400$$


---

$$\text{Sum} = 8900 \text{ and } \sqrt{8900} = 94.34 \text{ nearly}$$

(3)

$$24^2 = 576 \div 2 = 288 \text{ and } \sqrt{288} = 16.97.$$

(4)

$$36^2 = 1296$$

$$20^2 = 400$$


---

$$\text{Difference} = 896 \text{ and } \sqrt{896} = 29.933.$$

(5)

$$40^2 = 1600$$

$$14^2 = 196$$


---

$$\text{Difference} = 1404 \text{ and } \sqrt{1404} = 37.469.$$

$$40^2 = 1600$$

$$26^2 = 676$$


---

$$\text{Difference} = 924 \text{ and } \sqrt{924} = 30.397.$$

$$37.469 + 30.397 = 67.866 \text{ and } 67.866 \div 3 = 22.622.$$

(6)

$$1760 \text{ sq. yds.} = 15840 \text{ sq. ft. and } \sqrt{15840} = 125.857.$$

(7)

$$\sqrt{141376} = 376.$$

(8)

$$3^2 = 9$$

$$3^2 = 9$$


---

$$\text{Sum} = 18 \text{ and } \sqrt{18} = 4.24264.$$

(9)

$$16^2 = 256$$

$$12^2 = 144$$

---


$$\text{Sum} = 400 \text{ and } \sqrt{400} = 20$$

(10)

$$3^2 + 3^2 + 3^2 = 27 \text{ and } \sqrt{27} = 5.196.$$

(11)

$$(\frac{1}{10})^2 = \frac{1}{100} \text{ and } (1)^2 = 1.$$

$$\text{Then } \frac{1}{100} : 1 :: 450 : \frac{450}{\frac{1}{100}} = 45000.$$

(12)

$$1 \text{ sq. acre} = 160 \text{ sq. perches. } 160 \div 3.1416 = 50.929462 \text{ and } \sqrt{50.929462} = 7.136.$$

EXERCISE 141—Page 311.

(1)

|                            |   |   |
|----------------------------|---|---|
|                            |   | 62712728317(3973)                         |
|                            |   | 27  |
|                            |   | <hr style="width: 10%; margin: 0 auto;"/> |
| $3^2 \times 300 =$         | 2700                                      | 35712                                     |
| $3 \times 9 \times 30 =$   | 810                                       |   |
| $9^2 =$                    | 81  |   |
|                            | <hr style="width: 10%; margin: 0 auto;"/> |   |
|                            | 3591                                      | 32319                                     |
|                            | <hr style="width: 10%; margin: 0 auto;"/> |   |
| $39^2 \times 300 =$        | 456300                                    | 3393728                                   |
| $39 \times 7 \times 30 =$  | 8190                                      |   |
| $7^2 =$                    | 49  |   |
|                            | <hr style="width: 10%; margin: 0 auto;"/> |   |
|                            | 464539                                    | 3251773                                   |
|                            | <hr style="width: 10%; margin: 0 auto;"/> |   |
| $397^2 \times 300 =$       | 47282700                                  | 141955317                                 |
| $397 \times 3 \times 30 =$ | 35730                                     |   |
| $3^2 =$                    | 9   |   |
|                            | <hr style="width: 10%; margin: 0 auto;"/> |   |
|                            | 47318439                                  | 141955317                                 |

(2)

|                           |       |             |
|---------------------------|-------|-------------|
|                           |       | 1953125(125 |
|                           |       | 1           |
|                           |       | —           |
| $1^2 \times 300 =$        | 300   | 953         |
| $1 \times 2 \times 30 =$  | 60    |             |
| $2^2 =$                   | 4     |             |
|                           | —     |             |
|                           | 364   | 728         |
|                           | —     | —           |
| $12^2 \times 300 =$       | 43200 | 225125      |
| $12 \times 5 \times 30 =$ | 1800  |             |
| $5^2 =$                   | 25    |             |
|                           | —     |             |
|                           | 45025 | 225125      |

(3)

|                            |         |                 |
|----------------------------|---------|-----------------|
|                            |         | 1076890625(1025 |
|                            |         | 1               |
|                            |         | —               |
| $1^2 \times 300 =$         | 300     | 76              |
| $10^2 \times 300 =$        | 30000   | 76890           |
| $10 \times 2 \times 30 =$  | 600     |                 |
| $2^2 =$                    | 4       |                 |
|                            | —       |                 |
|                            | 30604   | 61208           |
|                            | —       | —               |
| $102^2 \times 300 =$       | 3121200 | 15682625        |
| $102 \times 5 \times 30 =$ | 15300   |                 |
| $5^2 =$                    | 25      |                 |
|                            | —       |                 |
|                            | 3136525 | 15682625        |

(4)

 $\cdot 697864103(\cdot 887$ 

512

---

185864

$8^2 \times 300 = 19200$

$8 \times 8 \times 30 = 1920$

$8^2 = 64$

---

21184

---

169472

$88^2 \times 300 = 2323200$

$88 \times 7 \times 30 = 18480$

$7^2 = 49$

---

2341729

---

16392103

---

16392103

(5)

 $\cdot 102503 \cdot 232(46 \cdot 8$ 

64

---

38503

$4^2 \times 300 = 4800$

$4 \times 6 \times 30 = 720$

$6^2 = 36$

---

5556

---

33336

$46^2 \times 300 = 634800$

$46 \times 8 \times 30 = 11040$

$8^2 = 64$

---

645904

---

5167 \cdot 232

---

5167 \cdot 232

(6)

$$\begin{array}{rcl}
 5^2 \times 300 & = & 7500 \\
 5 \times 6 \times 30 & = & 900 \\
 6^2 & = & 36 \\
 \hline
 & & 8436 \\
 \hline
 56^2 \times 300 & = & 940800 \\
 56 \times 4 \times 30 & = & 6720 \\
 4^2 & = & 16 \\
 \hline
 & & 947536 \\
 \hline
 564^2 \times 300 & = & 95428800 \\
 564 \times 2 \times 30 & = & 33840 \\
 2^2 & = & 4 \\
 \hline
 & & 95462644
 \end{array}$$

$$\begin{array}{r}
 \dot{1}79\dot{5}97\cdot\dot{0}69288(56\cdot42 \\
 125 \\
 \hline
 54597 \\
 \hline
 50616 \\
 \hline
 3981\cdot069 \\
 \hline
 3790\cdot144 \\
 \hline
 190\cdot925288 \\
 \hline
 190\cdot925288
 \end{array}$$

(7)

$$\begin{array}{rcl}
 7^2 \times 300 & = & 14700 \\
 7 \times 8 \times 30 & = & 1680 \\
 8^2 & = & 64 \\
 \hline
 & & 16444 \\
 \hline
 78^2 \times 300 & = & 1825200 \\
 78 \times 5 \times 30 & = & 11700 \\
 5^2 & = & 25 \\
 \hline
 & & 1836925
 \end{array}$$

$$\begin{array}{r}
 \dot{4}83\cdot\dot{7}36625(7\cdot85 \\
 343 \\
 \hline
 140\cdot736 \\
 \hline
 131\cdot552 \\
 \hline
 9\cdot184625 \\
 \hline
 9\cdot184625
 \end{array}$$

(8)

$$\begin{array}{r}
 8^2 \times 300 = 19200 \\
 8 \times 6 \times 30 = 1440 \\
 6^2 = 36 \\
 \hline
 20676
 \end{array}
 \qquad
 \begin{array}{r}
 \cdot 636056 \cdot 86 \\
 512 \\
 \hline
 124056 \\
 124056
 \end{array}$$

## EXERCISE 142.—Page 312.

(1)

$$\sqrt[3]{.105263157894} = .4721 \text{ and } \sqrt[3]{.105263157894} = .4721.$$

(2)

$$\sqrt[3]{.176470588235} = .5609 \text{ and } \sqrt[3]{.176470588235} = .5609.$$

(3)

$$\frac{1}{3} \text{ of } 2\frac{1}{2} = \frac{5}{6} = .83333333 \text{ and } \sqrt[3]{.83333333} = .941.$$

(4)

$$28\frac{1}{4} = 28.75 \text{ and } \sqrt[3]{28.75} = 3.063$$

(5)

$$32\frac{3}{4} = 32.75 \text{ and } \sqrt[3]{32.75} = 3.198.$$



## EXERCISE 143.—Page 313.

(1)

One million = 33233344 senary.

|                                  |        |              |
|----------------------------------|--------|--------------|
|                                  |        | 33233344(244 |
|                                  |        | 12           |
|                                  |        | <hr/>        |
| $2^2 = 4 \times 300 =$           | 2000   | 21233        |
| $2 \times 30 = 100 \times 4 =$   | 400    |              |
| $4^2 =$                          | 24     |              |
|                                  | <hr/>  |              |
|                                  | 2424   | 14544        |
|                                  | <hr/>  |              |
| $24^2 = 1104 \times 300 =$       | 332000 | 2245344      |
| $24 \times 30 = 1200 \times 4 =$ | 5200   |              |
| $4^2 =$                          | 24     |              |
|                                  | <hr/>  |              |
|                                  | 341224 | 2245344      |

(2)

|                                     |            |                        |
|-------------------------------------|------------|------------------------|
|                                     |            | 6131271·000000(165·32. |
|                                     |            | 1                      |
|                                     |            | <hr/>                  |
| $1^2 \times 300 =$                  | 300        | 5131                   |
| $1 \times 30 \times 6 =$            | 220        |                        |
| $6^2 =$                             | 44         |                        |
|                                     | <hr/>      |                        |
|                                     | 564        | 4270                   |
|                                     | <hr/>      |                        |
| $16^2 = 304 \times 300 =$           | 111400     | 641271                 |
| $16 \times 30 = 520 \times 5 =$     | 3220       |                        |
| $5^2 =$                             | 31         |                        |
|                                     | <hr/>      |                        |
|                                     | 114651     | 600115                 |
|                                     | <hr/>      |                        |
| $165^2 = 32571 \times 300 =$        | 12015300   | 41154·000              |
| $165 \times 30 = 5370 \times 3 =$   | 20350      |                        |
| $3^2 =$                             | 11         |                        |
|                                     | <hr/>      |                        |
|                                     | 12035661   | 36131·423              |
|                                     | <hr/>      |                        |
| $1653^2 = 3272071 \times 300 =$     | 1205625300 | 3022·355000            |
| $1653 \times 30 = 54010 \times 2 =$ | 130020     |                        |
| $2^2 =$                             | 4          |                        |
|                                     | <hr/>      |                        |
|                                     | 1205755324 | 2413·732650            |
|                                     | <hr/>      |                        |
|                                     |            | 406·422130             |

(3)

$$10\dot{2}21\dot{0}1\dot{2} \cdot 10\dot{2}000000$$

$$1 \overline{)112 \cdot 012} = \text{root.}$$

|  |                     |                            |
|--|---------------------|----------------------------|
| $1 \times 1000 =$                                | 1000                | 2221                       |
| $1 \times 1 \times 100 =$                        | 100                 |                            |
| $1^2 =$  | 1                   |                            |
|  | <u>1101</u>         | 1101                       |
| $11^2 = 121 \times 1000 =$                       | 121000              | 1120012                    |
| $11 \times 100 = 1100 \times 2 =$                | 2200                |                            |
| $2^2 =$  | 11                  |                            |
|  | <u>200211</u>       | 1101122                    |
| $112^2 = 21021 \times 1000 =$                    | 21021000            | 11120 \cdot 102            |
| $1120^2 = 2102100 \times 1000 =$                 | 2102100000          | 11120 \cdot 102000         |
| $1120 \times 100 = 112000 \times 1 =$            | 112000              |                            |
| $1^2 =$  | 1                   |                            |
|  | <u>2102212001</u>   | 2102 \cdot 212001          |
| $11201^2 = 211010101 \times 1000 = 211010101000$ |                     | 2010 \cdot 112222000       |
| $11201 \times 100 = 1120100 \times 2 =$          | 10010200            |                            |
| $2^2 =$  | 11                  |                            |
|  | <u>211020111211</u> | 1122 \cdot 111000122       |
|  |                     | <u>111 \cdot 001221101</u> |

(4)

|   |                       |   |
|---|-----------------------|---|
|   |                       | $t e t e t \cdot 000000 (e 7 \cdot t 2 .$ |
|   |                       | $92 e$                                    |
| $e^2 = t 1 \times 300 =$                | 26300                 | $18 e e e t$                              |
| $e \times 30 = 290 \times 7 =$          | 1730                  |   |
| $7^2 =$                                 | 41                    |   |
|   | <u>27 t 71</u>        | <u>167217</u>                             |
| $e 7^2 = e 221 \times 300 =$            | 2966300               | $249 t 3 \cdot 000$                       |
| $e 7 \times 30 = 2 t 90 \times t =$     | 24 e 60               |   |
| $t^2 =$                                 | 84                    |   |
|   | <u>298 e 324</u>      | <u>24154 \cdot 7 e 4</u>                  |
| $e 7 t^2 = e 39544 \times 300 =$        | 29 e 441000           | $84 t \cdot 408000$                       |
| $e 7 t \times 30 = 2 t e 60 \times 2 =$ | 59 e 00               |   |
| $2^2 =$                                 | 4                     |   |
|   | <u>29 e 49 t e 04</u> | <u>57 t \cdot 979 t 08</u>                |
|   |                       | <u>28 e \cdot 64 t 1 e 4</u>              |

(5)

|                                   |             |                                |
|-----------------------------------|-------------|--------------------------------|
|                                   |             | 421030·441200000(44·004<br>224 |
| $4^2=31 \times 300=$              | 14300       | 142030                         |
| $4 \times 30=220 \times 4=$       | 1430        |                                |
| $4^2=$                            | 31          |                                |
|                                   | 21311       | 141244                         |
| $44^2=4301 \times 300=$           | 2340300     | 231·441                        |
| $440^2=430100 \times 300=$        | 234030000   | 231·441200                     |
| $4400^2=43010000 \times 300=$     | 23403000000 | 231·441200000                  |
| $4400 \times 30=242000 \times 4=$ | 2123000     |                                |
| $4^2=$                            | 31          |                                |
|                                   | 23410123031 | 210·141102224                  |
|                                   |             | 21·300042221                   |

## EXERCISE 144.—Page 314.

(2)

$$3^3 : 6^3 :: 4 \text{ lb.} : \text{Ans.} = 32 \text{ lbs.}$$

(3)

$$1^3 : (\frac{1}{2})^3 :: \$120 : \text{Ans.} = \$5145.$$

(4)

$$\begin{aligned} (70)^3 : (62\frac{2}{3})^3 &:: 180 \text{ lbs.} : \text{Ans.} \\ 343000 : 241804\frac{367}{125} &:: 180 : \text{Ans.} = \\ 180 \times \frac{241804\frac{367}{125}}{343000} &= 1015.1 \text{ lbs.} \end{aligned}$$

(5)

$$973^3 = 921167317$$

$$45^3 = 91125$$

$$62^3 = 238328$$

$$30^3 = 27000$$

$$80^3 = 512000$$

$$20^3 = 8000$$

$$\begin{aligned} 921167317 - (91125 + 238328 + 27000 + 512000 + 8000) &= \\ 920290864 \text{ and } \sqrt[3]{920290864} &= 972.69. \end{aligned}$$

(6)

$$\begin{aligned} 8 \text{ feet } 3 \text{ inches} &= 99 \text{ inches, } 3 \text{ feet} = 36 \text{ inches, and } 2 \text{ feet } 7 \\ &\text{ inches} = 31 \text{ inches.} \end{aligned}$$

$$99 \times 36 \times 31 = 110484 \text{ and } \sqrt[3]{110484} = 47.9843.$$

(7)

After the first has wound off her portion, there will remain  $\frac{1}{2}$  of the thread.

Then the whole ball : part remaining :: cube of diameter of whole ball : cube of diameter of part remaining.

That is,  $1 : \frac{1}{8} :: 3^3 : x^3$ , and hence  $x = 3 \times \sqrt[3]{\frac{1}{8}} = 3 \times \sqrt[3]{.75} = .90856 \times 3 = 2.72568 =$  diameter of the ball after the first has wound off her portion.

Similarly after the second has wound off her portion, there will remain  $\frac{1}{8}$  of the ball, and after the third has taken her portion,  $\frac{1}{8}$  of the ball.

Hence  $1 : \frac{1}{8} :: 3^3 : x^3$ , whence  $x = 3 \times \sqrt[3]{\frac{1}{8}} = 3 \times \sqrt[3]{.5} = 3 \times .79370 = 2.38110 =$  diameter after the second has taken her portion.

$1 : \frac{1}{8} :: 3^3 : x^3$ , whence  $x = 3 \times \sqrt[3]{\frac{1}{8}} = 3 \times \sqrt[3]{.25} = 3 \times .62996 = 1.88988 =$  diameter after the third has taken her portion.

|                       |                     |                |
|-----------------------|---------------------|----------------|
| Hence 1st takes off 3 | — 2.72568 =         | .27432 inches. |
| 2nd “ “               | 2.72568 — 2.38110 = | .34458 “       |
| 3rd “ “               | 2.38110 — 1.88988 = | .49122 “       |
| 4th “ “               | remaining           | 1.88988 “      |

EXERCISE 145—Page 315.

(1)

$$\sqrt{19987173376} = 141376, \text{ and } \sqrt{141376} = 376.$$

(2)

$$\sqrt[3]{308915776} = 676, \text{ and } \sqrt{676} = 26.$$

(3)

$$\sqrt[3]{40353607} = 343, \text{ and } \sqrt[3]{343} = 7.$$

(4)

$$\sqrt[3]{387420489} = 729, \sqrt[3]{729} = 9, \text{ and } \sqrt{9} = 3.$$

(5)

$$\sqrt[3]{134217728} = 512, \sqrt[3]{512} = 8, \text{ and } \sqrt[3]{8} = 2.$$


---

## EXERCISE 148—Page 321.

(1)

The mantissa of the logarithm of 8193 (the first four digits) = .913443, and the next following mantissa is .913496.

Then from .913496

Subtract.. .913443

Difference, 53; and  $53 \times 217$  (remaining digits of given number) = 11501, from which we cut off three digits, since we multiplied by a number of three digits, and since the highest digit cut off is not less than 5, we add unity to the part retained, which gives us 12.

Then mantissa of logarithm of first four digits .913443

Add, 12

Mantissa of logarithm of given number, .913455

To which attach the characteristic 6 and required logarithm = 6.913455.

The mantissa of the logarithm of 7392 (the first four digits) = .868762, and the next following mantissa is .868821.

Then from .868821

Subtract.. .868762

Difference, 59; and  $59 \times 45$  (remaining digits of given number) = 2655, from which we cut off two digits, since we multiplied by a number of two digits, and since the highest digit cut off is not less than 5, we add unity to the part retained, which gives us 27.

Then mantissa of logarithm of first four digits, .868762

Add, 27

Mantissa of logarithm of given number, .868789

(Continued on next page.)

(1 continued.)

To which attach the characteristic 1 and required logarithm =  
1.868789.

The mantissa of the logarithm of 8437 (the first four digits)  
= .926188, and the next following mantissa is .926240.

Then from .926240

Subtract.. .926188

Difference, 52; and  $52 \times 42$  (remaining digits of given  
number) = 2184, from which we cut off two digits, since we  
multiplied by a number of two digits, and since the highest digit  
cut off is not less than 5, we add unity to the part retained,  
which becomes 22.

Then mantissa of logarithm of first four digits .926188

Add, 22

Mantissa of logarithm of given number, .926210

To which attach the characteristic 1 and required logarithm =  
1.926210.

(2)

The mantissa of the logarithm of 2345 = .370143, and the next  
following mantissa is .370328.

Then from .370328

Subtract.. .370143

Difference, 185; and  $185 \times 64 = 11840$ , from which we  
cut off two digits, since we multiplied by a number of two  
digits, which gives us 118.

Then mantissa of logarithm of 2345 = .370143

Add, 118

Mantissa of logarithm of given number = .370261

To which attach the characteristic 4 and required logarithm =  
4.370261.

(Continued on next page.)

(2 continued.)

The mantissa of the logarithm of 1007 = .003029, and the next following mantissa is .003461.

Then from .003461

Subtract.. .003029

---

Difference, 432; and  $432 \times 013 = 5616$ , from which we cut off three digits, since we multiplied by a number of three digits, and since the highest digit cut off is not less than 5, we add unity to the part retained, which gives us 6.

Then mantissa of logarithm of 1007 = .003029

Add, 6

---

Mantissa of logarithm of given number .003035

To which attach the characteristic 3, and required logarithm =  
 $\overline{3}.003035$ .

(2)

Mantissa of logarithm of 5237 ..... .719083

Difference from column D = 83; and  $83 \times 6 = 498$

from which we cut off 1 digit and add..... 50

---

And also attach the characteristic 1, and required

logarithm = ..... 1.719133

Mantissa of logarithm of 1294..... .111934

Difference from column D = 335; and  $335 \times 76 =$

25460 from which we cut off two digits and add, 255

---

And also attach the characteristic 2 and required

logarithm = ..... 2.112189



(4)

|                          |            |   |         |
|--------------------------|------------|---|---------|
| Mantissa of logarithm of | ·0004713   | = | ·673297 |
| P. P. corresponding to   | ·00000009  | = | 83      |
| P. P. " to               | ·000000008 | = | 74      |
| <hr/>                    |            |   |         |
| Sum, = ·6733874          |            |   |         |

Therefore required mantissa = ·673387 and required logarithm  
= 4·673387.

|                          |         |   |         |
|--------------------------|---------|---|---------|
| Mantissa of logarithm of | 9136000 | = | ·960756 |
| P. P. corresponding to   | 700     | = | 33      |
| P. P. " to               | 10      | = | 5       |
| P. P. " to               | 2       | = | 9       |
| <hr/>                    |         |   |         |
| Sum, = ·96078959         |         |   |         |

Therefore required mantissa = ·960790 and required logarithm  
= 6·960790.

(5)

|                          |         |   |         |
|--------------------------|---------|---|---------|
| Mantissa of logarithm of | 4·23400 | = | ·626751 |
| P. P. corresponding to   | 20      | = | 20      |
| P. P. " to               | 9       | = | 92      |
| <hr/>                    |         |   |         |
| Sum, = ·6267802          |         |   |         |

Therefore required logarithm is 0·626780.

|                          |        |   |         |
|--------------------------|--------|---|---------|
| Mantissa of logarithm of | 763·1  | = | ·882581 |
| P. P. corresponding to   | ·02    | = | 11      |
| P. P. " to               | ·009   | = | 51      |
| P. P. " to               | ·0008  | = | 46      |
| P. P. " to               | ·00009 | = | 40      |
| <hr/>                    |        |   |         |
| Sum, = ·882597600        |        |   |         |

Therefore required logarithm is 2·882598.

## EXERCISE 149.—Page 323.

(1)

Given logarithm,  $\cdot 137139$ Next lower in table,  $\cdot 137037 = \log.$  of 1371.

|            |                   |                                |
|------------|-------------------|--------------------------------|
| Difference | <u>          </u> | 102, Tabular difference = 316. |
|------------|-------------------|--------------------------------|

Then  $1020000 \div 316$  gives 3227 for digits in 5th, 6th, 7th, and 8th places.

Hence the digits of the natural number are 13713227; and since the characteristic is 4, i.e., one less than the number of digits to the left of the decimal point the required number is 13713·227.

Given logarithm,  $\cdot 718134$ Next lower in table,  $\cdot 718086 = \log.$  of 5225.

|             |                   |                              |
|-------------|-------------------|------------------------------|
| Difference, | <u>          </u> | 48, Tabular difference = 83. |
|-------------|-------------------|------------------------------|

Then  $48000 \div 83$  gives 578 for digits in 5th, 6th, and 7th places.

Hence the digits of the natural number are 5225578, and since the characteristic is 0, i.e., one less than the number of digits to the left of the decimal point, the required number is 5·225578.

Given logarithm,  $\cdot 635421$ Next lower in table,  $\cdot 635383 = \log.$  of 4319.

|             |                   |                               |
|-------------|-------------------|-------------------------------|
| Difference, | <u>          </u> | 38, Tabular difference = 101. |
|-------------|-------------------|-------------------------------|

Then  $38000 \div$  gives 376 for digits in 5th, 6th, and 7th places.

Hence the digits of the natural number are 4319376, and since the characteristic is  $\bar{4}$ , i.e., one more than the number of ciphers between the decimal point and the first figure to the right, the required number is  $\cdot 0004319376$ .

(2)

Given log.  $\cdot 921686 = \log.$  of 8350.

And since the characteristic is 2, i.e., one less than the number of digits to the left of the decimal point, the required number is 835.

Given logarithm,  $\cdot 922165$ Next lower in table,  $\cdot 922154 = \log.$  of 8359.Difference =  $\underline{\hspace{1cm}}$  11, Tabular difference = 52.

Then  $11000 \div 52$  gives 211 for digits in 5th, 6th, and 7th places.

Hence the digits of the natural number are 8359211; and since the characteristic is  $\bar{1}$ , i.e., one more than the number of ciphers between the decimal point and first figure to the right, the required number is  $\cdot 8359211$ .

(3)

Given logarithm,  $\cdot 407968$ Next lower in table,  $\cdot 407901 = \log.$  of 2558.Difference, =  $\underline{\hspace{1cm}}$  67

Highest P. P. not greater than 67 = 51 corresponds to 3  
 $\underline{\hspace{1cm}}$  for 5th place.

160  
 Highest P. P. not greater than 160 = 153 corresponds to 9  
 $\underline{\hspace{1cm}}$  for 6th place.

70  
 Highest P. P. not greater than 70 = 68 corresponds to  
 $\underline{\hspace{1cm}}$  4 for 7th place.  
 2

Therefore digits of required number are 2558394; and since the characteristic is 5, there must be six digits to the left of the decimal point.

Hence required number is 2558394.

(Continued on next page.)

(3 continued.)

Given logarithm,  $\cdot 408386$   
 Next lower in table,  $\cdot 408240 = \log. \text{ of } 2560.$

Difference, =  $\underline{\hspace{1cm}} 146$

Highest P.P. not greater than 146 =  $\underline{\hspace{1cm}} 136$  corresponds to 8  
 in 5th place.

100

Highest P.P. not greater than 100 =  $\underline{\hspace{1cm}} 85$  corresponds to 5  
 in 6th place.

150

Highest P.P. not greater than 150 =  $\underline{\hspace{1cm}} 136$  corresponds to 8  
 in 7th place.

140

Highest P.P. not greater than 140 =  $\underline{\hspace{1cm}} 136$  corresponds to 8  
 in 8th place.

Therefore digits of required number are 25608588 ; and since the characteristic is 7, there must be eight digits to the left of the decimal point.

Hence required number is 25608588.

Given logarithm,  $\cdot 416369$   
 Next lower in table,  $\cdot 416308 = \log. \text{ of } 2608.$

Difference, =  $\underline{\hspace{1cm}} 61$

Highest P.P. not greater than 61 =  $\underline{\hspace{1cm}} 49$  corresponds to 3  
 in 5th place.

12

Therefore digits of required number are 26083 ; and since the characteristic is  $\bar{3}$ , there must be two ciphers between the decimal point and first figure.

Hence required number is  $\cdot 0026083.$

(4)

|                                     |  |                        |
|-------------------------------------|--|------------------------|
| Given logarithm,                    | $\cdot 877777$                           |                        |
| Next lower in table,                | $\cdot 877774 = \log. \text{ of } 7547.$ |                        |
| Difference, =                       | <u>3</u>                                 |                        |
| There is no P.P. not greater than 3 |  | 0 corresponds to 0 in  |
|                                     | $\cdot$ <u>30</u>                        | 5th place.             |
| Highest P.P. not greater than 30 =  |  | 29 corresponds to 5 in |
|                                     | <u>10</u>                                | 6th place.             |
| Highest P.P. not greater than 10 =  |  | 6 corresponds to 1     |
|                                     | <u>40</u>                                | in 7th place.          |
| Highest P.P. not greater than 40 =  |  | 35 corresponds to 6    |
|                                     | <u>50</u>                                | in 8th place.          |
| Highest P.P. not greater than 50 =  |  | 46 corresponds to 8    |
|                                     | <u>4</u>                                 | in 9th place.          |

Therefore digits of required number are 754705168 ; and since the characteristic is 4, there must be five digits to the left of the decimal point.

Hence required number is 75470.5168.

|                                    |  |                     |
|------------------------------------|--|---------------------|
| Given logarithm,                   | $\cdot 555555$                           |                     |
| Next lower in table,               | $\cdot 555457 = \log. \text{ of } 3593.$ |                     |
| Difference, =                      | <u>98</u>                                |                     |
| Highest P.P. not greater than 98 = |  | 98 corresponds to 8 |
|                                    |  | in 5th place.       |

Therefore digits of required number are 35938 ; and since the characteristic is 0, there must be one digit to the left of the decimal point.

Hence required number is 3.5938.

## EXERCISE 150.—Page 324.

(1)

$$10 - 5.631642 = 4.368358.$$

$$10 - 0.714000 = 9.286000.$$

(2)

$$10 - \overline{3}.123456 = 12.876544.$$

$$10 - \overline{7}.213149 = 16.786851.$$

(3)

$$10 - 6.124357 = 3.875643 \text{ and } 10 - \overline{2}.000837 = 11.999163.$$

## EXERCISE 151.—Page 325.

(1)

$$\text{Logarithm of } 61 = 1.785330$$

$$\text{" } 22 = 1.342423$$

$$\text{" } 65 = 1.812913$$

$$\text{Sum} = 4.940666 = \text{logarithm of } 87230.$$

(2)

$$\text{Logarithm of } 52 = 1.716003$$

$$\text{" } 734 = 2.865696$$

$$\text{" } 6 = 0.778151$$

$$\text{Sum} = 5.359850$$

$$5.359835 = \text{logarithm of } 229000$$

$$15 =$$

8

Ans. 229008

(3)

$$\text{Logarithm of } 35.86 = 1.554610$$

$$\text{" } 2.1046 = 0.323169$$

$$\text{" } .8372 = \bar{1}.922829$$

$$\text{" } .00294 = \bar{3}.468347$$

$$\text{Sum} = \bar{1}.268955$$

$$\bar{1}.268812 = \text{logarithm of } .185700$$

$$143 =$$

61

$$\text{Ans. } .185761$$

(4)

$$\text{Log. of } .00008764 = \bar{5}.942702$$

$$\text{" } .86359 = \bar{1}.936308$$

$$\text{Sum} = \bar{5}.879010$$

$$\bar{5}.878981 = \text{logarithm of } .000075680$$

$$29 =$$

5

$$\text{Ans. } .000075685$$

## EXERCISE 152.—Page 326.

(1)

$$\text{Logarithm of } .6734 = \bar{1}.828273$$

$$\text{" } .0009278 = \bar{4}.967454$$

$$\text{Difference} = 2.860819$$

$$2.860817 = \text{logarithm of } .7258000$$

$$2 =$$

33

$$\text{Ans. } .7258033$$

(2)

$$\text{Logarithm of } 437.89 = 2.641365$$

$$\text{" } 62.735 = 1.797510$$

$$\text{Difference} = .843855 = \text{logarithm of } 6.98$$

(3)

$$\text{Logarithm of } 93.217 = 1.969495$$

$$\text{" } .0007132 = \overline{4.853211}$$

$$\text{Difference} = 5.116284$$

$$5.116276 = \text{logarithm of } 130700.0$$

$$8 = \quad \quad \quad 2.4$$

$$\text{Ans. } 130702.4$$

(4)

$$\text{Logarithm of } 23 = 1.361728$$

$$\text{" } 189 = 2.276462$$

$$\text{" } 2.748 = 0.439017$$

$$\text{Sum} = 4.077207$$

$$\text{Logarithm of } 9835267 = 6.992786$$

$$4.077207$$

$$\text{Difference} = 2.915579$$

$$2.915558 = \text{logarithm of } 823.300$$

$$21 = \quad \quad \quad 39$$

$$\text{Ans. } 823.339$$

### EXERCISE 153.—Page 326.

(1)

$$\text{Logarithm of } 5 = 0.698970.$$

$$\text{Then } 0.698970 \times 5 = 3.494850 = \text{logarithm of } 3125.$$



(2)

Logarithm of 1.073 = .030600.

Then  $\cdot 030600 \times 6 = \cdot 183600 = \text{logarithm of } 1\cdot 5261$ .

(3)

Logarithm of .0279 =  $\bar{2}\cdot 445604$ .Then  $\bar{2}\cdot 445604 \times 4 = \bar{7}\cdot 782416 = \text{logarithm of } \cdot 00000060592$ .

(4)

Logarithm of 1.111 = .045714.

Then  $\cdot 045714 \times 11 = \cdot 502854 = \text{logarithm of } 3\cdot 1831$ .

## EXERCISE 154.—Page 327.

(1)

Logarithm of 913426000 = 8.960673.

 $8\cdot 960673 \div 7 = 1\cdot 2800961 = \text{logarithm of } 19\cdot 0588$ .

(2)

Logarithm of 1.61342 = .207747.

 $\cdot 207747 \div 11 = \cdot 01888609 = \text{logarithm of } 1\cdot 0444$ .

(3)

Logarithm of .000007139 =  $\bar{5}\cdot 853637 = \bar{10} + 4\cdot 853637$ . $(\bar{10} + 4\cdot 853637) \div 5 = \bar{2}\cdot 970727 = \text{logarithm of } \cdot 0934817$ .

(4)

Logarithm of .002147 =  $\bar{3}\cdot 331832 = \bar{7} + 4\cdot 331832$ . $(\bar{7} + 4\cdot 331832) \div 7 = \bar{1}\cdot 618831 = \text{logarithm of } \cdot 41575$ .

## EXERCISE 155.—Page 328.

(1)

$$14000 = 7 \times 2 \times 1000 \therefore \log. 14000 = (\log. 7) + (\log. 2) + (\log. 1000).$$

$$\text{Log. } 7 = 0.845098$$

$$\text{Log. } 2 = 0.301030$$

$$\text{Log. } 1000 = 3$$

$$\text{Sum,} \quad \underline{\hspace{1cm}} = 4.146128 = \log. 14000$$

$$4.9 = 7^2 \div 10 \therefore \log. 4.9 = (\log. 7) \times 2 - (\log. 10).$$

$$\text{Log. } 7 = 0.845098 \times 2 = 1.690196$$

$$\text{Log. } 10 = \quad \quad \quad 1$$

$$\text{Difference} = \underline{\hspace{1cm}} = .690196 = \log. 4.9$$

$$.00196 = 49 \times 4 \div 100000 = 7^2 \times 2^2 \div 100000$$

$$\therefore \log .00196 = (\log. 7) \times 2 + (\log. 2) \times 2 - (\log. 100000).$$

$$\text{Log. } 7 = 0.845098 \times 2 = 1.690196$$

$$\text{Log. } 2 = 0.301030 \times 2 = 0.602060$$

$$\text{Sum} = \underline{\hspace{1cm}} = 2.292256$$

$$\text{Log. of } 100000 = 5 \text{ and } 2.292256 - 5 = \bar{3}.292256 = \log \text{ of } .00196.$$

$$\text{Since } 5 = 10 \div 2, \text{ the logarithm of } 5 = \log. 10 - \log. 2 = 1 - 0.301030 = 0.698970.$$

$$1750 = 5^2 \times 7 \times 10 \therefore \log. 1750 = (\log. 5) \times 2 + (\log. 7) + (\log. 10)$$

$$\text{Log. } 5 = 0.698970 \times 2 = 1.397940$$

$$\text{Log. } 7 = \quad \quad \quad .845098$$

$$\text{Log. } 10 = \quad \quad \quad 1$$

$$\text{Sum,} \quad \underline{\hspace{1cm}} = 3.243038 = \log. \text{ of } 1750.$$

$$1428.571428 = \frac{1}{7} \times 10000 \therefore \log. 1428.571428 = (\log. \frac{1}{7}) + \log. 10000.$$

(1 continued.)

$$\text{Log. } \frac{1}{7} = (\text{log. } 1) - (\text{log. } 7) = 0 - 0.845098 = \bar{1}.154902$$

$$\text{Log. } 10000 = 4$$

$$\therefore \text{log. of } 1428.571428 = \text{sum} = 3.154902$$

$$.00000112 = 2^4 \times 7 \div 100000000 \therefore \text{log. } .00000112 =$$

$$(\text{log. } 2) \times 4 + (\text{log. } 7) - (\text{log. } 100000000).$$

$$\text{Log. } 2 = 0.301030 \times 4 = 1.204120$$

$$\text{Log. } 7 = 0.845098$$

$$\text{Sum} = 2.049218 = \text{and log. } 100000000 = 8$$

$$2.049218 - 8 = \bar{6}.049218 = \text{log. } .00000112$$

$$3.0625 = 1\frac{9}{16} \therefore \text{log. } 3.0625 = (\text{log. } 49) - (\text{log. } 16) =$$

$$(\text{log. } 7) \times 2 - (\text{log. } 2) \times 4.$$

$$\text{Log. } 7 = 0.845098 \times 2 = 1.690196$$

$$\text{Log. } 2 = 0.301030 \times 4 = 1.204120$$

$$\text{Difference} = 0.486076 = \text{log. of } 3.0625.$$

(2)

$$49\frac{1}{2} = \frac{9^2}{2} = 3^2 \times 11 \times \frac{1}{2} \therefore \text{log. } 49\frac{1}{2} = (\text{log. } 3) \times 2 + (\text{log. } 11)$$

$$+ (\text{log. } \frac{1}{2}).$$

$$\text{Log. } 3 = 0.477121 \times 2 = 0.954242$$

$$\text{Log. } 11 = 1.041393$$

$$\text{Log. } \frac{1}{2} = \bar{1}.698970$$

$$\text{Sum} = 1.694605 = \text{log. of } 49\frac{1}{2}.$$

$$363 = 11^2 \times 3 \therefore \text{log. } 363 = (\text{log. } 11) \times 2 + (\text{log. } 3).$$

$$\text{Log. } 11 = 1.041393 \times 2 = 2.082786$$

$$\text{Log. } 3 = 0.477121$$

$$\text{Sum} = 2.559907 = \text{log. of } 363.$$

Log. .5 or  $\frac{1}{2} = 1.698970$ , and by altering the characteristic we get 0.698970 for log. of 5.

(Continued on next page.)

(2 continued.)

$$4\cdot09 = 4\frac{1}{11} = \frac{45}{11} = 3^2 \times 5 \div 11 \therefore \log. 4\cdot09 = (\log. 3) \times 2 + (\log. 5) - (\log. 11).$$

$$\text{Log. 3} = \cdot477121 \times 2 = 0\cdot954242$$

$$\text{Log. 5} = \cdot698970$$

---


$$1\cdot653212$$

$$\text{Log. 11} = 1\cdot041393 \text{ and } 1\cdot653212 - 1\cdot041393 = 0\cdot611819 = \log. \text{ of } 4\cdot09.$$

$$2\cdot4 = 2\frac{4}{9} = \frac{2^2}{9} = 11 \times 2 \div 9 \therefore \log. 2\cdot4 = (\log. 11) + (\log. 2) - (\log. 3) \times 2.$$

$$\text{Log. 2} = (\log. 10) - (\log. 5) = 1 - 0\cdot698970 = 0\cdot301030.$$

$$\text{Log. 11} = 1\cdot041393$$

$$\text{Log. 2} = 0\cdot301030$$

---


$$1\cdot342423$$

$$\text{Log. 3} = 0\cdot477121 \times 2 = 0\cdot954242 \text{ and } 1\cdot342423 - 0\cdot954242 = 0\cdot388181 = \log. \text{ of } 2\cdot4.$$

$$392\cdot72 = 392\frac{3}{11} = \frac{4320}{11} = 2^4 \times 3^3 \times 10 \div 11 \therefore \log. 392\cdot72 = (\log. 2) \times 4 + (\log. 3) \times 3 + (\log. 10) - (\log. 11).$$

$$\text{Log. 2} = 0\cdot301030 \times 4 = 1\cdot204120$$

$$\text{Log. 3} = 0\cdot477121 \times 3 = 1\cdot431363$$

$$\text{Log. 10} = 1$$

---


$$\text{Sum} = 3\cdot635483$$

$$\text{Log. 11} = 1\cdot041393 \text{ and } 3\cdot635483 - 1\cdot041393 = 2\cdot594090 = \log. \text{ of } 392\cdot72.$$

$$293333\frac{1}{3} = \frac{880000}{3} = 2^3 \times 11 \times 10000 \div 3 \therefore \log. 293333\frac{1}{3} = (\log. 2) \times 3 + (\log. 11) + (\log. 10000) - (\log. 3).$$

$$\text{Log. 2} = 0\cdot301030 \times 3 = 0\cdot903090$$

$$\text{Log. 11} = 1\cdot041393$$

$$\text{Log. 10000} = 4$$

---


$$\text{Sum} = 5\cdot944483$$

(Continued on next page.)

(2 continued.)

$$\text{Log. } 3 = 0.477121 \text{ and } 5.944483 - 0.477121 = 5.467362 = \text{log. of } 293333\frac{1}{3}.$$

$$19.965 = 11^3 \times 5 \times 3 \div 1000 \therefore \text{log. } 19.965 = (\text{log. } 11) \times 3 + (\text{log. } 5) + (\text{log. } 3) - (\text{log. } 1000).$$

$$\text{Log. } 11 = 1.041393 \times 3 = 3.124179$$

$$\text{Log. } 5 = 0.698970$$

$$\text{Log. } 3 = 0.477121$$

$$\text{Sum} = 4.300270$$

$$\text{Log. } 1000 = 3 \text{ and } 4.300270 - 3 = 1.300270 = \text{log. of } 19.965.$$

## EXERCISE 156—Page 336.

(1)

Here we have given the first term 4, the number of terms 17 and the sum of the series 884, to find  $l$ , the last term.

$$\text{Then } l = \frac{2s}{n} - a = \frac{884 \times 2}{17} - 4 = 104 - 4 = 100.$$

(2)

Here we have given the first term 21, the last term 497 and the number of terms 41, to find the common difference.

$$\text{Then } d = \frac{l - a}{n - 1} = \frac{497 - 21}{41 - 1} = \frac{476}{40} = \frac{119}{10} = 11 \frac{9}{10}.$$

(3)

Here we have given  $a$ ,  $l$ , and  $d$ , to find  $n$ , and since  $a = 12$ ,  $l = 96$ , and  $d = 6$ , we have

$$n = \frac{l - a}{d} + 1 = \frac{96 - 12}{6} + 1 = \frac{84}{6} + 1 = 14 + 1 = 15.$$

(4)

Here we have given  $l$ ,  $d$ , and  $s$ , to find  $n$ , and since  $l = 14$ ,  $d = 1$ , and  $s = 105$ , we have

$$n = \frac{2l + d}{2d} + \sqrt{\left(\frac{2l + d}{2d}\right)^2 - \frac{2s}{d}} = \frac{2 \times 14 + 1}{2 \times 1} + \sqrt{\left(\frac{2 \times 14 + 1}{2 \times 1}\right)^2 - \frac{2 \times 105}{1}} = 14\frac{1}{2} + \sqrt{(\frac{29}{2})^2 - 210} = 14\frac{1}{2} + \sqrt{8\frac{1}{4} - 210} = 14\frac{1}{2} + \sqrt{\frac{1}{4}} = 14\frac{1}{2} + \frac{1}{2} = 15.$$

(5)

Here we have given  $a$ ,  $d$ , and  $s$ , to find  $l$ , and since  $a = \frac{2}{3}$ ,  $d = \frac{2}{3}$ , and  $s = 1180$ , we have

$$l = -\frac{1}{2}d + \sqrt{2ds + (a - \frac{1}{2}d)^2} = -\frac{1}{2} \text{ of } \frac{2}{3} + \sqrt{2 \times \frac{2}{3} \times 1180 + (\frac{2}{3} - \frac{1}{2} \times \frac{2}{3})^2} = -\frac{1}{3} + \sqrt{1732\frac{2}{3} + (\frac{1}{3})^2} = -\frac{1}{3} + \sqrt{1732\frac{2}{3} + \frac{1}{9}} = -\frac{1}{3} + \sqrt{1732\frac{7}{9}} = -\frac{1}{3} + \frac{118}{3} = 39\frac{1}{3}.$$

(6)

Here we have given  $a$ ,  $l$ , and  $s$ , to find  $d$ , and since  $a = 8$ ,  $l = 170$ , and  $s = 4895$ , we have

$$d = \frac{(l + a)(l - a)}{2s - l - a} = \frac{(170 + 8)(170 - 8)}{2 \times 4895 - 170 - 8} = \frac{178 \times 162}{9790 - 178} = \frac{28836}{9612} = 3.$$

(7)

Here we have given  $a$ ,  $l$ , and  $d$ , to find  $n$ , and since  $a = 5$ ,  $l = 27\frac{1}{2}$ , and  $d = 2\frac{1}{4}$ , we have

$$n = \frac{l - a}{d} + 1 = \frac{27\frac{1}{2} - 5}{2\frac{1}{4}} + 1 = \frac{22\frac{1}{2}}{2\frac{1}{4}} + 1 = \frac{45}{2} + 1 = 10 + 1 = 11$$

(8)

Here we have given  $a$ ,  $l$ , and  $n$ , to find  $s$ , and since  $a = 2$ ,  $l = 478$ , and  $n = 86$ , we have

$$s = (a + l) \frac{n}{2} = (2 + 478) \frac{86}{2} = 480 \times 43 = 20640.$$

(9)

Here we have given  $a$ ,  $l$ , and  $d$ , to find  $s$ , and since  $a = 2$ ,  $l = 998$ , and  $d = 6$ , we have

$$s = \frac{(l+a)(l-a)}{2d} + \frac{l+a}{2} = \frac{(998+2)(998-2)}{2 \times 6} + \frac{998+2}{2} =$$

$$\frac{1000 \times 996}{12} + \frac{1000}{2} = 83000 + 500 = 83500.$$

(10)

Here we have given  $a$ ,  $n$ , and  $d$ , to find  $l$ , and since  $a = 5$ ,  $n = 11$ , and  $d = 2\frac{1}{4}$ , we have

$$l = a + (n-1)d = 5 + (11-1)2\frac{1}{4} = 5 + (10 \times 2\frac{1}{4}) = 5 +$$

$$4\frac{5}{2} = \frac{5.5}{2} = 27\frac{1}{2}.$$

(11)

Here we have given  $l$ ,  $d$ , and  $n$ , to find  $s$ , and since  $l = 199$ ,  $d = 11$ , and  $n = 19$ , we have

$$s = \{2l - (n-1)d\} \frac{n}{2} = \{2 \times 199 - (19-1)11\} \frac{19}{2} =$$

$$\{398 - (18 \times 11)\} \frac{19}{2} = 200 \times \frac{19}{2} = 1900.$$

(12)

Here we have given  $s$ ,  $a$ , and  $l$ , to find  $n$ , and since  $s = 39840$ ,  $a = 2$ , and  $l = 478$ , we have

$$n = \frac{2s}{l+a} = \frac{2 \times 39840}{478+2} = \frac{79680}{480} = 166.$$

(13)

Here we have given  $s$ ,  $l$ , and  $a$ , to find  $d$ , and since  $s = 83500$ ,  $l = 998$ , and  $a = 2$ , we have

$$l = \frac{(l+a)(l-a)}{2s-l-a} = \frac{(998+2)(998-2)}{(2 \times 83500) - 998 - 2} = \frac{1000 \times 996}{167000 - 1000} = \frac{996000}{166000} = 6$$

(14)

Here we have given  $s$ ,  $a$ , and  $d$ , to find  $n$ , and since  $s = 360$ ,  $a = 2$ , and  $d = 2$ , we have

$$n = \frac{d-2a}{2d} + \sqrt{\frac{2s}{d} + \left(\frac{2a-d}{2d}\right)^2} = \frac{2-(2 \times 2)}{2 \times 2} + \sqrt{\frac{2 \times 260}{2} + \left(\frac{(2 \times 2)-2}{2 \times 2}\right)^2} = -\frac{1}{2} + \sqrt{260 + \left(\frac{1}{2}\right)^2} = -\frac{1}{2} + \sqrt{260\frac{1}{4}} = -\frac{1}{2} + 16.13226 = 15.63226 \text{ days} = 15 \text{ days, } 15 \text{ hours, } 10 \text{ minutes, } 27.264 \text{ seconds.}$$

(15)

Here we have given  $s$ ,  $a$ , and  $d$ , to find  $l$ , and since  $s = 83500$ ,  $a = 2$ , and  $d = 6$ , we have

$$l = -\frac{1}{2}d + \sqrt{2ds + \left(a - \frac{1}{2}d\right)^2} = -\frac{1}{2} \times 6 + \sqrt{2 \times 6 \times 83500 + \left(2 - \frac{1}{2} \times 6\right)^2} = -3 + \sqrt{1002000 + (2-3)^2} = -3 + \sqrt{1002001} = -3 + 1001 = 998.$$

(16)

Here we have given  $s$ ,  $n$ , and  $l$ , to find  $a$ , and since  $s = \$1125$ ,  $n = 18$ , and  $l = 120$ , we have

$$a = \frac{2s}{n} - l = \frac{2 \times 1125}{18} - 120 = 125 - 120 = 5.$$



(17)

Here we have given  $a$ ,  $l$ , and  $n$ , to find  $d$ , and since  $a = 5$ ,  $l = 27\frac{1}{2}$ , and  $n = 11$  we have

$$d = \frac{l - a}{n - 1} = \frac{27\frac{1}{2} - 5}{11 - 1} = \frac{22\frac{1}{2}}{10} = 2\frac{1}{4}.$$

(18)

Here we have  $a$ ,  $d$ , and  $n$  given, to find  $s$ , and since to deposit one stone he must walk 5 yards, and the distance travelled for each succeeding stone is 5 yards, therefore  $a = 5$ ,  $d = 5$ , and  $n = 220$ .

$$\begin{aligned} \text{Then } s &= \{2a + (n - 1)d\} \frac{n}{2} = \{2 \times 5 + (220 - 1)5\} \frac{220}{2} \\ &= \{10 + (219 \times 5)\} 110 = \\ 1105 \times 110 &= 121550 \text{ yards} = 69\frac{1}{8} \text{ miles.} \end{aligned}$$

(19)

Here we have  $s$ ,  $n$ , and  $l$  given, to find  $a$ , and since  $s = 39840$ ,  $n = 166$ , and  $l = 478$ , we have

$$a = \frac{2s}{n} - l = \frac{2 \times 39840}{166} - 478 = 480 - 478 = 2.$$

(20)

Here we have  $n$ ,  $a$ , and  $d$  given, to find  $s$ , and since  $n = 12$ ,  $a = 4$ , and  $d = 2$ , we have

$$\begin{aligned} s &= \{2a + (n - 1)d\} \frac{n}{2} = \{2 \times 4 + (12 - 1)2\} \frac{12}{2} = \{8 + (11 \times 2)\} 6 = \\ 30 \times 6 &= 180. \end{aligned}$$

(21)

Here we have given  $a$ ,  $l$ , and  $n$ , to find  $s$ , and  $a = 1$ ,  $l = 24$ , and  $n = 24$ .

$$\text{Then } s = (a + l) \frac{n}{2} = (1 + 24) \frac{24}{2} = 25 \times 12 = 300.$$

## EXERCISE 157—Page 342.

(1)

Here  $n = 11$ ,  $a = £1024$ , and  $r = 1\frac{1}{2}$ .Then  $l = ar^n - 1 = 1024 \times (\frac{3}{2})^{10} = 1024 \times \frac{59049}{64} = £59049$ 

$$s = \frac{rl - a}{r - 1} = \frac{\frac{3}{2} \times 59049 - 1024}{\frac{3}{2} - 1} = \frac{177147 - 1024}{\frac{1}{2}} = \frac{176123}{\frac{1}{2}} = £175099 = \text{whole fortune.}$$

(2)

Here  $a = 7$ ,  $l = 1240029$  and  $s = 1860040$ .

$$\text{Then } r = \frac{s - a}{s - l} = \frac{1860040 - 7}{1860040 - 1240029} = \frac{1860033}{620011} = 3.$$

(3)

Here  $n = 12$ ,  $a = £1$ , and  $l = £2048$ .

$$\text{Then } r = \left( \frac{l}{a} \right)^{\frac{1}{n-1}} = \left( \frac{2048}{1} \right)^{\frac{1}{12-1}} = \sqrt[11]{2048} = 2.$$

$$s = \frac{rl - a}{r - 1} = \frac{(2 \times 2048) - 1}{2 - 1} = 4096 - 1 = £4095.$$

(4)

Here  $r = \frac{3}{2}$ ,  $n = 8$ , and  $l = 106\frac{103}{12}$ .

$$\text{Then } s = \frac{l(r^n - 1)}{(r - 1)r^{n-1}} = \frac{106\frac{103}{12} \times [(\frac{3}{2})^8 - 1]}{(\frac{3}{2} - 1)(\frac{3}{2})^7} = \frac{54675 \times \frac{6305}{256}}{\frac{1}{2} \times \frac{2187}{128}} = \frac{25 \times 6305}{512} = 307\frac{11}{12}.$$

(5)

Here  $a = 1$ ,  $n = 7$ , and  $r = 3$ .

$$\text{Then } s = \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (3^7 - 1)}{3 - 1} = \frac{2186}{2} = 1093.$$

(6)

Here  $a = 1$ ,  $l = 10077696$ , and  $n = 10$ .

$$\begin{aligned} \text{Then } s &= \frac{l^{\frac{n}{n-1}} - a^{\frac{n}{n-1}}}{l^{\frac{1}{n-1}} - a^{\frac{1}{n-1}}} = \frac{(10077696)^{\frac{10}{9}} - 1^{\frac{10}{9}}}{(10077696)^{\frac{1}{9}} - 1^{\frac{1}{9}}} = \\ &= \frac{\sqrt[9]{(10077696)^{10}} - 1}{\sqrt[9]{10077696} - 1} = \frac{\sqrt[9]{(216)^{10}} - 1}{\sqrt[9]{216} - 1} = \frac{6^{10} - 1}{6 - 1} = \frac{60466176 - 1}{5} \\ &= \frac{60466175}{5} = 12093235. \end{aligned}$$

(7)

Here  $a = 6$ ,  $l = 3072$ , and  $s = 6138$ .

$$\text{Then } r = \frac{s - a}{s - l} = \frac{6138 - 6}{6138 - 3072} = \frac{6132}{3066} = 2.$$

(8)

Here  $r = 2$ ,  $n = 11$ , and  $s = 20470$ .

$$\begin{aligned} \text{Then } l &= \frac{(r-1)sr^{n-1}}{r^n - 1} = \frac{(2-1) \times 20470 \times 2^{10}}{2^{11} - 1} = \frac{20470 \times 1024}{2047} \\ &= 10240. \end{aligned}$$

(9)

Here  $a = 1s.$ ,  $n = 12$ , and  $r = 2$ .

$$\text{Then } s = \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (2^{12} - 1)}{2 - 1} = 4095s. = £204 \text{ } 15s.$$

(10)

Here  $a = 1$  farthing,  $r = 2$  and  $n = 32$ .

$$\text{Then } s = \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (2^{32} - 1)}{2 - 1} = 4294967295 \text{ far.} = £4473924 \text{ } 5s. \text{ } 3\frac{1}{4}d.$$

(11)

Here  $a = 4$ ,  $l = 78732$ , and  $n = 10$ .

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{78732}{4}\right)^{\frac{1}{10-1}} = \sqrt[9]{19683} = 3.$$

(12)

Here  $a = 5$ ,  $r = 2$ , and  $n = 7$ .

$$\text{Then } l = ar^{n-1} = 5 \times 2^{7-1} = 5 \times 2^6 = 5 \times 64 = 320.$$

(13)

Here  $a = 5$ ,  $l = 327680$ , and  $r = 4$ .

$$\text{Then } s = \frac{rl - a}{r - 1} = \frac{(327680 \times 4) - 5}{4 - 1} = 436905.$$

(14)

Here  $a = 1$ ,  $r = 2$ , and  $n = 64$ .

$$\text{Then } s = \frac{a(r^n - 1)}{r - 1} = \frac{1 \times (2^{64} - 1)}{2 - 1} = 18446744073709551615 \text{ gr.}$$

$$18446744073709551615 \div (7680 \times 64) = 37529996894754 \text{ bush.}$$

$$\$1.50 \times 37529996894754 = \$56294995342131$$

(15)

Here  $r = 3$ ,  $n = 10$ , and  $s = 295240$ .

$$\text{Then } l = \frac{(r-1)sr^{n-1}}{r^n - 1} = \frac{(3-1) \times 295240 \times 3^9}{3^{10} - 1} = \frac{2 \times 295240 \times 19683}{59048} = 196830.$$

(16)

Here  $a = 1$ ,  $l = 2048$ , and  $n = 12$ .

$$\text{Then } s = \frac{l^{\frac{n}{n-1}} - a^{\frac{n}{n-1}}}{l^{\frac{1}{n-1}} - a^{\frac{1}{n-1}}} = \frac{2048^{\frac{12}{12-1}} - 1^{\frac{12}{12-1}}}{2048^{\frac{1}{12-1}} - 1^{\frac{1}{12-1}}} = \frac{\sqrt[11]{(2048)^{12} - 1}}{\sqrt[11]{2048 - 1}} = \frac{2^{12} - 1}{2 - 1} = 2^{12} - 1 = 4095.$$

17)

Here  $a = 5$ ,  $r = 4$ , and  $n = 9$ .

$$\text{Then } l = ar^{n-1} = 5 \times 4^{9-1} = 5 \times 4^8 = 5 \times 65536 = 327680.$$

## EXERCISE 156.—Page 344.

(1)

Here  $a = \frac{2}{7}$ , and  $r = \frac{3}{5}$ .

$$\text{Then } s = \frac{a}{1-r} = \frac{\frac{2}{7}}{1-\frac{3}{5}} = \frac{\frac{2}{7}}{\frac{2}{5}} = \frac{5}{7}.$$

(2)

Here  $a = 4$ , and  $r = \frac{1}{2}$ ,

$$\text{Then } s = \frac{a}{1-r} = \frac{4}{1-\frac{1}{2}} = \frac{4}{\frac{1}{2}} = 8.$$

(3)

Here  $a = \frac{79}{100}$ , and  $r = \frac{1}{100}$ .

$$\text{Then } s = \frac{a}{1-r} = \frac{\frac{79}{100}}{1-\frac{1}{100}} = \frac{\frac{79}{100}}{\frac{99}{100}} = \frac{79}{99}.$$

(4)

Here  $a = \frac{1234}{10000}$ , and  $r = \frac{1}{10000}$ .

$$\text{Then } s = \frac{a}{1-r} = \frac{\frac{1234}{10000}}{1-\frac{1}{10000}} = \frac{\frac{1234}{10000}}{\frac{9999}{10000}} = \frac{1234}{9999};$$

## EXERCISE 159.—Page 345.

(1)

Since there are 9 means and 2 extremes the number of terms is 11.

$$\text{Then } d = \frac{l-a}{n-1} = \frac{92-2}{11-1} = \frac{90}{10} = 9.$$

1st term = 2; 2nd =  $2 + 9 = 11$ ; 3rd =  $11 + 9 = 20$ ; 4th =  $20 + 9 = 29$ ; 5th =  $29 + 9 = 38$ ; 6th =  $38 + 9 = 47$ ; and so on.

And series is 2, 11, 20, 29, 38, 47, 56, 65, 74, 83, 92.

(2)

Since there are 4 means and 2 extremes the number of terms is 6.

$$\text{Then } d = \frac{l-a}{n-1} = \frac{50-7}{6-1} = \frac{43}{5} = 8\frac{3}{5}.$$

1st term = 7; 2nd =  $7 + 8\frac{3}{5} = 15\frac{3}{5}$ ; 3rd =  $15\frac{3}{5} + 8\frac{3}{5} = 24\frac{1}{5}$ ; 4th =  $24\frac{1}{5} + 8\frac{3}{5} = 32\frac{4}{5}$ ; 5th =  $32\frac{4}{5} + 8\frac{3}{5} = 41\frac{2}{5}$ ; and 6th =  $41\frac{2}{5} + 8\frac{3}{5} = 50$ .

And series is 7,  $15\frac{3}{5}$ ,  $24\frac{1}{5}$ ,  $32\frac{4}{5}$ ,  $41\frac{2}{5}$ , 50.

(3)

Since there are 8 means and 2 extremes the number of terms is 10.

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{4096}{4096}\right)^{\frac{1}{10-1}} = (512)^{\frac{1}{9}} = \frac{1}{2}.$$

1st term = 4096; 2nd =  $4096 \times \frac{1}{2} = 2048$ ; 3rd =  $2048 \times \frac{1}{2} = 1024$ ; 4th =  $1024 \times \frac{1}{2} = 512$ ; 5th =  $512 \times \frac{1}{2} = 256$ , and so on.

And the means are 2048, 1024, 512, 256, 128, 64, 32, and 16.

(4)

Since there are 7 means and 2 extremes the number of terms is 9.

$$\text{Then } r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = (23514624)^{\frac{1}{8}} = (1679616)^{\frac{1}{8}} = 6.$$

1st term = 14; 2nd =  $14 \times 6 = 84$ ; 3rd =  $84 \times 6 = 504$ ;  
4th =  $504 \times 6 = 3024$ ; 5th =  $3024 \times 6 = 18144$ , and so on.

And the means are 84, 504, 3024, 18144, 108864, 653184, and 3919104.

## EXERCISE 160.—Page 347

(1)

Assume 4 to be the number of men.

Then  $2 \times 4 = 8 =$  number of women.

And  $8 \times 3 = 24 =$  number of children.

6d.  $\times 4 = 24$ d. = amount received by the men.

4d.  $\times 8 = 32$ d. = “ “ “ women.

2d.  $\times 24 = 48$ d. = “ “ “ children.

Sum, = 104d., but it should, by question, = 78d.

$$\text{Then } 104 : 78 :: 4 : \frac{78 \times 4}{104} = 3 = \text{number of men.}$$

$3 \times 2 = 6 =$  number of women, and  $6 \times 3 = 18 =$  number of children.

(2)

Assume £8 to be the price of the harness.

Then  $£8 \times 2 = 16 =$  price of horse.

And  $£8 + £16 = £24 \times 2 = 48 =$  “ chaise.

Sum, = £ 72, but it should by question = £60.

$$\text{Then } £72 : £60 :: £8 : \frac{8 \times 60}{72} = £6 \ 13 \ 4 = \text{price of harness.}$$

$£6 \ 13 \ 4 \times 2 = 13 \ 6 \ 8 =$  “ horse.

$£6 \ 13 \ 4 + £13 \ 6 \ 8 = £20 \times 2 = 40 \ 0 \ 0 =$  “ chaise.



(3)

Assume 20 as C's age.

Then  $20 \times 3 = 60 = \text{B's age.}$ And  $60 \times 2 = 120 = \text{A's age.}$ 

Sum = 200, but by question it should = 140.

$$20 \times 140$$

Then  $200 : 140 :: 20 : \frac{200}{140} = 14 = \text{C's age.}$  $14 \times 3 = 42 = \text{B's age, and } 42 \times 2 = 84 = \text{A's age.}$ 

(4)

Assume 100.

One fourth of 100 = 25 and remainder =  $100 - 25 = 75$ .One fifth of 75 = 15 and remainder =  $75 - 15 = 60$ , but it should be by the question = 72.

$$100 \times 72$$

Then  $60 : 72 :: 100 : \frac{100 \times 72}{60} = 120$ .

(5)

A can do the work in 7 days  $\therefore$  he will do  $\frac{1}{7}$  of it in 1 day.B " " 5 "  $\therefore$  "  $\frac{1}{5}$  " "C " " 6 "  $\therefore$  "  $\frac{1}{6}$  " "Then all working together will do  $\frac{1}{7} + \frac{1}{5} + \frac{1}{6} = \frac{107}{210}$  in 1 day.Therefore to do the whole work it will take them  $\frac{1}{\frac{107}{210}} = \frac{210}{107} =$  $1\frac{103}{107}$  days.

(6)\*

A and B working together can do it in 10 days  $\therefore$  they will do  $\frac{1}{10}$  of it in 1 day.A can do it in 15 days  $\therefore$  he will do  $\frac{1}{15}$  of it in 1 day.Therefore  $\frac{1}{10} - \frac{1}{15} = \frac{1}{30} = \text{amount done by B in 1 day.}$ Then if he does  $\frac{1}{30}$  in 1 day, it will take him 30 days to do the whole.

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\*The mode of working these questions by position is so simple that they cannot trouble any one; it has therefore been thought advisable to work them by simple analysis.

(7)\*

The first pipe empties the whole of it in 1 hour.

The second pipe empties  $\frac{1}{2}$  of it in 1 hour.

The third pipe empties  $\frac{1}{3}$  of it in 1 hour.

Then all these pipes running together will empty  $1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$  in 1 hour.

Therefore to empty the cistern it will take  $1 \div \frac{11}{6} = \frac{6}{11}$  hours.

(8)

Assume 84

One third of 84 = 28

One sixth of 84 = 14

One seventh of 84 = 12

Sum = 54, but by question it should = 27.

Then  $54 : 27 :: 84 : \frac{84 \times 27}{54} = 42$ .

(9)

All 5 mills working together will grind  $7 + 5 + 4 + 3 + 1 = 20$  bushels in 1 hour.

Therefore to grind 500 bushels it will take them  $500 \div 20 = 25$  hours.

(10)\*

One pipe fills  $\frac{1}{2}$  of the cistern in 1 hour, and the other empties  $\frac{1}{8}$  of it in 1 hour.

Then  $\frac{1}{2} - \frac{1}{8} = \frac{1}{8} =$  part of the cistern filled in 1 hour when both are left open.

And if  $\frac{1}{8}$  of it is filled in 1 hour, the whole will be filled in  $\frac{1}{\frac{1}{8}} = 8$  hours.

---

\* See note on page 227.

## EXERCISE 161.—Page 352.

(1)

Assume 60 for father's age, then  $15 =$  son's.

$$\begin{array}{r}
 5 \\
 \hline
 5 \overline{)55} \\
 \hline
 11 \\
 10 \\
 \hline
 -1
 \end{array}$$

$$\begin{array}{r}
 5 \\
 \hline
 10
 \end{array}$$

Assume 100 for father's age, then  $25 =$  son's.

$$\begin{array}{r}
 5 \\
 \hline
 5 \overline{)95} \\
 \hline
 19 \\
 20 \\
 \hline
 +1
 \end{array}$$

$$\begin{array}{r}
 5 \\
 \hline
 20
 \end{array}$$

Errors. Assumed numbers.

$$\begin{array}{rclcl}
 -1 & \times & 100 & = & 100 \\
 +1 & \times & 60 & = & 60 \\
 \hline
 \end{array}$$

Sum of errors  $= 2$  Sum of products  $= 160$ 

Therefore result required  $= 160 \div 2 = 80 =$  father's age, and  
 $\frac{1}{4}$  of 80  $= 20 =$  son's age.

(2)

Assume 80

$$\begin{array}{r}
 34 \\
 \hline
 46 \\
 3 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 138 \\
 80 \\
 \hline
 \end{array}$$

$$58$$

$$\frac{1}{4} \text{ of } 80 = 20$$

$$+ 38$$

Assume 44

$$\begin{array}{r}
 34 \\
 \hline
 10 \\
 3 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 30 \\
 44 \\
 \hline
 \end{array}$$

$$-14$$

$$\frac{1}{4} \text{ of } 44 = 11$$

$$-25$$

(Continued on next page.)

(2 continued.)

Errors. Assumed numbers.

$$\begin{array}{rclcl} -25 & \times & 80 & = & 2000 \\ +38 & \times & 44 & = & 1672 \\ \hline \end{array}$$

$$\text{Sum of errors} = 63 \qquad \text{Sum of products} = 3672$$

$$\text{Therefore result required} = 3672 \div 63 = 58\frac{2}{3}.$$

(4)

$$\begin{array}{rcl} \text{Assume } 18 & \text{and } 7 \\ \text{One half of } 18 = \frac{18}{2} & 2 \times 7 = 14 \\ \hline 9 & & 14 \\ -5 & & \hline \end{array}$$

$$\begin{array}{rcl} \text{Assume } 22 & \text{and } 3 \\ \text{One half of } 22 = \frac{22}{2} & 2 \times 3 = 6 \\ \hline 11 & & 6 \\ +5 & & \hline \end{array}$$

Errors. Assumed numbers.

$$\begin{array}{rclcl} -5 & \times & 22 & = & 110 \\ +5 & \times & 18 & = & 90 \\ \hline \end{array}$$

$$\text{Sum of errors} = 10 \qquad \text{Sum of products} = 200$$

$$\text{Then } 200 \div 10 = 20 = \text{one number, and } 25 - 20 = 5 \\ = \text{other number.}$$

(5)

|           |     |
|-----------|-----|
| A.        | B.  |
| Suppose 8 | 6   |
| 22½       | 9   |
| —         | 12  |
| 180       | 15  |
| 132       | 18  |
| —         | 21  |
| 8)48      | 24  |
| —         | 27  |
| + 6       | —   |
| 6         | 132 |
| —         |     |
| 36        |     |
| 72        |     |
| —         |     |

$$3)36$$

$$12$$

$$9 - 6 = 3 = \text{difference of errors.}$$

|           |    |
|-----------|----|
| A.        | B. |
| Suppose 6 | 6  |
| 22½       | 9  |
| —         | 12 |
| 135       | 15 |
| 81        | 18 |
| —         | 21 |
| 6)54      | —  |
| —         | 81 |
| + 9       |    |
| 8         |    |
| —         |    |
| 72        |    |

(7)

Assume 30.

$$\begin{aligned} \frac{1}{2} \text{ of } 30 &= 15; \frac{1}{4} \text{ of } 30 = 7\frac{1}{2}; \\ \frac{1}{6} \text{ of } 30 &= 5; \text{ and } \frac{1}{8} \text{ of } 30 = 3\frac{3}{4}; \\ 15 \times 7\frac{1}{2} \times 5 \times 3\frac{3}{4} &= 3375; \\ 3375 - 6998\frac{1}{2} &= -3623\frac{1}{2} = \text{error.} \end{aligned}$$

Assume 60.

$$\begin{aligned} \frac{1}{2} \text{ of } 60 &= 30; \frac{1}{4} \text{ of } 60 = 15; \\ \frac{1}{6} \text{ of } 60 &= 10; \text{ and } \frac{1}{8} \text{ of } 60 = 7\frac{1}{2}. \\ 30 \times 15 \times 10 \times 7\frac{1}{2} &= 54000. \\ 54000 - 6998\frac{1}{2} &= +47001\frac{1}{2} = \text{error.} \\ 30^4 &= 810000, \text{ and } 60^4 = 12960000 \\ -3623\frac{1}{2} \times 12960000 &= 46959264000 \\ +47001\frac{1}{2} \times 810000 &= 38071296000 \end{aligned}$$

$$\text{Sum} = 50625 \qquad \text{Sum} = 85030560000$$

$$85030560000 \div 50625 = 1679616$$

4th root = square root of square root.

$$\sqrt{1679616} = 1296, \text{ and } \sqrt{1296} = 36 = \text{required number.}$$

NOTE.—For reason why we multiply by the 4th powers of the assumed numbers and then take the 4th root of the quotient, see Arith. page 353, Example 11.

It may, however, perhaps be clearer from the following illustration:

Let  $x$  = the number required.

$$\begin{aligned} & \begin{array}{ccccccc} x & x & x & x & x^4 \\ \text{Then } - & \times & - & \times & - & \times & - \\ & 2 & 4 & 5 & 6 & 240 \end{array} \\ & \therefore x^4 = 1679616 \\ & \therefore x = \sqrt[4]{1679616} = 36. \end{aligned}$$

(8)

Suppose A had 9s. at first.

Then  $9 + 1 = 10$ ;  $10 \div 2 = 5$ ;  $5 + 1 = 6$  = what B had at first.

$6 + 1 = 7$ , but should =  $9 - 1 = 8$ .

$$\text{Error} = 7 - 8 = -1.$$

Suppose A had 11s. at first.

Then  $11 + 1 = 12$ ;  $12 \div 2 = 6$ ;  $6 + 1 = 7$  = what B had at first.

$7 + 1 = 8$ , but should =  $11 - 1 = 10$ .

$$\text{Error} = 8 - 10 = -2.$$

(Continued on next page.)

(8 continued.)

Errors.

$$- 2 \times 9 = 18$$

$$- 1 \times 11 = 11$$

$$\text{Diff.} = 1 \quad \text{diff.} = 7$$

$7 \div 1 = 7 =$  shillings A had at first.

$7 + 1 = 8$ ;  $8 \div 2 = 4$ ;  $4 + 1 = 5 =$  shillings B had at first.

(9)

Assume 24 and 6.

$$\frac{24}{2} + \frac{24}{3} + \frac{24}{6} = 24.$$

$$\frac{6}{2} + \frac{3}{4} \text{ of } 6 + \frac{6}{4} = 9.$$

$$24 - 9 = + 15 = \text{error.}$$

Assume 20 and 10.

$$\frac{20}{2} + \frac{20}{3} + \frac{20}{6} = 20.$$

$$\frac{10}{2} + \frac{3}{4} \text{ of } 10 + \frac{10}{4} = 15.$$

$$20 - 15 = + 5 = \text{error.}$$

Errors.

$$+ 15 \times 20 = 300$$

$$+ 5 \times 24 = 120$$

$$\text{Diff.} = 10 \quad \text{diff.} = 180$$

$$180 \div 10 = 18 = \text{one number.}$$

$$30 - 18 = 12 = \text{other number.}$$

(10)

Suppose 1st horse to be worth £20.

$$20 + 50 = 70; 70 \div 2 = £35 = \text{value of 2nd horse.}$$

$$35 + 50 = 85, \text{ but it should equal } 60, \text{ i.e. } (20 \times 3).$$

$$\text{Then } 60 - 85 = - 25 = \text{error.}$$

Suppose 1st horse to be worth £60.

$$£60 + £50 = £110; £110 \div 2 = £55 = \text{worth of 2nd horse.}$$

$$55 + 50 = 105, \text{ but it should equal } 180, \text{ i.e. } (60 \times 3).$$

$$180 - 105 = + 75 = \text{error.}$$

Errors.

$$+ 75 \times 20 = 1500$$

$$- 25 \times 60 = 1500$$

$$\text{Sum} = 100$$

$$\text{Sum} = 3000$$

$$3000 \div 100 = £30 = \text{value of 1st horse.}$$

$$£20 + £50 = £80; £80 \div 2 = £40 = \text{value of 2nd horse.}$$

(11)

Suppose there were 11 beggars.

$$11 \times 4 = 44; 44 + 6 = 50 = \text{number of pence he had.}$$

$$11 \times 6 = 66; 66 - 12 = 54 = \quad " \quad " \quad "$$

$$54 - 50 = + 4 = \text{error.}$$

Suppose there were 12 beggars.

$12 \times 4 = 48$ ;  $48 + 6 = 54 =$  pence he had.

$12 \times 6 = 72$ ;  $72 \div 12 = 60 =$  pence he had.

$$60 - 54 = +6 = \text{error.}$$

### Errors.

$$+ 6 \times 11 = 66$$

$$+ 4 \times 12 = 48$$

Diff. = 2    diff. = 18, and  $18 \div 2 = 9$  = number of beggars.

EXERCISE 162.—Page 357.

(1)

Here  $P = \$713.29$ ,  $r = .045$ , and  $t = 14$ .

Then  $A = P(1+r)^t$ , or  $\log. A = \log. P + \log. (1+r) \times t$   
 $= 2.853267 + (.019116 \times 14) = 3.120891 = \log. \text{ of } Ans.$

Hence amount = \$1320.96.

(2)

Here  $n = 7$ ,  $r = .015$ .

log.  $n$ .      .845098

Then  $t = \frac{\log \frac{10000}{10000 - 10000 \cdot 1.006466^t}}{\log(1 + r)} = 130.698$  payments, and

$$130.698 \div 4 = 32.674 \text{ years} = 32 \text{ years } 8 \text{ months } 2 \text{ days.}$$

(3)

Here  $A = \$1111.11$ ,  $P = 111.11$ , and  $r = .08$ .

$$\log. \mathcal{A} - \log. P \quad 3.045757 - 2.045753 \quad 1.000004$$

$$\text{Then } t = \frac{\log .01}{\log .(1+r)} = \frac{-1.30103}{-.033424} = 38.9224$$

$$= 29.918 \text{ years} = 29 \text{ years } 11 \text{ months.}$$

(4)

Here  $A = \$3333.33$ ,  $P = \$222.22$ , and  $t = 120$ .

$$\text{Then } r = \sqrt[t]{\frac{A}{P}} - 1; \text{ or } \log. (1 + r) = \frac{\log. A - \log. P}{t} =$$

$$\frac{3.522878 - 2.346783}{120} = \frac{1.176095}{120} = .0098007. \text{ Hence } 1 + r$$

$$= 1.0228, r = .0228, \text{ and rate per cent.} = 2\frac{7}{25}\%.$$

(5)

Here  $n = 2$  and  $r = .07$ .

$$\text{Then } t = \frac{\log. n.}{\log. (1 + r)} = \frac{0.301030}{0.029384} = 10.2446 \text{ years} = 10 \text{ yrs.}$$

$$2 \text{ months } 28 \text{ days.}$$

(6)

Here  $A = \$100$ ,  $r = .0225$ , and  $t = 28$ .

$$\text{Then } P = \frac{A}{(1 + r)^t}, \text{ or } \log P = \log. A - \log. (1 + r) \times t.$$

$$\text{Log. } P = 2 - (0.009664 \times 28) = 2 - 0.270592 = 1.729408.$$

Hence  $P = \$53.63$ .

(7)

Here  $P = \$2468.13$ ,  $r = .0375$ , and  $t = 26$ .

$$\text{Then } A = P (1 + r)^t, \text{ or } \log. A = \log. P + \log. (1 + r) \times t.$$

$$\text{Log. } A = 3.392368 + (0.015988 \times 26) = 3.392368 + 0.415688$$

$$= 3.808056.$$

Hence  $A = \$6427.705$ .

(8)

Here  $A = \$7137.40$ ,  $r = .0425$ , and  $t = 22$ .

$$\text{Then } P = \frac{A}{(1 + r)^t}, \text{ or } \log. P = \log. A - \log. (1 + r) \times t.$$

$$\text{Log. } P = 3.853540 - (0.018076 \times 22) = 3.853540 - 0.397672$$

$$= 3.455868.$$

Hence  $P = \$2856.723$ .



(16)

Here  $n = 19$ , and  $r = .0525$ .

$$\text{Then } t = \frac{\log. n}{\log. (1 + r)} = \frac{1.278754}{0.022222} = 57.5445 \text{ payments} = 28.7722 \text{ years} = 28 \text{ years } 9 \text{ months } 8 \text{ days.}$$

## EXERCISE 163.—Page 360.

(1)

Here  $r = .03$ ,  $a = 500$ ,  $A = 8365$ .

$$\begin{aligned} & \sqrt{\left\{ \frac{8rA}{a} + (2-r)^2 \right\}} - (2-r) \\ \text{Formula IV. } t &= \frac{\sqrt{\left\{ \frac{8 \times .03 \times 8365}{500} + (2 - .03)^2 \right\}} - (2 - .03)}{2r} \\ &= \frac{\sqrt{\left\{ \frac{2007.6}{500} + 3.8809 \right\}} - 1.97}{2 \times .03} \\ &= \frac{\sqrt{4.0152 + 3.8809} - 1.97}{.06} = \frac{\sqrt{7.8961} - 1.97}{.06} \\ &= \frac{2.81 - 1.97}{.06} = \frac{.84}{.06} = \frac{84}{6} = 14 \text{ payments} = 7 \text{ years.} \end{aligned}$$

(2)

Here  $a = 112.50$ ,  $r = .015$ ,  $t = 44$ .

$$\begin{aligned} & \text{Formula I. } A = at \left( 1 + \frac{(t-1)r}{2} \right) \\ &= 112.50 \times 44 \left( 1 + \frac{(44-1) \times .015}{2} \right) = 4950 \times 1.3225 \\ &= \$6546.375. \end{aligned}$$

(3)

Here  $a = 300$ ,  $A = 1680$ , and  $t = 5$ .

$$\begin{aligned} \text{Formula III. } r &= \frac{2(A - at)}{at(t-1)} = \frac{2\{1680 - (300 \times 5)\}}{300 \times 5(5-1)} \\ &= \frac{2(1680 - 1500)}{300 \times 5 \times 4} = \frac{2 \times 180}{6000} = \frac{360}{6000} = .06 \\ \therefore \text{Rate per cent} &= .06 \times 100 = 6. \end{aligned}$$

(4)

Here  $A = 2080$ ,  $r = .04$ , and  $t = 16$ .

$$\begin{aligned} \text{Formula II. } a &= \frac{2A}{t\{2 + (t-1)r\}} = \frac{2 \times 2080}{16\{2 + (16-1) \cdot 04\}} \\ &= \frac{4160}{16 \times \{2 + (15 \times .04)\}} = \frac{4160}{16 \times 2.6} = \frac{4160}{41.6} = 100 \\ &= \$100 = 1 \text{ payment or rent for half a year, hence yearly} \\ &\quad \text{rent} = \$100 \times 2 = \$200. \end{aligned}$$

**EXERCISE 164.—Page 366.**

(1)

Here  $r = .04$ , and  $v = \$3000$ .Then  $a = vr = 3000 \times .04 = \$120$ .

(2)

Here  $a = 563$ , and  $v = 11260$ 

$$\begin{aligned} \text{Then } r &= \frac{a}{v} = \frac{563}{11260} = \frac{1}{20} = .05, \text{ and hence rate} \\ &\quad \text{per cent.} = 5. \end{aligned}$$

(3)

Here  $a = 75$ ,  $r = .05$ , and  $s = 14$ .

$$\begin{aligned} \text{Then } v &= \frac{r(1+r)s}{\log. v = \log. 75 - \{(\log. 1.05 \times 14) + \log. .05\}} \\ &= \frac{.05 \times (1.05)^{14}}{1.875061 - (0.021189 \times 14 + \log. .05)} \\ &= \frac{.05 \times (1.05)^{14}}{1.875061 - (0.296646 + .698970)} \\ &= \frac{.05 \times (1.05)^{14}}{2.879445} \end{aligned}$$

$\therefore v =$  nat. number corresponding to the logarithm 2.879445,  
which is \$757.608.

(4)

Here  $a = \$90$ ,  $r = .04$ ,  $t = 12$ ,  $s = 7$ , and  $\therefore s + t = 19$ .

$$\begin{aligned} \text{Formula VIII. } v &= \frac{a}{r} \left\{ \frac{1}{(1+r)^t} - \frac{1}{(1+r)^{s+t}} \right\} \\ &= \frac{90}{.04} \left\{ \frac{1}{(1.04)^{12}} - \frac{1}{(1.04)^{19}} \right\} = \frac{9000}{4} \left\{ \frac{1}{1.60101} - \frac{1}{2.10682} \right\} \\ &= 2250 \times (.624605 - .474649) = 2250 \times .149956 \\ &= \$337.401. \end{aligned}$$

(5)

Here  $a = 1500$ , and  $r = .05$ .

$$\begin{aligned} \text{Formula IX. } v &= \frac{a}{r} = \frac{1500}{.05} = \frac{150000}{5} = \$30000 \\ &= 20 \times 1500 \text{ or 20 years' purchase.} \end{aligned}$$

(6)

Here  $a = 22$ ,  $v = 308.64166$ , and  $r = .04$ .

$$\begin{aligned} \text{Then Formula VII. } t &= \frac{\log. a - \log. (a - vr)}{\log. (1+r)} \\ &= \frac{\log. 22 - \log. (22 - 308.6416 \times .04)}{\log. (1.04)} \\ &= \frac{1.342423 - \log. (9.65425)}{0.017033} = \frac{1.342423 - 0.984707}{0.017033} \\ &= \frac{0.357716}{0.017033} = \frac{357716}{17033} = 21 + \end{aligned}$$

(7)

Here  $a = 154$ ,  $t = 19$ , and  $r = .05$ .

$$\begin{aligned} \text{Formula V. } v &= \frac{a}{r} \left\{ 1 - \frac{1}{(1+r)^t} \right\} \\ &= \frac{154}{.05} \times \left\{ 1 - \frac{1}{(1.05)^{19}} \right\} = \frac{15400}{5} \times \left\{ 1 - \frac{1}{2.5269} \right\} \\ &= 3080 \times (1 - .39574) = 3080 \times .60426 = \$1861.12 + \end{aligned}$$

(8)

Here  $A = 600$ ,  $t = 40$ , and  $r = .0375$ .

$$\begin{aligned} \text{Formula II. } a &= \frac{Ar}{(1+r)^t - 1} = \frac{600 \times .0375}{(1.0375)^{40} - 1} \\ &= \frac{22.5}{22.5} = \frac{2250000}{4.36034 - 1} = \frac{2250000}{3.36034} \\ &= £6.6957 = £6 \text{ } 13\text{s. } 10\frac{3}{4}\text{d.} + \end{aligned}$$

(9)

Here  $a = 8$ ,  $A = 187.315625$ , and  $r = .03$ .

$$\begin{aligned} \text{Formula IV. } t &= \frac{\log. (Ar + a) - \log. a}{\log. (1+r)} \\ &= \frac{\log. (187.315625 \times .03 + 8) - \log. 8}{\log. 1.03} \\ &= \frac{\log. 5.61946875 + 8 - \log. 8}{\log. 1.03} \\ &= \frac{1.134160 - 0.903090}{0.012837} = \frac{0.231070}{0.012837} = 18. \end{aligned}$$

(10)

Here  $a = 74$ ,  $r = .04$ , and  $t = 30$ 

$$\begin{aligned} \text{Formula I. } A &= a \left\{ (1+r)^t - 1 \right\} \div r = \frac{74 \times \{ (1.04)^{30} - 1 \}}{.04} \\ &= \frac{74}{.04} \times (3.24332 - 1) = \frac{7400}{4} \times 2.24332 = \$4150.142 \end{aligned}$$

By Table, page 362. Amount of \$1 for 30 years, at 4 per cent.  
= \$56.08494

Then  $\$56.08494 \times 74 = \$4150.28$ .

## EXERCISE 165—Page 367.

## EXAMINATION PROBLEMS.

## FIRST SERIES.

(2)

$\$7580 \times .19 = \$1440.20$ , and  $\$7580 - \$1440.20 = \$6139.80$ .  
D is to have one third as much as A, B, and C together, therefore he will have one-fourth of the whole.  $\frac{1}{4}$  of  $\$6139.80 = \$1534.95 = D$ 's share.

$\$6139.80 - \$1534.95 = \$4604.85 =$  amount to be divided among A, B, and C.

B is to have  $\$90.90$  more than C.

A is to have  $\$111.11 + \$90.90 = 202.01$  " " "

$\$292.91$

$\$4604.85 - \$292.91 = \$4311.94 =$  three times C's share.

$\$4311.94 \div 3 = \$1437.31\frac{1}{3} =$  C's share.

$\$1437.31\frac{1}{3} + \$90.90 = \$1528.21\frac{1}{3} =$  B's share.

$\$1528.21\frac{1}{3} + \$111.11 = \$1639.32\frac{1}{3} =$  A's share.

(3)

A and B working together can do the work in 96 hours, therefore in one hour they will do  $\frac{1}{96}$  of it.

A by himself can do the work in 192 hours; therefore in 1 hour he can do  $\frac{1}{192}$  of it.  $\frac{1}{96} - \frac{1}{192} = \frac{1}{192} =$  part B can do in one hour. Therefore he will require as many hours to finish it as  $\frac{1}{192}$  is contained times in the whole, i. e.  $1 \div \frac{1}{192} = 192$  hours. Then  $192 \div 14 = 13\frac{1}{2}$  days.

(4)

$\pounds 179 \text{ 14s. } 8\frac{1}{2}\text{d.} = \$718.94\frac{7}{8} = \$718.94583.$

$\$718.94583 \div .00000048 = \$71894583333.3 \div 48 =$   
 $\$1497803819.4444.$

(5)

|    |  |                        |       |
|----|--|------------------------|-------|
| 77 |  | 44..18..30..77..56..27 |       |
| 30 |  | 4..18..30              | 8..27 |
| 36 |  | 2..8                   | 4..9  |

$77 \times 30 \times 36 = 83160 = 1. \text{ c. m.}$

Q

(6)

Here  $n = 20$ , and  $r = .0525$ .

$$\text{Then } t = \frac{n-1}{r} = \frac{20-1}{.0525} = \frac{19}{.0525} = 361.9048 \text{ years} =$$

361 years 10 months 25 days.

(7)

7342163 octenary = 710e57 duodenary, and 61351 nonary = 1e454 duodenary.

$$710e57 \div 1e454 = 40.38 \text{ duodenary.}$$

(8)

$$783\frac{1}{2} = 3\frac{1}{2} + 10 \times 8 + 10 \times 10 \times 7.$$

|      |     |      |      |          |                |      |     |      |      |                |
|------|-----|------|------|----------|----------------|------|-----|------|------|----------------|
| lbs. | oz. | dwt. | grs. |          |                | lbs. | oz. | dwt. | grs. |                |
| 43   | 3   | 17   | 11   | $\times$ | $3\frac{1}{2}$ | =    | 151 | 7    | 11   | $2\frac{1}{2}$ |
|      |     |      |      |          | 10             |      |     |      |      |                |

|     |   |    |    |          |    |   |      |   |    |    |
|-----|---|----|----|----------|----|---|------|---|----|----|
| 433 | 2 | 14 | 14 | $\times$ | 8  | = | 3465 | 9 | 16 | 16 |
|     |   |    |    |          | 10 |   |      |   |    |    |

|      |   |   |    |          |   |   |       |    |   |                 |
|------|---|---|----|----------|---|---|-------|----|---|-----------------|
| 4332 | 3 | 5 | 20 | $\times$ | 7 | = | 30325 | 11 | 0 | 20              |
|      |   |   |    |          |   |   | 33943 | 4  | 8 | $14\frac{1}{2}$ |

(9)

Here  $a = 1$ , and  $r = \frac{1}{2}$ .

$$\text{Then } S = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2.$$

(10)

$$\frac{1}{2} \text{ of } \frac{2}{3} \text{ of } 192 \div \frac{2\frac{1}{2}}{3} = 64 \div \frac{\frac{5}{2}}{3} = 64 \div \frac{\frac{5}{2}}{\frac{2}{3}} = 64 \div \frac{5}{4} = 64 \times \frac{4}{5} = 129\frac{3}{5}.$$

(11)

Logarithm of 129140163 = 8.111061.  
 $8.111061 \div 17 = .477121 = \text{logarithm of } 3.$

(12)

Suppose 48

18

66

84

— 18

Suppose 36

18

54

63

— 9

Errors. Assumed numbers.

— 18  $\times$  36 = 648— 9  $\times$  48 = 432

Difference of errors = 9

9)216 = sum of products.

24

SECOND SERIES.

(13)

B is to have \$69.18 more than C.

A is to have \$69.18 + \$93.40 = \$162.58 “ “ “

\$231.76

 $\$897.43 - \$231.76 = \$665.67 = \text{Amount to be divided}$   
 equally amongst A, B, and C.
 $\$665.67 \div 3 = \$221.89 = \text{C's share.}$  $\$221.89 + \$69.18 = \$291.07 = \text{B's “}$  $\$291.07 + \$93.40 = \$384.47 = \text{A's “}$ 

(14)

|                |   |                |   |                           |
|----------------|---|----------------|---|---------------------------|
| 7 lbs. wheat   | = | 9 lbs. rye     | } | $x = 9$                   |
| 5 “ rye        | = | 8 “ oats       |   | $x = 8$                   |
| 13 “ oats      | = | 21 “ buckwheat |   | $13 = 21 \quad x$         |
| 27 “ buckwheat | = | 20 “ barley    |   | $3 \quad 27 = 20 \quad 4$ |
| 24 “ barley    | = | 26 “ peas      |   | $3 \quad 24 = 26 \quad 2$ |
| 11 “ peas      | = | 35 “ potatoes  |   | $11 = 35$                 |
| x “ potatoes   | = | 16 “ wheat     | } | $x = 16$                  |

$4 \times 2 \times 35 \times 16 = 4480$   
*Ans.*  $\frac{4480}{3 \times 11} = 135\frac{1}{3}$ .

(15)

$$\frac{2}{3} \text{ of } 4\frac{1}{2} \text{ of } 7\frac{1}{2} \text{ of } \frac{9}{19\frac{1}{2}} \text{ of } \frac{5}{9} \text{ of } 3 \text{ oz. 4 drs. 2 scr. 5 grs.} = \frac{2}{3} \text{ of } \frac{9}{2} \text{ of } \frac{3}{6} \text{ of } \frac{1}{3} \text{ of } \frac{5}{9} \text{ of } 1725 \text{ grs.} = 10350 \text{ grs.}$$

$$\frac{6}{11} \text{ of } \frac{6}{3} \text{ of } \frac{2}{4} \text{ of } \frac{3}{13} \text{ of } 6\frac{1}{2} \text{ times } 7 \text{ lbs. 3 oz.} = \frac{6}{11} \text{ of } \frac{7}{11} \text{ of } \frac{1}{12} \text{ of } \frac{3}{13} \text{ of } \frac{1}{2} \text{ of } 41760 \text{ grs.} = 62640 \text{ grs.}$$

$$10350 \div 62640 = .165229.$$

(16)

Dissimilar.                      Similar.                      Similar and Coterminous.

$$623.42793 = 623.42793793 = 623.42793793793$$

$$93.4267192 = 93.4267192 = 93.42671929292$$

$$\text{Difference} = 530.00121864500$$

(17)

$$\$1.00 - \$0.046 = \$0.954, \text{ and } \$7493 \div 0.954 = \$7854.29.$$

(18)

|                   |   |            |  |
|-------------------|---|------------|--|
| 36 : 20 weeks     | } | :: 18 men: |  |
| 6 : 5 days        |   |            |  |
| 9 : 11 hours      |   |            |  |
| 11 : 24 cellars   |   |            |  |
| 20 : 22 feet long |   |            |  |
| 16 : 22 feet wide |   |            |  |
| 5 : 4 feet deep   |   |            |  |

$$= \frac{11 \times 22}{9} = 26\frac{2}{9}.$$

(19)

$$\frac{1}{2} \text{ of } \frac{3}{8} \text{ of } \frac{4}{7} = \frac{6}{35}; \text{ and if } \frac{6}{35} \text{ of a certain number} = \frac{72}{35}, \frac{1}{35} = \frac{12}{35}$$

$$\text{and } \frac{35}{35} = \frac{12}{35} \times 35 = 12.$$

$$(\{[(\{[(12 \times 12\frac{1}{2}) + 31] \times 3\} - 33) \times 300] \div 17\} \times 9)$$

$$= 81000$$

(20)

$$\begin{array}{r|l} 1176 & 480..768..848..1176 \\ 32 & 20..32..29 \\ 145 & 5 \quad 29 \\ \hline 1176 \times 32 \times 145 & = 5456640. \end{array}$$



(21)

$$\begin{array}{r}
 838)171347(204 \\
 \underline{1676} \\
 3747 \\
 3352 \\
 \underline{\phantom{0000}} \\
 395)838(2 \\
 \underline{790} \\
 17598)46090(2 \\
 \underline{35196} \\
 10894)17598(1 \\
 \underline{10894} \\
 6704)10894(1 \\
 \underline{6704} \\
 4190)6704(1 \\
 \underline{4190} \\
 2514)4190(1 \\
 \underline{2514} \\
 1673)2514(1 \\
 \underline{1676} \\
 838)1676(2 \\
 \underline{1676} \\
 48)395(8 \\
 \underline{384} \\
 11)48(4 \\
 \underline{44} \\
 4)11(2 \\
 \underline{8} \\
 3)4(1 \\
 \underline{3} \\
 1)3 \\
 \underline{1} \\
 3
 \end{array}$$

As no number greater than unity will divide all of them without a remainder, they have no G. C. M.

(22)

$$\begin{array}{l}
 \$12000 \times 4 = \$48000 \\
 \$12000 + \$8000 = \$20000 \times 2 = \$40000 \\
 \hline
 \$88000 = \text{product of A's} \\
 \quad \text{stock and time.} \\
 \$25000 \times 3 = \$75000 \\
 \$25000 - \$10000 = \$15000 \times 3 = \$45000 \\
 \hline
 \$120000 = \text{product of B's} \\
 \quad \text{stock and time.} \\
 \$35000 \times 2 = \$70000
 \end{array}$$

(Continued on next page.)

(22 continued.)

$$\frac{2}{3} \text{ of } \$35000 = \$10000. \quad \$35000 - \$10000 = \$25000 \times 4 = \overline{\$100000}$$

= product of C's stock and time.

$$\$88000 + \$120000 + \$170000 = \$378000 = \text{sum of the products of stocks and times.}$$

$$\text{Then } \$378000 : \$88000 :: \$15000 : \frac{15000 \times 88000}{378000} = \$3492.06$$

= A's share.

$$\$378000 : \$170000 :: \$15000 : \frac{15000 \times 170000}{378000} = \$6746.03$$

= C's share.

$$\$15000 - (\$3492.06 + \$6746.03) = \$4761.91 = \text{B's share.}$$

(23)

A's gain in 5 months = \$125 ∴ his gain for 9 months

$$= 1\frac{1}{5} \times \$125 \dots\dots\dots = \$225$$

B's gain in 6 months = \$125 ∴ his gain for 9 months

$$= 1\frac{1}{2} \times \$125 \dots\dots\dots = \$187\frac{1}{2}$$

C's gain in 9 months ∴ = \$125

$$\text{Sum} = \overline{\$537\frac{1}{2}}$$

$$\text{Then } \$537\frac{1}{2} : \$225 :: \$400 : \frac{400 \times 225}{537\frac{1}{2}} = \$167\frac{1}{4} = \text{A's stock.}$$

$$\$537\frac{1}{2} : \$187\frac{1}{2} :: \$400 : \frac{400 \times 187\frac{1}{2}}{537\frac{1}{2}} = \$139\frac{3}{4} = \text{B's stock.}$$

$$\$537\frac{1}{2} : \$125 :: \$400 : \frac{400 \times 125}{537\frac{1}{2}} = \$93\frac{1}{4} = \text{C's stock.}$$

(24)

 $\frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \frac{1}{12} = \frac{5}{24} = \frac{10}{48} = \text{part of the cistern filled in one hour when the four pipes are left open.}$ 
 $\frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \frac{1}{12} = \frac{10}{48} = \frac{38}{48} = \text{part of the cistern emptied in one hour when the four are left open.}$ 
 $\frac{38}{48} - \frac{10}{48} = \frac{28}{48} = \text{part of the cistern which remains filled after the eight pipes have been left open for one hour. And if } \frac{10}{48} \text{ of the cistern are emptied in one hour, it will take } 1 \div \frac{10}{48} = 2\frac{2}{5} \text{ hours to empty the whole of it.}$

## THIRD SERIES.

(26)

As often as the first receives 4 the second receives 3, therefore as often as the first receives 6 the second receives  $4\frac{1}{2}$ . Then  $6 + 4\frac{1}{2} + 7 = 17\frac{1}{2}$ .

loaves.

$$2310 \times 6$$

$$17\frac{1}{2}:6 :: 2310: \frac{2310 \times 6}{17\frac{1}{2}} = 792 \text{ loaves} = \text{number the first receives.}$$

$$17\frac{1}{2}$$

$$2310 \times 4\frac{1}{2}$$

$$17\frac{1}{2}:4\frac{1}{2} :: 2310: \frac{2310 \times 4\frac{1}{2}}{17\frac{1}{2}} = 594 \text{ " " second "}$$

$$17\frac{1}{2}$$

$$2310 \times 7$$

$$17\frac{1}{2}:7 :: 2310: \frac{2310 \times 7}{17\frac{1}{2}} = 924 \text{ " " third "}$$

$$17\frac{1}{2}$$

(27)

To produce a mixture worth 8 cents a pound, we require 4 lbs. @ 12 cents, 4 @ 4 cents, 1 @ 5 cents, and 3 @ 9 cents, or 3 lbs @ 12 cents, 1 @ 4 cents, 4 @ 5 cents, and 4 @ 9 cents, lbs.lbs.lbs. lbs.lbs.lbs.

$$\text{Then } 4:72::4:72 \text{ lbs. @ 4 cts. or } 3:72::1:24 \text{ lbs. @ 4 cts.}$$

$$4:72::1:18 \text{ lbs. @ 5 cts. } 3:72::4:96 \text{ lbs. @ 5 cts.}$$

$$4:72::3:54 \text{ lbs. @ 9 cts. } 3:72::4:96 \text{ lbs. @ 9 cts.}$$

(28)

$$\text{Here } A = \$4444.44, r = .0444, \text{ and } t = 4.3\frac{1}{3}$$

$$A \quad \$4444.44 \quad \$4444.44$$

$$\text{Then } P = \frac{A}{1+rt} = \frac{\$4444.44}{1+(.0444 \times 4.3\frac{1}{3})} = \frac{\$4444.44}{1.19289\frac{1}{3}} = \$3725.764.$$

(29)

$$\$1.00 - \$0.0225 = \$0.9775. \quad \$23470 \div 0.9775 = \$24010.23.$$

(30)

$$\text{Here } A = \$7493.47, r = .07, \text{ and } t = 8.$$

$$A \quad 7493.47 \quad 7493.47$$

$$\text{Then } P = \frac{A}{1+rt} = \frac{7493.47}{1+(.07 \times 8)} = \frac{7493.47}{1.56} = \$4803.5064.$$

(31)

 $\$17460 \div 1.03125 = \$16930.909 = \text{sum to be invested.}$ 
 $16930.909 \div 2.95 = 5739.29 \text{ yds. cloth.}$ 
 $16930.909 \times .02\frac{1}{2} = \$423.27272 = \text{ad valorem duty.}$ 
 $\$17460 + \$1347.90 + \$479.40 + \$169.83 + \$423.27272 =$   
 $\$19880.40272 = \text{whole cost.}$ 
 $\$25000 - \$19880.40272 = \$5119.59728 = \text{whole gain.}$ 
 $\text{Then } \$19880.40272 : \$100 :: \$5119.59728 : \frac{5119.59728 \times 100}{19880.40272} =$   
 $25.75 = 25\frac{3}{4} \text{ per cent.}$ 

(32)

| V.        | III.       | VIII.     | XII.      |
|-----------|------------|-----------|-----------|
| 134234    | = 21122021 | = 12701   | = 3281    |
| 5         | 3          | 8         | 12        |
| <hr/>     | <hr/>      | <hr/>     | <hr/>     |
| 8         | 7          | 10        | 38        |
| 5         | 3          | 8         | 12        |
| <hr/>     | <hr/>      | <hr/>     | <hr/>     |
| 44        | 22         | 87        | 464       |
| 5         | 3          | 8         | 12        |
| <hr/>     | <hr/>      | <hr/>     | <hr/>     |
| 222       | 68         | 696       | 5569 den. |
| 5         | 3          | 8         |           |
| <hr/>     | <hr/>      | <hr/>     |           |
| 1113      | 206        | 5569 den. |           |
| 5         | 3          |           |           |
| <hr/>     | <hr/>      |           |           |
| 5569 den. | 618        |           |           |
|           | 3          |           |           |
|           | <hr/>      |           |           |
|           | 1956       |           |           |
|           | 3          |           |           |
|           | <hr/>      |           |           |
|           | 5569 den.  |           |           |

(33)

 $\frac{2}{7} \text{ of } 4\frac{1}{2} \text{ of } \frac{9\frac{3}{4}}{\frac{1\frac{3}{4}}{20}} \text{ of } \frac{1}{15} \text{ of } \frac{1}{9} \text{ of } £43 \text{ 18s. } 11\frac{1}{2}\text{d.} =$   
 $\$175.79\frac{1}{6}.$ 

(Continued on next page.)

(33 continued.)

$$\frac{2}{7} \text{ of } \frac{9}{2} \text{ of } \frac{32}{13} \text{ of } \frac{1}{16} \text{ of } \frac{7}{9} \text{ of } \$175.79\frac{1}{6}, = \frac{3}{7} \text{ of } \frac{9}{2} \text{ of } \frac{15}{1} \text{ of } \frac{1}{15} \text{ of } \frac{7}{9} \text{ of } \$175.79\frac{1}{6} = \frac{2}{3} \text{ of } \$175.79\frac{1}{6} = \$263.6875.$$

$$3\frac{2}{9} \text{ of } \frac{1}{17\frac{1}{2}} \text{ of } .56 \text{ of } 1.75 \text{ of } 6\frac{1}{2} \text{ times } \$97.18 =$$

$$3\frac{2}{9} \text{ of } \frac{1}{3\frac{1}{2}} \text{ of } \frac{56}{100} \text{ of } \frac{175}{100} \text{ of } 6\frac{1}{2} \text{ times } \$97.18; 6\frac{1}{2} \text{ times } \$97.18 = \$631.67.$$

$$\frac{35}{9} \text{ of } \frac{2}{35} \text{ of } \frac{56}{100} \text{ of } \frac{175}{100} \text{ of } \$631.67 = \frac{49}{9 \times 25} \text{ of } \$631.67 = \frac{25}{25} \text{ of } \$631.67 = \frac{49}{25} \text{ of } \$631.67$$

$$\frac{49}{25} \text{ of } \$631.67 = \$137.5636.$$

$$\text{Then } \$263.6875 - \$137.5636 = \$126.1239 = \text{difference.}$$

(34)

$$\frac{1}{13} = 1 \div 13 \therefore \log. \frac{1}{13} = \log. 1 - \log. 13 = 0 - 1.113943 = -1.113943.$$

$$19.5 = 3 \times 13 \times 5 \div 10 \therefore \log. 19.5 = \log. 3 + \log. 13 + \log. 5 - \log. 10.$$

$$\log. 3 = 0.477121$$

$$\log. 13 = 1.113943$$

$$\log. 5 = \log. 10 - \log. 2 = 1 - 0.301030 \therefore \log. 5 = 0.698970$$

$$\text{Sum} = 2.290034$$

$$\text{From which take } \log. 10 = 1$$

$$\text{Rem.} = 1.290034$$

$$= \log. 19.5.$$

$$1125 = 5^3 \times 3^2 \therefore \log. 1125 = (\log. 5) \times 3 + (\log. 3) \times 2.$$

$$\log. 5 = 0.698970 \times 3 = 2.096910$$

$$\log. 3 = 0.477121 \times 2 = 0.954242$$

$$\text{Sum} = 3.051152 = \log. \text{ of } 1125,$$

(Continued on next page.)

(34 continued.)

$$28\cdot1\dot{6} = 28\frac{1}{6} = \frac{13^2}{6} = 13^2 \div 6 \therefore \log. 28\cdot1\dot{6} = (\log. 13) \times 2 - (\log. 2 + \log. 3.)$$

$$\log. 13 = 1\cdot113943 \times 2 = 2\cdot227886$$

$$(\log. 2 + \log. 3) = (0\cdot301030 + 0\cdot477121) = 0\cdot778151$$

$$\text{Diff.} = \underline{\underline{1\cdot449735}}$$

$$= \log. 28\cdot1\dot{6}.$$

$$65000 = 13 \times 5 \times 1000 \therefore \log. 65000 = \log. 13 + \log. 5 + \log. 1000.$$

$$\log. 13 = 1\cdot113943$$

$$\log. 5 = 0\cdot698970$$

$$\log. 1000 = 3$$

$$\text{Sum} = \underline{\underline{4\cdot812913}} = \log. \text{ of } 65000.$$

$$\log. \cdot0005 = \log. 5 \text{ with characteristic changed to } -4 = \underline{\underline{4\cdot698970}}.$$

$$152\cdot1 = 3^2 \times 13^2 \div 10 \therefore \log. 152\cdot1 = (\log. 3) \times 2 + (\log. 13) \times 2 - \log. 10.$$

$$\log. 3 = 0\cdot477121 \times 2 = 0\cdot954242$$

$$\log. 13 = 1\cdot113943 \times 2 = 2\cdot227886$$

$$\text{Sum} = \underline{\underline{3\cdot182128}}$$

$$\text{From which take } \log. 10 = 1$$

$$\text{Diff.} = \underline{\underline{2\cdot182128}} = \log. 152\cdot1$$

$$8\cdot112 = 2^4 \times 13^2 \times 3 \div 1000 \therefore \log. 8\cdot112 = (\log. 2) \times 4 + (\log. 13) \times 2 + \log. 3 - \log. 1000.$$

$$\log. 2 = 0\cdot301030 \times 4 = 1\cdot204120$$

$$\log. 13 = 1\cdot113943 \times 2 = 2\cdot227886$$

$$\log. 3 = \underline{\underline{0\cdot477121}}$$

$$\text{Sum} = \underline{\underline{3\cdot909127}}$$

$$\text{From which take } \log. 1000 = 3$$

$$\text{Diff.} = \underline{\underline{0\cdot909127}} = \log. 8\cdot112.$$

(35)

|                                    |           |                                |
|------------------------------------|-----------|--------------------------------|
|                                    |           | XII.                           |
| $t^2 \times 300 = 21000$           |           | $871tet \cdot 72 (t8 \cdot t2$ |
| $t \times 8 \times 30 = 1800$      |           | $6e4$                          |
| $8^2 = 54$                         |           | <hr/>                          |
|                                    | 22854     | $179tet$                       |
|                                    |           | <hr/>                          |
|                                    |           | $159768$                       |
|                                    |           | <hr/>                          |
|                                    |           | $20352720$                     |
| $t 8^2 \times 300 = 2454000$       |           |                                |
| $t 8 \times t \times 30 = 22800$   |           |                                |
| $t^2 = 84$                         |           |                                |
|                                    | <hr/>     |                                |
|                                    | 2476884   | $1et372e4$                     |
|                                    |           | <hr/>                          |
| $t 8 t^2 \times 300 = 249961000$   |           | $517428000$                    |
| $t 8 t \times 2 \times 30 = 54500$ |           |                                |
| $2^2 = 4$                          |           |                                |
|                                    | <hr/>     |                                |
|                                    | 2499e5504 | $4977ttt08$                    |
|                                    |           | <hr/>                          |
|                                    |           | $3e8391e4$                     |

(36)

$\frac{1}{6} + \frac{1}{2} + \frac{1}{7} + 5$  years =  $\frac{11}{8}$  of life time + 5 years = age at birth of son.  
 $\frac{28}{8} - (\frac{11}{8} + 5) = \frac{17}{8}$  of his life time — 5 years = time he lived after birth of son.

$\frac{17}{8}$  of father's life time — 5 years — 4 years = age of son =  $\frac{1}{2}$  father's age.

$\frac{17}{8}$  of father's life time — 9 years =  $\frac{1}{2}$  father's age.

$\therefore$  9 years is the difference between  $\frac{17}{8}$  and  $\frac{11}{8}$  of father's age.

$\therefore$  9 years is equal to  $\frac{3}{8}$  of father's age.

If 9 years is  $\frac{3}{8}$  of his age,  $\frac{1}{8}$  will be the  $\frac{1}{3}$  of 9 which is 3 years.

If  $\frac{1}{8}$  is 3 years,  $\frac{28}{8}$  or the whole age will be  $3 \times 28 = 84$  years.

*Or by Position.*

Assume 42 for father's age at death, the son's age = 21.

$\frac{1}{6} + \frac{1}{2} + \frac{1}{7} + 5 = \frac{11}{8} + 5$ ;  $\frac{11}{8}$  of 42 =  $16\frac{1}{2}$  and  $16\frac{1}{2} + 5 = 21\frac{1}{2}$  = age of father when son was born.

$\therefore$  he lived after birth of his son  $42 - 21\frac{1}{2} = 20\frac{1}{2}$  years,

(Continued on next page.)

(36 continued.)

By the question he lived  $21 + 4 = 25$  years.The error  $25 - 20\frac{1}{2} = -4\frac{1}{2}$ .Assume 98 for father's age, then son's age  $= \frac{1}{2}$  of 98 = 49. $\frac{1}{6} + 1\frac{1}{2} + \frac{1}{7} + 5 = \frac{11}{8} + 5$ ;  $\frac{11}{8}$  of 98 =  $38\frac{1}{2}$ , and  $38\frac{1}{2} + 5 = 43\frac{1}{2}$   
= age of father at birth of son. $\therefore$  he lived after birth of his son  $98 - 43\frac{1}{2} = 54\frac{1}{2}$  years.But by the question he lived  $49 + 4$  years = 53 years.Then  $53 - 54\frac{1}{2} = -1\frac{1}{2}$  = error.

Errors.

$$-4\frac{1}{2} \times 98 = 441$$

$$+1\frac{1}{2} \times 42 = 63$$

$$\begin{array}{r} \text{Sum} = 6 \qquad \qquad 504 \end{array}$$

$$504 \div 6 = 84 = \text{father's age.}$$

(37)

| m.                                    | fur. | per. | yds. | ft. | in. |        | fur.                                | per. | yds.           |
|---------------------------------------|------|------|------|-----|-----|--------|-------------------------------------|------|----------------|
| 63                                    | 3    | 7    | 3    | 2   | 7   | $\div$ | 7                                   | 23   | $3\frac{1}{2}$ |
| 8                                     |      |      |      |     |     |        | 40                                  |      |                |
| <u>507</u>                            |      |      |      |     |     |        | <u>303</u>                          |      |                |
| 40                                    |      |      |      |     |     |        | 5 $\frac{1}{2}$                     |      |                |
| <u>20287</u>                          |      |      |      |     |     |        | <u>1518<math>\frac{1}{2}</math></u> |      |                |
| 5 $\frac{1}{2}$                       |      |      |      |     |     |        | 151 $\frac{1}{2}$                   |      |                |
| <u>101438</u>                         |      |      |      |     |     |        | <u>1670<math>\frac{1}{2}</math></u> |      |                |
| 10143 $\frac{1}{2}$                   |      |      |      |     |     |        | 3                                   |      |                |
| <u>111581<math>\frac{1}{2}</math></u> |      |      |      |     |     |        | <u>5010<math>\frac{1}{2}</math></u> |      |                |
| 3                                     |      |      |      |     |     |        | 12                                  |      |                |
| <u>334746<math>\frac{1}{2}</math></u> |      |      |      |     |     |        | <u>60129</u>                        |      |                |
| 12                                    |      |      |      |     |     |        |                                     |      |                |
| <u>4016965</u>                        |      |      |      |     |     |        |                                     |      |                |

(Continued on next page.)



(37 continued.)

60129)4016965(66·80578 times

360774

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409225

360774

---

484510

481032

---

347800

300645

---

471550

420903

---

506470

481032

(38)

 $6 \cdot 3 \div \cdot 000000274$ 

274)6300000000(22992700·72992700

548

---

820

548

---

2720

2466

---

2540

2466

---

740

548

---

1920

1918

---

2000

1918

---

820

548

2720

2466

---

2540

2466

---

740

548

---

1920

1918

200 remainder.

(39)

$$\frac{7}{8} \text{ yds.} : 6\frac{3}{4} \text{ yds.} :: \$1\frac{3}{4} : \frac{7}{8} \times \frac{11}{1} \times 1\frac{3}{4} = \frac{7 \times 11 \times 5}{8} = \$5.482.$$

(40)

$$I = Prt. = \$4237.71 \times .065 \times 1.67 = \$460.0034205.$$

(41)

$$t = \frac{A - P}{Pr} = \frac{\$1000 - \$674.30}{\$674.30 \times .085} = \frac{325.70}{57.3155} = 5.68258 \text{ years} =$$

5 years 8 months 5.7288 days.

(42)

By Table, page 260, the amount of \$1 for 14 payments at 4 per cent. is \$1.73168.

$$\text{Then } \$1.73168 \times 813.71 = \$1409.0853328 = \text{Amount.}$$

$$\text{Subtract } 813.71$$

$$\text{Difference} = 595.3753328 = \text{Interest.}$$

(43)

|                            |   |    |   |   |
|----------------------------|---|----|---|---|
| \$300                      | × | 0  | = | 0                                       |
| 700                        | × | 4  | = | 2800                                    |
| 750                        | × | 7  | = | 5250                                    |
| 850                        | × | 9  | = | 7650                                    |
| 400                        | × | 13 | = | 5200                                    |
| 1300                       | × | 19 | = | 24700                                   |
| <hr style="width: 100%;"/> |   |    |   |   |
| 4300                       | ) |    |   | 45600 (10 months 18 $\frac{2}{3}$ days. |
|                            |   |    |   | 4300                                    |
|                            |   |    |   | <hr style="width: 100%;"/>              |
|                            |   |    |   | 2600                                    |
|                            |   |    |   | 30                                      |
|                            |   |    |   | <hr style="width: 100%;"/>              |
|                            |   |    |   | 78000 = days                            |
|                            |   |    |   | 4300                                    |
|                            |   |    |   | <hr style="width: 100%;"/>              |
|                            |   |    |   | 35000                                   |
|                            |   |    |   | 34400                                   |
|                            |   |    |   | <hr style="width: 100%;"/>              |
|                            |   |    |   | 4398                                    |

(44)

23 per cent. of \$4200 =  $\frac{23}{100}$  of 4200 = \$966.00, and \$4200 — \$966.00 = \$3234.00. E has half as much as A, B, C, and D together; therefore E has *one-third* of \$3234.00, which is \$1078.00.

Deducting E's share, \$1078, from \$3234, the whole sum to be divided, there remains \$2156 to be divided among A, B, C and D. Now D gets a certain amount; C gets \$42.11 more than D; B gets \$61.34 ( $42.11 + 19.23$ ) more than D; and A gets \$78.44 ( $61.34 + 17.10$ ) more than D. Together they get, then, *four times* D's share, together with \$42.11 + \$61.34 + \$78.44, or, in other words, four times D's share, together with \$181.89.

That is, four times D's share, together with \$181.89 is equal to \$2156.

Hence \$2156.00 — \$181.89 = \$1974.11 = four times D's share. Then \$1974.11  $\div 4$  = \$493.5275 = D's share.

Add 42.11

Sum \$535.6375 = C's share.

Add 19.23

Sum \$554.8675 = B's share.

Add 17.10

Sum \$571.9675 = A's share.

(45)

$$P = \frac{A}{1+rt} = \frac{\$3786.80}{1+1.76} = \frac{3786.80}{2.76} = \frac{378680}{276} = \$1372.02898 +$$

(46)

$$\left\{ (3\frac{3}{7} - 2\frac{7}{10}) \times .46 \div \frac{2}{3} \text{ of } .142857 \right\} \div 8\frac{1}{2} \text{ times } (\frac{1}{2} + \frac{1}{7} + \frac{1}{10} - \frac{337}{2310})$$

$$\left\{ (.73 \times .12345 \div \frac{679}{130}) + \frac{2}{7} + 9\frac{3}{8} + 17\frac{4}{11} \right\} \div 27.4922077$$

$$\left\{ (3\frac{3}{7} - 2\frac{7}{10}) \times \frac{46}{90} \div \frac{2}{3} \text{ of } \frac{1}{7} \right\} \div \frac{17}{2} \times (\frac{35}{10} + \frac{10}{7} + \frac{14}{10} - \frac{337}{2310})$$

$$\left\{ (\frac{66}{90} \times \frac{13333}{90000} \div \frac{679}{130}) + \frac{2}{7} + 9\frac{3}{8} + 17\frac{4}{11} \right\} \div 27.4922077$$

(Continued on next page.)

$$*rt = .16 \times 11 = 1.76.$$

(46 continued.)

$$\begin{aligned}
 &= \frac{(\frac{51}{70} \times \frac{46}{99} \times \frac{5}{2} \times \frac{7}{1}) \times \frac{2}{17} \times \frac{2310}{1610}}{\left\{ (\frac{11}{15} \times \frac{679}{3500}) \times \frac{750}{679} + 27\frac{51}{385} \right\} \div 27.4922077} \\
 &= \frac{\frac{391}{66} \times \frac{2}{17} \times \frac{2310}{1610}}{1} = \frac{1}{(10 + 27\frac{51}{385}) \div 27.4922077} = \frac{27\frac{79}{70} \div 27.4922077}{1} \\
 &= \frac{1}{27.4922077 \div 27.4922077} = \frac{1}{1} = 1
 \end{aligned}$$

(47)

312312302 quaternary = 224690 decimal scale.

2312132 quaternary = 11678 decimal scale.

Sum = 236368

4234 quinary = 569 decimal, and  $569 \times 23011 = 13093259$ . $236368 \times 13093259 = 3094827443312$ . $555 + 444 + 333 + 222 + 111$  senary = 2553 senary = 645 decimal. $3094827443312 - 645 = 3094827442667$ .

6542 septenary = 2333 decimal.

 $3094827442667 \div 2333 = 1326544124\frac{375}{333}$  den.

X.

VIII.

 $1326544124 = 11704272374$ 

X.

VIII.

 $1375 = 2537$ 

X.

VIII.

 $2333 = 4435$ 

X.

VIII.

 $\therefore 1326544124\frac{375}{333} = 11704272374\frac{2537}{4435}$ 

(48)

 $\cdot 1 = \frac{1}{10}$  and  $(\frac{1}{10})^2 = \frac{1}{100} = 01$  $\cdot 1 = \frac{1}{9}$  and  $(\frac{1}{9})^2 = \frac{1}{81} = \cdot 012345679$ .



(56)

$$P = \frac{A}{(1+r)^t}; \log. P = \log. A - \log. (1+r) \times t = \log. 8899.77 \\ - \log. (1.06) \times 22 = 3.949378 - 0.025306 \times 22 \\ = 3.949378 - 0.556732 = 3.392646, \text{ and } \log. 3.392646 \\ = \$2469.71.$$

By Table, page 260, amount of \$1 at 6 per cent. for 22 payments = 3.60354.

Then  $\$8899.77 \div 3.60354 = \$2469.73$  nearly.

(57)

Let the 1st number be 2. Then  $2 \times 2 = 4$

$$1\frac{1}{3} \times 3 = 4$$

$$10 - (2 + 1\frac{1}{3}) = 10 - 3\frac{1}{3} = 6\frac{2}{3} \times 4 = 26\frac{2}{3}, \text{ but it should equal 4.}$$

$$\text{Therefore } 26\frac{2}{3} - 4 = + 22\frac{2}{3} = \text{error.}$$

Let  $1\frac{1}{3}$  be the 1st number; then  $1\frac{1}{3} \times 2 = 3$

$$1 \times 3 = 3$$

$$10 - (1\frac{1}{3} + 1) = 10 - 2\frac{1}{3} = 7\frac{2}{3} \times 4 = 30, \text{ but it should } = 3.$$

$$\text{Therefore } 30 - 3 = + 27 = \text{error.}$$

Errors.

$$+ 27 \times 2 = 54$$

$$+ 22\frac{2}{3} \times 1\frac{1}{3} = 34$$

$$\text{Diff.} = 4\frac{1}{3} \text{ diff.} = 20, \text{ and } 20 \div 4\frac{1}{3} = 4\frac{8}{13} = \text{1st number.}$$

$$4\frac{8}{13} \times 2 = 9\frac{3}{13} = \text{1st product.}$$

$$\text{Second number} = 9\frac{3}{13} \div 3 = 3\frac{1}{13} \times 3 = 9\frac{3}{13} = \text{2nd product.}$$

$$10 - 7\frac{9}{13} = 2\frac{4}{13} \times 4 = 9\frac{3}{13} = \text{3rd product.}$$

(58)

Suppose A has 40; then B has  $110 - 40 = 70$ , and C has  $130 - 70 = 60$ .

A and C together have  $40 + 60 = 100$ , but it should be 120.

$$\text{Therefore } 100 - 120 = - 20 = \text{error.}$$

Suppose A has 80; then B has  $110 - 80 = 30$ , and C has  $130 - 30 = 100$ .

A and C together have  $80 + 100 = 180$ , but they should have 120.

$$\text{Therefore } 180 - 120 = + 60 = \text{error.}$$

(Continued on next page.)

(58 continued.)

Errors.

$$\begin{array}{rclcl} + 60 & \times & 40 & = & 2400 \\ - 20 & \times & 80 & = & 1600 \\ \hline \end{array}$$

Sum = 80

Sum = 4000

$$4000 \div 80 = 50 = \text{number A has.}$$

$$\text{Then B has } 110 - 50 = 60, \text{ and C has } 130 - 60 = 70.$$

$$50 + 60 + 70$$

$$\frac{\quad}{3} = 60 = \text{each man's share when equally divided.}$$

(59)

$$\begin{aligned} \text{Formula I, p. 333. } l &= a + (n - 1) d = 7 + (47 - 1) \times 4 \\ &= 7 + (46 \times 4) = 7 + 184 = 191. \end{aligned}$$

$$\begin{aligned} \text{Formula VI, p. 333. } s &= \left\{ 2a + (n - 1) d \right\} \frac{n}{2} \\ &= \left\{ 2 \times 7 + (93 - 1) \times 4 \right\} \frac{93}{2} = \left\{ 14 + (92 \times 4) \right\} \frac{93}{2} \\ &= (14 + 368) \times \frac{93}{2} = \frac{382 \times 93}{2} = 17763. \end{aligned}$$

(60)

$$t = \frac{\log. n}{\log. (1 + r)} = \frac{\log. 21}{\log. (1.07)} = \frac{1.322219}{0.029384} = 44.997 \text{ years.}$$

## SIXTH SERIES.

(61)

B gets \$196.87 more than C, and A gets \$387 + \$196.87 = \$583.87 more than C, therefore together they get *three* times C's share, together with \$196.87 + \$583.87, i. e. three times C's share, together with \$780.74; but together they get \$3700.

Therefore \$3700 = three times C's share, together with \$780.74, or \$3700 - \$780.74 = \$2919.26 = three times C's share.

Hence \$2919.26  $\div$  3 = \$973.08 $\frac{2}{3}$  = C's share.

$$\begin{array}{r} \text{Add} \quad 196.87 \\ \hline \end{array}$$

Sum = \$1169.95 $\frac{2}{3}$  = B's share.

$$\begin{array}{r} \text{Add} \quad 387.00 \\ \hline \end{array}$$

Sum = \$1556.95 $\frac{2}{3}$  = A's share.

(62)

$$5716 = 2^2 \times 1429$$

$$1 \quad \dots \quad 2 \quad \dots \quad 4$$

$$1 \quad \dots \quad 1429$$

$$1 \quad \dots \quad 2 \quad \dots \quad 4 \quad \dots \quad 1429 \quad \dots \quad 2858 \quad \dots \quad 5716$$

(63)

$$\{ (17\frac{7}{12} - 10\frac{5}{60}) - (.4 + \frac{1}{6} + .9 - \frac{1}{2}) \} \div (.8378 \div \frac{1}{2} \text{ of } 31)$$

$$.6322632 \times \frac{1}{2} \text{ of } 9\frac{1}{4} \div (\frac{1}{5} \text{ of } 4\frac{1}{2} \text{ of } \frac{1}{11} \text{ of } 85\frac{1}{2} \div 101)$$

$$6\frac{3}{5} - 1 \div (\frac{8378}{9999} \times \frac{3}{31})$$

$$= \frac{6322}{9999} \times \frac{1}{2} \times \frac{37}{4} \div (\frac{1}{5} \times \frac{37}{9} \times \frac{1}{11} \times \frac{2161}{3161} \times \frac{101}{101})$$

$$5\frac{3}{5} \times \frac{2390}{3370} \times \frac{3}{2}$$

$$= \frac{6322}{9999} \times \frac{1}{2} \times \frac{37}{4} \times \frac{5}{1} \times \frac{9}{37} \times \frac{11}{11} \times \frac{37}{3161} \times \frac{101}{1}$$

$$\frac{28}{5} \times \frac{37}{31} \times \frac{3}{2}$$

$$= \frac{2161 \times 37 \times 5 \times \frac{1}{37} \times \frac{37}{3161} \times \frac{101}{1}}{14 \times 37}$$

$$14 \times 37$$

$$14 \times 37$$

$$5$$

$$5$$

$$= \frac{5}{\frac{1}{4} \times \frac{5}{1} \times \frac{37}{1}} = \frac{5}{5 \times 37} = \frac{56}{25} = 2\frac{6}{25}$$

$$4$$

(64)

Each child gets 1 child's share,  $\therefore$  17 children get 17 shares.

Each woman gets *three* times a child's share,  $\therefore$  4 women get 12 shares.

Each man gets *six* times a child's share,  $\therefore$  3 men get 18 shares.

And together they get 47 times a child's share.

Therefore  $\$7200 \div 47 = \$153.19\frac{7}{17}$  = a child's share.

$$\$153.19\frac{7}{17} \times 3 = \$459.57\frac{21}{17}$$

$$\$153.19\frac{7}{17} \times 6 = \$919.14\frac{42}{17}$$

(65)

$25400 = 2^3 \times 5^2 \times 127$ . Adding unity to each index and multiplying the results, we get  $4 \times 3 \times 2 = 24$ .



(66)

$$\frac{2}{3} \text{ of } 4\frac{1}{2} \text{ of } \frac{9\frac{3}{4}}{1\frac{1}{4}} \text{ of } \frac{1}{6} \text{ of } £3 \text{ } 16\text{s. } 11\frac{1}{2}\text{d.} = \frac{2}{3} \times \frac{9}{4} \times \frac{66 \times 14}{7 \times 11} \times \frac{1}{6}$$

$$\times \$15 \cdot 39\frac{1}{6} = 6 \text{ times } \$15 \cdot 39\frac{1}{6} = \$92 \cdot 35.$$

$$\frac{3}{11} \text{ of } 4\frac{3}{5} \text{ of } \frac{19\frac{1}{2}}{3\frac{1}{4}} \text{ of } \frac{25}{117} \text{ of } \frac{1}{2\frac{1}{3}} \text{ of } \cdot 85 \text{ of } \frac{1}{42\frac{1}{2}} \text{ of } \$1783$$

$$\frac{13}{13}$$

$$= \frac{3}{11} \times \frac{23}{6} \times \frac{39}{19} \times \frac{25}{117} \times \frac{1}{2\frac{1}{3}} \times \frac{85}{100} \times \frac{2}{3\frac{1}{2}} \text{ of } \$1783.$$

$$= \frac{3}{11} \times \frac{23}{6} \times \frac{78}{19} \times \frac{55}{117} \times \frac{11}{23} \times \frac{85}{100} \times \frac{2}{85} \times \frac{1783}{1}$$

$$= \$17 \cdot 83 \times 4 = \$71 \cdot 32. \quad \$92 \cdot 35 - \$71 \cdot 32 = \$21 \cdot 03.$$

(67)

$$\left. \begin{array}{l} 7 : 13 = 7 \div 13 = \cdot 538 \\ 9 : 16 = 9 \div 16 = \cdot 562 \\ 8 : 15 = 8 \div 15 = \cdot 533 \\ 10 : 19 = 10 \div 19 = \cdot 526 \end{array} \right\} \begin{array}{l} \text{Therefore } 9 : 16 \text{ is the} \\ \text{greatest, and } 10 : 19 \text{ is} \\ \text{the least.} \end{array}$$

$$\text{Compound ratio} = \frac{7}{13} \times \frac{9}{16} \times \frac{8}{15} \times \frac{10}{19} = \frac{21}{247} = 21 : 247.$$

(68)

$$67 \cdot 432 = 67 \frac{432}{1000} = \frac{66758}{1000} \text{ and } 7 \cdot 9036 = 7 \frac{9036}{1000} = \frac{78957}{1000}$$

$$\frac{66758}{990} \div \frac{78957}{9990} = \frac{66758}{990} \times \frac{111}{78957} = \frac{7410138}{868527} = 8.5318452,$$

(69)

9 per. 9 yds. 7 ft. 120 in. = 365628 inches

 $\frac{1}{2}$  of  $\frac{2}{3}$  of  $\frac{2}{7}$  of 35 acres 2 roods =  $\frac{3}{35}$  of 35 acres 2 roods =  $\frac{3}{35}$  of  
 222678720 inches

$$\frac{365628}{\frac{3}{35} \text{ of } 222678720} = \frac{2559396}{133607232} = 0.019156118,$$

(70)

Dissimilar.

Similar.

17.0342

17.03424242

27.06357

27.06357575

98.123456

98.123456456

829.6423

829.642342342

986.1234298

986.1234298429

9.876342

9.876342876342

813.9864234567 813.9864234567

Similar and Coterminous.

17.0342424242424242

27.0635757575757575

98.123456456456456

829.642342342342342

986.1234298429842984

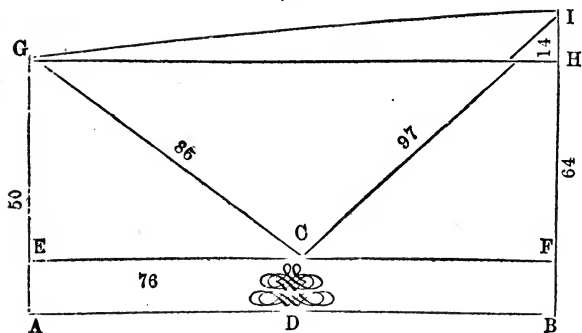
9.876342876342876342

813.986423456745674567

4 carried

2781.849813156689829957

(71)



$$EG = \sqrt{86^2 - 76^2} = \sqrt{1620} = 40.249 \text{ feet}$$

Height of Statue  $CD=AG-EG=50-40.249=9.751$  ft.  $=BF$

$$FI = BI - BF = 64 - 9.751 = 54.249 \text{ feet}$$

$$CF = \sqrt{CI^2 - FI^2} = \sqrt{97^2 - 54.249^2} = \sqrt{6466.045999} = 80.411 \text{ feet}$$

$$GH = EF = EC + CF = 76 + 80.411 = 156.411 \text{ feet and}$$

$$HI = 64 - 50 = 14 \text{ feet}$$

$$GI = \sqrt{GH^2 + HI^2} = \sqrt{156.411^2 + 14^2} = \sqrt{24660.400921} = 157.036 \text{ feet.}$$

(72)

The mixture = spirits + water =  $\frac{1}{2}$  of mixture + 25 gal. +  $\frac{1}{3}$  of mixture - 5 gal. =  $\frac{1}{2} + \frac{1}{3} + 20$  gal. =  $\frac{5}{6} + 20$  gal. Then 20 gal. =  $\frac{1}{6}$  of the mixture, and therefore the mixture contained  $6 \times 20 = 120$  gal.

Then  $\frac{1}{2}$  of 120 = 60 + 25 = 85 gal. = spirits }  
 $\frac{1}{2}$  of 120 = 40 — 5 = 35 gal. = water }

## SEVENTH SERIES.

(73)

$$\begin{array}{r}
 \begin{array}{r}
 \dot{4} \dot{0} \dot{1} \dot{2} \dot{4} \dot{1} \cdot \dot{3} \dot{4} \dot{2} \dot{4} \quad (422 \cdot 32 \\
 31 \\
 \hline
 132 \quad ) \quad 412 \\
 \quad \quad 314 \\
 \hline
 1342 \quad ) \quad 4341 \\
 \quad \quad 3234 \\
 \hline
 13443 \quad ) \quad 110234 \\
 \quad \quad 101434 \\
 \hline
 140012) \quad 330024 \\
 \quad \quad 330024
 \end{array}
 \end{array}$$

(74)

Suppose father's age = 60, the son's age now =  $60 \div 5 = 12$ ,  
 and son's age four years ago =  $12 - 4 = 8$ . But the son's  
 age four years ago should, by the question, have been  $60 \div$   
 $7 = 8\frac{4}{7}$ .

Therefore  $8 - 8\frac{4}{7} = -\frac{4}{7} = \text{error}$ .

Suppose father's age = 35; then son's age now =  $35 \div 5 = 7$ ,  
 and age four years ago =  $7 - 4 = 3$ .

But son's age four years ago should, by question, have been  $35$   
 $\div 7 = 5$ .

Therefore  $3 - 5 = -2 = \text{error}$ .

Errors.

$$-2 \times 60 = 120$$

$$-\frac{4}{7} \times 35 = 20$$

$$\text{diff. } 1\frac{1}{2} \quad \text{diff. } = 100$$

$$100 \div \frac{1}{2} = 70 = \text{father's and son's age} = 70 \div 5 = 14.$$

(75)

$$72347 \div 0032 = \frac{72275}{99900} \div \frac{32}{9900} =$$

$$\frac{72275}{55500} \times \frac{11}{32} = \frac{795025}{3552} = 223.82460585$$

(76)

Logarithm of 97294764.372 is 7.988089

$$7.988089 \div 11 = 0.726139$$

$$\text{Log. } 0.726189 = 5.32341 = 11\text{th root of } 97294764.372.$$

(77)

Assume  $43\frac{1}{2}$  for the greater number

$$7\frac{1}{2} : 3\frac{1}{2} :: 43\frac{1}{2} : \frac{43\frac{1}{2} \times 3\frac{1}{2}}{7\frac{1}{2}} = 21 \text{ the less ;}$$

$$43\frac{1}{2} - 21 = 22\frac{1}{2} \text{ but it should } = 30$$

Therefore error =  $22\frac{1}{2} - 30 = -7\frac{1}{2}$ .

Assume  $72\frac{1}{2}$  for the greater number

$$7\frac{1}{4} : 3\frac{1}{2} :: 72\frac{1}{2} : \frac{72\frac{1}{2} \times 3\frac{1}{2}}{7\frac{1}{4}} = 35 = \text{the less}$$

$72\frac{1}{2} - 35 = 37\frac{1}{2}$ , but it should  $= 30$

Therefore error =  $37\frac{1}{2} - 30 = + 7\frac{1}{2}$ .

### Errors.

$$+ 7\frac{1}{2} \times 43\frac{1}{2} = 326\frac{1}{4}$$

$$- 7\frac{1}{2} \times 72\frac{1}{2} = 543\frac{3}{4}$$

$$\text{Sum} = 15 \qquad \text{Sum} = 870$$

$$870 \div 15 = 58 \text{ greater}$$

$$58 \times 34$$

$$7\frac{1}{4} : 3\frac{1}{2} :: 58 : \frac{58 \times 7\frac{1}{4}}{3\frac{1}{2}} = 28 \text{ less.}$$

(78)

|           |                            |
|-----------|----------------------------|
| Assume 35 | 35, 16, 18, 28, 62, 68, 40 |
| Assume 16 | 16, 18, 4, 62, 9, 8        |
| Assume 9  | 4, 31, 9                   |
|           | 31                         |

$$l. c. m. = 35 \times 16 \times 9 \times 31 = 156240.$$

(79)

Here  $a = 1$ ,  $d = 6$ ,  $n = 101$ ,

$$s = \left\{ 2a + (n-1)d \right\} \frac{n}{2} = \left\{ 2 \times 1 + (101-1) \times 6 \right\} \frac{101}{2}$$

$$= (2 + 600) \frac{101}{2} = \frac{602 \times 101}{2} = 30401.$$

(80)

$$\frac{19}{7} \times \frac{11}{56} \times \frac{35}{121} \times \frac{113}{29} \times \frac{8}{44} \times \frac{41}{3} = \frac{113 \times 4 \times 5}{7 \times 7 \times 11 \times 3} = \frac{2284}{1617} = 2284 : 1617.$$

(82)

$$\left( \left\{ (9\frac{1}{2} + 4\frac{1}{2} + 3\frac{1}{2} - 16\frac{3}{4}) \times .54 \right\} \div 14 \right) \times 35 \text{ times } .142857$$

$$\left\{ .97 \times .24378 \times (1\frac{1}{4} \times 4\frac{10}{17}) \right\} \times (4\frac{3}{7} - 2\frac{4}{7})$$

$$\left( \left\{ (16\frac{5}{20} - 16\frac{10}{20}) \times \frac{5}{9} \right\} \div \frac{1}{7} \right) \times 35 \times \frac{1}{4}$$


---


$$\frac{88 \times 24378 \times 4\frac{1}{4} \times 4\frac{10}{17} \times (4\frac{6}{7} - 2\frac{4}{7})}{12\frac{1}{2} \times 7\frac{5}{7} \times 7\frac{1}{7} \times 3\frac{5}{7} \times \frac{1}{4}} = \frac{1}{187} = \frac{187}{187}$$


---


$$\frac{4\frac{1}{2} \times 16\frac{5}{20} \times 4\frac{1}{4} \times 16\frac{10}{20} \times 3\frac{8}{7}}{187} = \frac{187}{187}$$

(83)

Suppose the *hour* hand moves over 4 minutes, then since the minute hand moves 12 times as fast, it will have travelled over 48 minutes. But in order to overtake the hour hand, the minute hand must traverse the entire circle, 60 minutes, plus the 4 minutes we have supposed the hour hand to have moved forward, *i. e.* 64 minutes. Then 48 should equal 64, for we should find the same number by each process;  $48 - 64 = -16$  error.

Suppose hour hand moves over 6 minutes, the minute hand moves over  $6 \times 12 = 72$  minutes. But minute hand moves over  $60 + 6 = 66$  minutes.

Then  $72 - 66 = +6$  error.

(Continued on next page.)

(83 continued.)

Errors.

$$- 16 \times 6 = 96$$

$$+ 6 \times 4 = 24$$

---

 Sum 22    Sum 120

$120 \div 22 = 5\frac{5}{11}$  min. = minutes passed over by the hour hand,  
 hence space passed over by the minute hand  $= 5\frac{5}{11} \times 12$   
 $= 65\frac{5}{11}$  min. = 1 hour  $5\frac{5}{11}$  min. = time.

(84)

$$\text{Log. } 5 = \text{log. } 10 - \text{log. } 2 = 1 - 0.301030 = 0.698970$$

$$3850000 = 5 \times 7 \times 11 \times 10000.$$

$$\therefore \text{Log. } 3850000 = \text{log. } 5 + \text{log. } 7 + \text{log. } 11 + \text{log. } 10000$$

$$= 0.698970 + 0.845098 + 1.041393 + 4 = 6.585461.$$

$$3181.81 = 31.81 \times 100 = 31\frac{81}{100} \times 100 = 31\frac{81}{100} \times 100.$$

$$\therefore \text{Log. } 3181.81 = \text{log. } 5 + \text{log. } 7 + \text{log. } 1000 - \text{log. } 11$$

$$= 0.698970 + 0.845098 + 3 - 1.041393 = 3.502675$$

$$.0000154 = 2 \times 7 \times 11 \div 10000000$$

$$\therefore \text{Log. } .0000154 = \text{log. } 2 + \text{log. } 7 + \text{log. } 11 - \text{log. } 10000000$$

$$= 0.301030 + 0.845098 + 1.041393 - 7 = 5.187521.$$

$$\text{Log. } \frac{1}{77} = \text{log. } 1 - (\text{log. } 7 + \text{log. } 11) = 0 - (0.845098$$

$$+ 1.041393) = 0 - 1.886491 = 2.113509.$$

$$1.571428 = 1\frac{4}{7} = \frac{11}{7}.$$

$$\text{Log. } 1.571428 = \text{log. } 11 - \text{log. } 7 = 1.041393 - 0.845098$$

$$= 0.196295$$

$$93.17 = 9317 \div 100 = 11^3 \times 7 \div 100.$$

$$\therefore \text{Log. } 9317 = 3 \text{ times log. } 11 + \text{log. } 7 - \text{log. } 100 = 1.041393$$

$$\times 3 + 0.845098 - 2 = 1.969277.$$

## EIGHTH SERIES.

(85)

$$\text{Simple Interest} = Prt = \$700 \times .045 \times 3 = \$94.50.$$

$$\text{Amount Compound Interest} = P(1+r)^t = \$700 \times (1.045)^3$$

$$= \$700 \times 1.14116 = \$798.814 - \$700 = \$98.814 = \text{Comp}$$

$$\text{Int.}$$

$$\$98.814 - \$94.50 = \$4.314.$$

(86)

X's gain =  $\frac{1}{12}$ , and Z's =  $\frac{1}{3}$ ;  $\therefore$  Y's gain =  $1 - (\frac{1}{12} + \frac{1}{3})$   
 $= 1 - \frac{5}{12} = \frac{7}{12}$ .

X's gain is  $\frac{1}{12}$  for 3 months, therefore for 1 month it is  $\frac{1}{36}$ .

Y's gain is  $\frac{7}{12}$  for 9 months, " " "  $\frac{7}{108}$ .

Z's gain is  $\frac{1}{3}$  for 4 months, " " "  $\frac{1}{6}$ .

$\frac{1}{36} : \frac{1}{36} :: \$3024 : \$3024 \times \frac{1}{36} \times \frac{8}{1} = \$672 = \text{X's stock.}$

$\frac{1}{6} : \frac{1}{108} :: \$3024 : \$3024 \times \frac{1}{108} \times \frac{8}{1} = \$1120 = \text{Y's stock.}$

(87)

$$\frac{3}{8} \times \sqrt[3]{17} \div (1\frac{1}{2})^3 = \frac{3}{8} \times \sqrt[3]{\frac{16}{9}} \div (\frac{3}{2})^3 = \frac{3}{8} \times \frac{1}{3} \times \frac{8}{27} = \frac{1}{27}.$$

(88)

|                                |   |          |                          |
|--------------------------------|---|----------|--------------------------|
| $4^2 = 16 \times 300$          | = | 4800     | 80677568161 (4321 cubert |
| $4 \times 3 = 12 \times 30$    | = | 360      | 64                       |
| $3^2$                          | = | 9        | 16677                    |
|                                |   | 5169     | 15507                    |
| $43^2 = 1849 \times 300$       | = | 554700   | 1170568                  |
| $43 \times 2 = 86 \times 30$   | = | 2580     |                          |
| $2^2$                          | = | 4        |                          |
|                                |   | 557284   | 1114568                  |
| $432^2 = 186624 \times 300$    | = | 55987200 | 56000161                 |
| $432 \times 1 = 432 \times 30$ | = | 12960    |                          |
| $1^2$                          | = | 1        |                          |
|                                |   | 56000161 | 56000161                 |

(89)

$$7 = \left\{ 8 - 1 \begin{array}{l} \nearrow 3+4 \\ \searrow 1+6 \end{array} \right\} = 7$$

4 lbs. at 8d. }  
 1 lb. at 4d. } Make a mixture of 6 lbs. at 7d.  
 1 lb. at 6d. }

$$6 : 112 :: 4 : \frac{112 \times 4}{6} = 74\frac{2}{3} \text{ at 8d.}$$

(Continued on next page.)



(89 continued.)

$$6 : 112 :: 1 : \frac{112 \times 1}{6} = 18\frac{2}{3} \text{ at 4d.}$$

$$6 : 112 :: 1 : \frac{112 \times 1}{6} = 18\frac{2}{3} \text{ at 6d.}$$

(90)

Assume 40 as the sum of the three numbers.

Since  $1\text{st} + 2\text{nd} + 3\text{rd} = 40$ ,And  $1\text{st} + \frac{1}{2}(2\text{nd} + 3\text{rd}) = 34 \therefore \frac{1}{2}(2\text{nd} + 3\text{rd}) = 6 \dots \therefore 2\text{nd} + 3\text{rd} = 12$ And  $2\text{nd} + \frac{1}{3}(1\text{st} + 3\text{rd}) = 34 \therefore \frac{2}{3}(1\text{st} + 3\text{rd}) = 6 \dots \therefore 1\text{st} + 3\text{rd} = 9$ And  $3\text{rd} + \frac{1}{4}(1\text{st} + 2\text{nd}) = 34 \therefore \frac{3}{4}(1\text{st} + 2\text{nd}) = 6 \dots \therefore 1\text{st} + 2\text{nd} = 8$ Adding,  $2 \times (1\text{st} + 2\text{nd} + 3\text{rd}) = 29$  $\therefore 1\text{st} + 2\text{nd} + 3\text{rd} = 14\frac{1}{2}$ .

But the sum should equal 40.

Hence  $14\frac{1}{2} - 40 = -25\frac{1}{2}$ .

Assume 48 as the sum of the three numbers.

Since  $1\text{st} + 2\text{nd} + 3\text{rd} = 48$ .And  $1\text{st} + \frac{1}{2}(2\text{nd} + 3\text{rd}) = 34 \therefore \frac{1}{2}(2\text{nd} + 3\text{rd}) = 14 \dots \therefore 2\text{nd} + 3\text{rd} = 28$ And  $2\text{nd} + \frac{1}{3}(1\text{st} + 3\text{rd}) = 34 \therefore \frac{2}{3}(1\text{st} + 3\text{rd}) = 14 \dots \therefore 1\text{st} + 3\text{rd} = 21$ And  $3\text{rd} + \frac{1}{4}(1\text{st} + 2\text{nd}) = 34 \therefore \frac{3}{4}(1\text{st} + 2\text{nd}) = 14 \dots \therefore 1\text{st} + 2\text{nd} = 18\frac{2}{3}$ Adding,  $2 \times (1\text{st} + 2\text{nd} + 3\text{rd}) = 67\frac{1}{3}$  $\therefore 1\text{st} + 2\text{nd} + 3\text{rd} = 33\frac{5}{6}$ 

But the sum should equal 48.

Hence  $33\frac{5}{6} - 48 = -14\frac{1}{6} = \text{error}$ .

Errors.

$$-25\frac{1}{2} \times 48 = 1224$$

$$-14\frac{1}{6} \times 40 = 566\frac{2}{3}$$

$$\text{Diff.} = 11\frac{1}{2} \quad \text{Diff.} = 657\frac{1}{2}$$

$$657\frac{1}{2} \div 11\frac{1}{2} = 58 = \text{the sum of the three numbers,}$$

(Continued on next page.)

(90 continued.)

$$1\text{st} + \frac{1}{2}(2\text{nd} + 3\text{rd}) = 34 \therefore \frac{1}{2}(2\text{nd} + 3\text{rd}) = 58 - 34 = 24$$

$$\therefore 2\text{nd} + 3\text{rd} = 48.$$

$$2\text{nd} + \frac{1}{3}(1\text{st} + 3\text{rd}) = 34 \therefore \frac{2}{3}(1\text{st} + 3\text{rd}) = 58 - 34 = 24$$

$$\therefore 1\text{st} + 3\text{rd} = 36.$$

$$1\text{st} + 2\text{nd} + 3\text{rd} = 58, \text{ and } 2\text{nd} + 3\text{rd} = 48 \therefore 1\text{st} = 10.$$

$$1\text{st} + 2\text{nd} + 3\text{rd} = 58, \text{ and } 1\text{st} + 3\text{rd} = 36 \therefore 2\text{nd} = 22.$$

$$2\text{nd} + 3\text{rd} = 48, \text{ and } 2\text{nd} = 22 \therefore 3\text{rd} = 26.$$

(91)

4 means + 2 extremes = 6 terms.

$$\text{Formula IX, p. 333. } d = \frac{l - a}{n - 1} = \frac{40 - 1}{6 - 1} = \frac{39}{5} = 7\frac{4}{5}.$$

$$1, 8\frac{1}{5}, 16\frac{2}{5}, 24\frac{3}{5}, 32\frac{4}{5}, 40.$$

(92)

$$s = 1860040, l = 1240029, \text{ and } r = 3.$$

$$\text{Formula XI, p. 340. } a = rl - (r - 1)s = 1240029 \times 3 - (2 \times 1860040) = 3720087 - 3720080 = 7.$$

(93)

6 apples + 7 pears cost 33 pence  $\therefore$  2 apples +  $2\frac{1}{2}$  pears cost 11 pence.10 apples + 8 pears cost 44 pence  $\therefore$  2 apples +  $1\frac{1}{2}$  pears cost  $8\frac{1}{2}$  pence.Subtract, and  $2\frac{1}{2} - 1\frac{1}{2}$  pears cost 11d. —  $8\frac{1}{2}$ d.That is,  $\frac{1}{6}$  of a pear costs  $2\frac{1}{2}$ d.If  $\frac{1}{6}$  cost  $\frac{1}{6}$ d.,  $\frac{1}{12}$  will cost  $\frac{1}{12}$  of  $\frac{1}{6}$ d., which is  $\frac{1}{12}$ d.If  $\frac{1}{12}$  cost  $\frac{1}{12}$ d.,  $\frac{1}{6}$  will cost  $\frac{1}{6}$ d. = 3d.6 apples + 7 pears cost 33 pence, and 7 pears cost 21d.  $\therefore$  6 apples cost 12d. and 1 apple costs 2d.

(94)

$$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{\frac{57}{2}}{\frac{1}{2}} \times \frac{2}{3} \times \frac{3}{4} \times \frac{2}{3}$$

$$= \frac{1}{2} \times \frac{2}{4} \times \frac{3}{3} \times \frac{19}{12} \times \frac{2}{3} \times \frac{3}{5} \times \frac{2}{4} = \frac{19}{2 \times 4 \times 3 \times 2} = \frac{19}{12}$$

(95)

$\$10 = \frac{1}{2}$  of 2nd rem. —  $\$20 \therefore \frac{1}{2}$  of 2nd rem. =  $\$30 \therefore$  2nd rem. =  $\$40$ .

$\$40 = \frac{1}{2}$  of 1st rem. —  $\$30 \therefore \frac{1}{2}$  of 1st rem. =  $\$70 \therefore$  1st rem. =  $\$87.50$ .

$\$87.50 = \frac{1}{2}$  of original sum —  $\$50 \therefore \frac{1}{2}$  of original sum =  $\$137.50$   
 $\therefore$  original sum =  $\$137.50 \times 2 = \$275$ .

(96)

$a = 60, n = 17$ , and  $d = 4$ .

Formula VI, p. 333.  $s = \left\{ 2a + (n-1)d \right\} \frac{n}{2}$

$$= \left\{ 2 \times 60 + (17-1) \times 4 \right\} \frac{17}{2} = (120 + 64) \times \frac{17}{2}$$

$$= \frac{184 \times 17}{2} = \$1564 = \text{sum received for 17 years.}$$

Formula I, p. 333.  $l = a + (n-1)d = 60 + (17-1) \times 4$   
 $= 60 + 64 = \$124 = \text{wages for 17th year.}$

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### NINTH SERIES.

(98)

$\pounds 749 \text{ 16s. 5}\frac{1}{2}\text{d.} = \pounds 749.823958$ ;  $\pounds 1 \text{ sterling} = \$4.867$   
 $\pounds 749.823958 \times 4.867 = \$3649.3932$ .

(99)

2)177408

---

 2)88704

---

 2)44352

---

 2)22176

---

 2)11088

---

 2)5544

---

 2)2772

2)1386

---

 3)693

---

 3)231

---

 7)77

---

 11

---


$$2^3 \times 3^2 \times 7 \times 11.$$

(100)

Formula III, page 354,  $r = \sqrt[t]{P} - 1 \therefore r + 1 = \sqrt[t]{P}$

$$\text{Log. } (r + 1) = (\text{log. } A - \text{log. } P) \div t$$

$$\begin{aligned} \text{That is, log. } (r + 1) &= (\text{log. } 11111 \cdot 11 - \text{log. } 704) \div 11 \\ &= (4.045757 - 2.847573) \div 11 \\ &= 1.198184 \div 11 = 0.108925 \end{aligned}$$

Therefore  $r + 1$  = natural number corresponding to the logarithm 0.108925 which is 1.285.

Since  $r + 1 = 1.285$ ,  $r = .285$  = rate per unit and rate per cent.  $= .285 \times 100 = 28\frac{1}{2}$ .

(101)

If 9 be  $\frac{1}{3}$ ,  $\frac{1}{3}$  or the whole will equal  $9 \times 13 = 117$ .

(102)

3 gal. + 4 gal. + 7 gal. = 14 gal.

Hence 14 gal. : 292 gal. :: 3 gal. :  $\frac{292 \times 3}{14} = 62\frac{1}{2}$  of 1st kind.

14 gal. : 292 gal. :: 4 gal. :  $\frac{292 \times 4}{14} = 83\frac{2}{7}$  gal. of 2d. "

14 gal. : 292 gal. :: 7 gal. :  $\frac{292 \times 7}{14} = 146$  gal. of 3d. "

(103)

$$£\frac{1}{2} + £\frac{1}{2} + £\frac{1}{2} + £\frac{1}{2} = £1\frac{1}{2}$$

$$\text{Then } £1\frac{1}{2} : £500 :: £\frac{1}{2} : £500 \times \frac{1}{2} \times \frac{69}{77} = \frac{£15000}{77}$$

$$= £194 \text{ 16s. } 11\frac{2}{7}\text{d.}$$

$$£1\frac{1}{2} : £500 :: £\frac{1}{2} : £500 \times \frac{1}{2} \times \frac{69}{77} = \frac{£10000}{77}$$

$$= £129 \text{ 17s. } 4\frac{4}{7}\text{d.}$$

$$£1\frac{1}{2} : £500 :: £\frac{1}{2} : £500 \times \frac{1}{2} \times \frac{69}{77} = \frac{£7500}{77}$$

$$= £97 \text{ 8s. } 0\frac{4}{7}\text{d.}$$

$$£1\frac{1}{2} : £500 :: £\frac{1}{2} : £500 \times \frac{1}{2} \times \frac{69}{77} = \frac{£6000}{77}$$

$$= £77 \text{ 18s. } 5\frac{3}{7}\text{d.}$$

(104)

By Table, page 363, present value of annuity of \$1 at 6 per cent. for 23 payments = \$12 30338.

Hence present value of \$100 = \$12.30338  $\times$  100 = \$1230.338.

By Formula V, page 361,  $v = \frac{a}{r} \left\{ 1 - \frac{1}{(1+r)^t} \right\}$

$$= \frac{100}{.06} \times \left( 1 - \frac{1}{(1.06)^{23}} \right) = \frac{10000}{6} \times (1 - 0.261795)$$

$$= \frac{10000}{6} \times 0.738205 = \frac{738205}{6} = \$1230.34$$

(105)

Since each loses 1 hour per day for 24 days, the whole hours lost =  $24 \times 25$ .

Also, 5 men working 1 hour per day for 12 days make up  $5 \times 12 \times 1 = 60$  hours.

Hence they will each have to work as many hours per day as 60 hours is contained times in  $24 \times 25$  hours, i.e.  $\frac{24 \times 25}{60} = 10$  hours.

(106)

$$a = 5, s = 161 \text{ and } d = 6$$

Then Formula II, p. 333.  $l = -\frac{1}{2}d + \sqrt{2ds + (a - \frac{1}{2}d)^2} =$   
 $-\frac{1}{2} \text{ of } 6 + \sqrt{2 \times 6 \times 161 + (5 - \frac{1}{2} \text{ of } 6)^2} = -3 +$   
 $\sqrt{1932 + 4} = -3 + \sqrt{1936} = -3 + 44 = 41 \text{ years.}$

(107)

$$6^3 : 10^3 :: 1 \text{ day} : \frac{10^3 \times 1}{6^3} = \frac{1000}{216} = 4.629 \text{ days.}$$

$$* \text{ Log. } \frac{1}{(1.06)^{23}} = \log. 1 - \log. 1.06 \times 23 = 0 - 0.025306 \times 23$$

$$= 0 - 0.582038 = \bar{1}.417962$$

$$\therefore \frac{1}{(1.06)^{23}} = \text{natural number corresponding to the logarithm}$$

$$\bar{1}.417962, \text{ which is } 0.261795$$

(108)

For 12 months he was to receive £8 and a suit of clothes; for 7 months he received £2 13s. 4d. and the suit of clothes;  $\therefore$  for 5 months he would have received the difference between £8 and £2 13s. 4d., which is £5 6s. 8d.

Hence for 1 month he would have received £5 6s. 8d.  $\div$  5, which is £1 1s. 4d., and hence his wages for the year would have been, in money alone, £1 1s. 4d.  $\times$  12, *i.e.*, £12 16s. Therefore the suit of clothes was valued at £12 16s. — £8 = £4 16s.

## TENTH SERIES.

(109)

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}$ ; if  $\frac{13}{12}$  of a number = 48,  $\frac{1}{12}$  will =  $48 \div 13 = 3\frac{2}{13}$ . If  $3\frac{2}{13} = \frac{1}{12}$ ,  $\frac{1}{2}$ , or the whole number =  $3\frac{2}{13} \times 12 = 44\frac{4}{13}$ .

(110)

$$6^3 : 8^3 :: 600 : \frac{600 \times 8^3}{6^3} = \frac{600 \times 512}{216} = 1422\cdot2 \text{ lbs.}$$

(See Art. 33, sec. X.)

(111)

Part of ball remaining after 1st has taken off her share =  $\frac{3}{4}$

Then whole ball : remainder :: cube of diameter of whole : cube of diameter of remainder

$1 : \frac{3}{4} :: 5^3 : x^3$  hence  $x = \sqrt[3]{\frac{3}{4} \times 125} = \sqrt[3]{93\frac{3}{4}} = \sqrt[3]{93\cdot75} = 4\cdot542$   
 $\therefore$  Part taken off by 1st = 5 in. —  $4\cdot542$  in. =  $0\cdot458$  in.

After 2nd had taken off her portion  $\frac{1}{2}$  of the ball remained.

$1 : \frac{1}{2} :: 5^3 : x^3$ , hence  $x = \sqrt[3]{\frac{1}{2} \times 125} = \sqrt[3]{62\frac{1}{2}} = \sqrt[3]{62\cdot5} = 3\cdot968$  in.  
 $\therefore$  Part taken off by 2nd =  $4\cdot542 - 3\cdot968 = 0\cdot574$  in.

After 3rd had taken off her share there remained  $\frac{1}{4}$  of the ball.

$1 : \frac{1}{4} :: 5^3 : x^3$ , hence  $x = \sqrt[3]{\frac{1}{4} \times 125} = \sqrt[3]{31\cdot25} = 3\cdot149$  in.

$\therefore$  Part taken off by 3rd =  $3\cdot968 - 3\cdot149 = 0\cdot819$  inches

Remainder =  $3\cdot149$  = part taken off by 4th.

$$71214 \cdot 43 \div 12 \cdot 342 = 71214430 \div 12342$$

$$12342 \overline{) 71214430} \quad (5570 \cdot 238552$$

$$62831$$

72734

62831

88033

87625

3070·0

2468·4

500·50

371·36

118·130

111·067

7·0520

6·2831

·65780

·62831

·028480

·024684

·003685

$$5570 \cdot 238552 (71 \cdot 118 = \text{sq. rt.}$$

$$54$$

151) 170

151

1521) 18·23

15·21

15221) 3·0285

1·5221

152228) 1·406452

1·360051

·036411

NOTE.—Unless the quotient is carried out to *six places* of decimals, i.e., twice as many as are required in the root, the last figure in the root will be 6 or 7.

$$(113)$$

|     |   |                      |                       |   |                           |
|-----|---|----------------------|-----------------------|---|---------------------------|
| 1st | { | $\$60 \times 48 =$   | $\$2880$ for 1 month  | } | $= \$43280$ for 1 month.  |
|     |   | $\$800 \times 43 =$  | 34400 for 1 month     |   |                           |
|     |   | $\$1500 \times 4 =$  | 6000 for 1 month      |   |                           |
|     |   | Sum =                | $\$43280$             |   |                           |
| 2nd | { | $\$600 \times 48 =$  | $\$28800$ for 1 month | } | $= \$104400$ for 1 month. |
|     |   | $\$1800 \times 42 =$ | 75600 for 1 month     |   |                           |
|     |   | Sum =                | $\$104400$            |   |                           |

(Continued on next page.)

(113 continued).

|     |   |                     |            |   |                                   |
|-----|---|---------------------|------------|---|-----------------------------------|
| 3rd | { | $\$400 \times 48 =$ | $\$19200$  | } | $= \$103200 \text{ for 1 month.}$ |
|     |   | $\$500 \times 42 =$ | 21000      |   |                                   |
|     |   | $\$500 \times 36 =$ | 18000      |   |                                   |
|     |   | $\$500 \times 30 =$ | 15000      |   |                                   |
|     |   | $\$500 \times 24 =$ | 12000      |   |                                   |
|     |   | $\$500 \times 18 =$ | 9000       |   |                                   |
|     |   | $\$500 \times 12 =$ | 6000       |   |                                   |
|     |   | $\$500 \times 6 =$  | 3000       |   |                                   |
|     |   | <hr/>               |            |   |                                   |
|     |   | Sum =               | $\$103200$ |   |                                   |
| 4th | { | $\$900 \times 40 =$ | $\$36000$  | } | $= \$138600 \text{ for 1 month.}$ |
|     |   | $\$900 \times 34 =$ | 30600      |   |                                   |
|     |   | $\$900 \times 28 =$ | 25200      |   |                                   |
|     |   | $\$900 \times 22 =$ | 19800      |   |                                   |
|     |   | $\$900 \times 16 =$ | 14400      |   |                                   |
|     |   | $\$900 \times 10 =$ | 9000       |   |                                   |
|     |   | $\$900 \times 4 =$  | 3600       |   |                                   |
|     |   |                     |            |   |                                   |
|     |   | Sum =               | $\$138600$ |   |                                   |

\$43280

104400

103200

138600

4 years at \$1.25 per day

= \$1.25 × 4 × 365 = \$1825 = share of 5th.

\$389480 for one month.

\$20000 — \$1825 = \$18175 = sum to be divided among the four.

\$389480 : \$18175 :: \$43280 : \$2019.651 = share of 1st.

\$389480 : \$18175 :: \$104400 : \$4871.803 = " 2nd.

\$389480 : \$18175 :: \$103200 : \$4815.805 = " 3rd.

\$389480 : \$18175 :: \$138600 : \$5467.739 = " 4th.

(114)

Simple Interest, formula IX, p. 248.  $t = \frac{n-1}{r} = \frac{16-1}{.05} = \frac{15}{.05}$   
 $= \frac{1500}{5} = 300$  years.

Compound Interest, formula V, p. 354.  $t = \frac{\log. n}{\log. (1+r)}$   
 $= \frac{\log. 16}{\log. 1.05} = \frac{1.204120}{0.021189} = \frac{1204120}{21189} = 56.827$  years.



(115)

For every \$1 the first gave, the second gave \$3, and the third \$6.  $\$1 + \$3 + \$6 = \$10$ .

Hence the 1st gave \$1, the second \$3, and the third \$6 as often as \$10 is contained times in \$9202, which is  $920\frac{1}{5}$  times.

$$\$1 \times 920\frac{1}{5} = \$920.20 = \text{payment of 1st person.}$$

$$\$3 \times 920\frac{1}{5} = \$2760.60 = \quad \quad \quad \text{2nd} \quad \quad \quad "$$

$$\$6 \times 920\frac{1}{5} = \$5521.20 = \quad \quad \quad \text{3rd} \quad \quad \quad "$$

(116)

$$25 + 22 = 47 = \text{whole number of men.}$$

$$165 \div 47 = 3\frac{1}{7} = \text{acres cleared by each man.}$$

$$3\frac{1}{7} \times 22 = 77\frac{1}{7} \text{ acres} = \text{acres cleared by company of 22 men.}$$

$$165 \text{ acres} - 77\frac{1}{7} \text{ acres} = 87\frac{3}{7} \text{ acres} = \text{acres cleared by company of 25 men.}$$

1st company contains 3 more men than 2nd company and receives \$86 more.

Therefore \$86 pays 3 men. Hence each man gets  $\$86 \div 3 = \$28.66\frac{2}{3}$ .

Each man clears  $3\frac{1}{7}$  acres, and receives  $\$28.66\frac{2}{3}$  for it; therefore cost of 1 acre  $= \$28.66\frac{2}{3} \div 3\frac{1}{7} = \$8.49\frac{2}{3}$ .

(117)

$$15^2 = 225; 346 - 225 = 121 = \text{square of the less.}$$

$$\text{Hence less} = \sqrt{121} = 11.$$

(118)

$$\text{Formula V, page 248, } A = P(1 + rt) = \$1200 \times 1.95 = \$2340.00.$$

(119)

$$\begin{array}{l|l} 24 : & 496 \\ 9 : & 11 \\ 7 : & 4 \\ 465 : & 337\frac{1}{2} \\ 3\frac{3}{4} : & 5\frac{3}{8} \\ 2\frac{1}{2} : & 3\frac{1}{2} \end{array} \quad \begin{array}{l} \\ \\ \\ \therefore 5\frac{1}{2} : x \\ \\ \end{array}$$

(Continued on next page.)



## ELEVENTH SERIES.

(121)

$$\cdot 7 = \frac{7}{9}; \cdot 83 = \frac{83}{99}; \cdot 727 = \frac{727}{999}; \cdot 91325 = \frac{91325}{99990} = \frac{91316}{99990} = \frac{45658}{49995}$$

$$8 \cdot 671347 = 8 \frac{671347}{999900} = 8 \frac{671280}{999900} = 8 \frac{11188}{16665}$$

(122)

$$713 \text{ unden.} = 861 \text{ den.}; 291 \text{ unden.} = 342 \text{ den.}; 3t1 \text{ unden.} \\ = 474 \text{ den.}$$

291

$$\text{Then } 713 \text{ — unden.} = 861 \frac{342}{474} \text{ den.} = 861 \frac{7}{9} \text{ den.}$$

3t 1

$$12123 \text{ quat} = 411 \text{ den.}; 11223 \text{ quat.} = 363 \text{ den.}; 100000 \text{ quat.} \\ = 1024 \text{ den.}$$

$$\text{Then } 12123 \frac{11223}{100000} = 411 \frac{363}{1024} \text{ den.}$$

(123)

$$3\frac{3}{8} \text{ of } 2\frac{1}{8} \text{ of } 7\frac{1}{2} \text{ of } £1 = \frac{27}{8} \text{ of } \frac{1}{5} \text{ of } \frac{151}{20} \text{ of } £1$$

$$= £ \frac{48817}{800} \dots\dots\dots = £56 \quad 1 \quad 2\frac{1}{10}$$

$$9\frac{3}{7} \text{ of } 3\frac{3}{7} \text{ of } 1s. = \frac{66}{7} \text{ of } \frac{35}{9} \text{ of } 1s. = \frac{110}{3} s. \dots = \quad 1 \quad 16 \quad 8$$

$$8\frac{1}{4} \text{ of } 4\frac{1}{8} \text{ of } 1d. = \frac{33}{4} \text{ of } \frac{33}{8} \text{ of } 1d. = \frac{1089}{32} d. \dots = \quad 0 \quad 2 \quad 10\frac{1}{2}$$

$$\text{Sum} = £58 \quad 0 \quad 8\frac{31}{160}$$

$$\frac{1}{12} \text{ of } \frac{5}{14} \text{ of } \frac{3}{8} \text{ of } 3\frac{1}{2} d. = \frac{1}{12} \times \frac{5}{14} \times \frac{3}{8} \times \frac{7}{2} = \frac{55}{112} d.$$

$$£58 \text{ os. } 8\frac{31}{160} d. = \frac{2228501}{160} d.$$

$$\frac{2228501}{160} \div \frac{55}{112} = \frac{2228501}{160} \times \frac{112}{55} = \frac{202521}{5} \times \frac{1}{2} = \frac{210364}{25} \\ = 32414 \cdot 56.$$

(124)

$$\begin{array}{l|l} 24 : 90 & \\ 2\frac{1}{2} : 4\frac{1}{5} & \\ 12\frac{1}{2} : 9\frac{2}{3} & \\ 4\frac{7}{8} : 4\frac{1}{2} & \\ 3\frac{1}{5} : 2\frac{1}{2} & \end{array} \quad \therefore 139\frac{1}{2} : x$$

(Continued on next page.)

(124 continued.)

$$\begin{array}{cccccccccccc}
 9 & & & & & & & & & & & \\
 18 & 7 & & 3 & & 43 & & & & & & \\
 50 & 21 & 29 & 5 & 5 & 559 & 1 & 2 & 2 & 8 & & \\
 \hline 1 & 5 & 3 & 2 & 2 & 4 & 24 & 5 & 25 & 39 & & \\
 & & & & & & 8 & & 5 & 13 & & \\
 & & 5 & 9 \times 7 \times 29 \times 43 & & 78561 & & & & & & \\
 \times \frac{5}{18} = \frac{2 \times 4 \times 5 \times 4}{160} = \frac{78561}{160} = 491 \frac{1}{160}.
 \end{array}$$

(125)

\$182 is  $\frac{91}{100}$  of buying price  $\therefore \$182 \div 91 = \$2 = \frac{1}{100}$  of buying price  $\therefore$  buying price  $= \$2 \times 100 = \$200$ .

To realize a profit of 7 per cent., he must receive \$1.07 for every \$1 the goods cost; but they cost him \$200, therefore he must sell for  $\$1.07 \times 200 = \$214$ .

(126)

$$\text{Simple Interest } t = \frac{n-1}{r} = \frac{11\frac{1}{2}-1}{.06} = \frac{10.5}{.06} = \frac{1050}{6} = 175 \text{ years.}$$

$$\begin{aligned}
 \text{Compound Interest } t &= \frac{\log. n}{\log. (1+r)} = \frac{\log. 11\frac{1}{2}}{\log. 1.06} = \frac{1.060693}{0.025306} \\
 &= \frac{1060693}{25306} = 41.914 \text{ years.}
 \end{aligned}$$

(127)

An acre contains 4 roods  $= 160$  sq. perches.

$\therefore 160 \div 15\frac{1}{2} = 10\frac{10}{31}$  perches  $=$  length.

(128)

35 yards  $= 32$  metres  $\therefore 1$  yd.  $= \frac{32}{35}$  of a metre.

$$\begin{aligned}
 69 \frac{1}{2} \text{ miles} &= 69 \frac{1}{2} \times \frac{1760}{1} \text{ yards} = 69 \frac{1}{2} \times \frac{1760}{1} \times \frac{32}{35} \text{ metres} \\
 &= \frac{217}{22} \times \frac{1760}{1} \times \frac{32}{35} = 217 \times 16 \times 32 = 111104 \text{ metres.}
 \end{aligned}$$

(129)

7 means  $\div$  2 extremes = 9 terms.Formula XIII, p. 340.  $r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}} = \left(\frac{19683}{3}\right)^{\frac{1}{8}} = (6561)^{\frac{1}{8}} = 3$ 

Hence means are 9, 27, 81, 243, 729, 2187, and 6561.

(130)

Formula XXI, p. 344.  $s = \frac{a}{1-r} = \frac{7}{1-\frac{1}{2}} = \frac{7}{\frac{1}{2}} = \frac{28}{3} = 9\frac{1}{3}$ .

(131)

Part remaining after 1st has received his share =  $\frac{3}{4}$ . $1 : \frac{3}{4} :: 60^2 : x^2$ ; whence  $x = \sqrt{3600 \times \frac{3}{4}} = \sqrt{900 \times 3} = 30\sqrt{3} = 1.732 \times 30 = 51.96$  inches.Hence 1st ground off  $60 - 51.96 = 8.04$  inches.Part remaining after 2nd had taken off his share =  $\frac{1}{2}$ . $1 : \frac{1}{2} :: 60^2 : x^2$ ; whence  $x = \sqrt{3600 \times \frac{1}{2}} = 30\sqrt{2} = 1.4142 \times 30 = 42.426$ .Hence 2nd ground off  $51.96 - 42.426 = 9.534$  inches.Part remaining after the 3rd had taken off his share =  $\frac{1}{4}$ . $1 : \frac{1}{4} :: 60^2 : x^2$ ; whence  $x = \sqrt{3600 \times \frac{1}{4}} = \sqrt{900} = 30$  inches.Hence 3rd ground off  $42.426 - 30$  inches = 12.426 inches,  
and the 4th ground off remaining 30 inches.

(132)

1 guinea = 21s.

1 half guinea = 10½s.

1 crown = 5s.

1 half crown = 2½s.

1 shilling = 1s.

Sixpence = ½s.

100 guineas = 2100 shillings.

 $2100 \div 40\frac{1}{2} = 51$  times and remainder, 69 half-shillings.69 half-shil. =  $\frac{69}{2}$ s. = £17½ = 17½s.

Sum = 40½s.

## TWELFTH SERIES.

(133)

$$\frac{3}{11} \text{ of } \frac{2}{9} \text{ of } \frac{4}{17} = \frac{8}{561}; \frac{2\frac{1}{2}}{4\frac{1}{4}} \text{ of } \frac{2}{5} = \frac{10}{17} \text{ of } \frac{2}{5} = \frac{4}{17}$$

$$\frac{8}{561} : \frac{4}{17} :: \$12\frac{4}{33} : \$12\frac{1}{3} \times \frac{4}{17} \times \frac{561}{8} = \frac{200}{33} \times \frac{4}{17} \times \frac{33}{561} \\ = \$200.$$

(134)

$$\text{By Formula III, page 354, } r = \sqrt[t]{\frac{A}{P}} - 1 \therefore r + 1 = \sqrt[t]{\frac{A}{P}} \\ \therefore \text{Log. } (1 + r) = (\log. A - \log. P) \div t \\ = (\log. 1679.40 - \log. 700.90) \div 5 \\ = (3.225154 - 2.845656) \div 5. \\ = 0.379498 \div 5 = 0.075899.$$

$\therefore 1 + r =$  nat. num. corresponding to the logarithm 0.075899 which is 1.19,  $\therefore r = .19 =$  rate per unit, and hence rate per cent. = 19.

(135)

Having paid 10 per cent. he had 90 per cent. remaining.

$$\frac{90}{100} \text{ or } \frac{9}{10} \text{ of his salary} = \$1250, \therefore \frac{1}{10} = \frac{1250}{9} = \$138\frac{2}{9}.$$

$$\text{If } \$138\frac{2}{9} = \frac{1}{10}, \text{ the whole} = \$138\frac{2}{9} \times 10 = \$1388.888.$$

(136)

21 children receive 21 times a child's share

21 women " 42 " "

21 men " 63 " "

— — — — —  
Together they receive 126 " "

£3 13s. 6d.  $\div$  126 = 7d. = a child's share.

7d.  $\times$  2 = 1s. 2d. = a woman's share.

7d. + 1s. 2d. = 1s. 9d. = a man's share.



(140 continued.)

 $\dot{9}59\dot{5}1\cdot\dot{2}57\dot{6}$  ( $309\cdot76 = \text{square root.}$ )

9

$$\begin{array}{r} 609) \ 5951 \\ \underline{5481} \end{array}$$

$$\begin{array}{r} 309\cdot76 \ (17\cdot6 = 17\frac{1}{2} = \text{fourth root.}) \\ \underline{1} \end{array}$$

$$\begin{array}{r} 618\cdot7) 470\cdot25 \\ \underline{433\cdot09} \end{array}$$

$$\begin{array}{r} 27) 209 \\ \underline{189} \end{array}$$

$$\begin{array}{r} 619\cdot46) 37\cdot1676 \\ \underline{37\cdot1676} \end{array}$$

$$\begin{array}{r} 34\cdot6) 2076 \\ \underline{2076} \end{array}$$

(141)

250

300

400

500

$$1450:250::\$520:\frac{\$520 \times 250}{1450} = \$89\frac{1}{2} = \text{contrib. on 1st village.}$$

$$1450:300::\$520:\frac{\$520 \times 300}{1450} = \$107\frac{1}{2} = \quad \quad \text{2nd} \quad \quad "$$

$$1450:400::\$520:\frac{\$520 \times 400}{1450} = \$143\frac{1}{2} = \quad \quad \text{3rd} \quad \quad "$$

$$1450:500::\$520:\frac{\$520 \times 500}{1450} = \$179\frac{9}{10} = \quad \quad \text{4th} \quad \quad "$$

(142)

By Table on p. 362, the amount of \$1 for 34 payments at 3 per cent. = \$57.73018.

$$\$57.73018 \times 260 = \$15009.84.$$

$$\text{By Formula I, page 361, } A = \frac{a \{ (1+r)^t - 1 \}}{r}$$

$$= \frac{a}{r} \left\{ (1+r)^t - 1 \right\} = \frac{260}{.03} \left\{ (1.03)^{34} - 1 \right\}$$

$$= \frac{26000}{3} \times (2.731855 - 1) = \frac{26000 \times 1.731855}{3} = \$15009.41$$



(143)

$$\text{By Formula IX, p. 333, } d = \frac{l-a}{n-1} = \frac{79-2}{6-1} = \frac{77}{5} = 15\frac{2}{5}.$$

Hence the series is 2,  $17\frac{2}{5}$ ,  $32\frac{4}{5}$ ,  $48\frac{1}{5}$ ,  $63\frac{3}{5}$ , and 79.

$$\text{Formula I, p. 333. } l = a + (n-1)d = 3 + (9-1) \times 4 \\ = 3 + (8 \times 4) = 3 + 32 = 35.$$

$$\text{Formula VI, p. 333. } s = \left\{ 2a + (n-1)d \right\} \frac{n}{2} \\ = \left\{ 2 \times 3 + (207-1) \times 4 \right\} \frac{207}{2} = \left\{ 6 + (206 \times 4) \right\} \frac{207}{2} \\ = (6 + 824) \times \frac{207}{2} = \frac{830 \times 207}{2} = 85905.$$

(144)

B travels 4 miles per day faster than A, and will therefore gain the circumference of the island in  $7\frac{3}{4} = 18\frac{1}{4}$  days.

C travels 10 miles per day faster than A, and will therefore gain the whole circumference of the island in  $7\frac{3}{10} = 7\frac{3}{10}$  days.

Now B cannot be with A except at the end of  $18\frac{1}{4}$  days or twice  $18\frac{1}{4}$  days, or three times  $18\frac{1}{4}$  days, or some other multiple of  $18\frac{1}{4}$  days.

Similarly C cannot be with A except at the end of  $7\frac{3}{10}$  days, or of some other multiple of  $7\frac{3}{10}$  days.

Therefore C and B will both be with A for the first time after the lapse of a number of days expressed by the least common multiple of  $18\frac{1}{4}$  and  $7\frac{3}{10}$ .

The greatest common factor of  $18\frac{1}{4}$  and  $7\frac{3}{10}$  is  $3\frac{1}{20}$ .

Hence the l. c. m. of  $7\frac{3}{10}$  and  $18\frac{1}{4}$  is  $\frac{7\frac{3}{10} \times 18\frac{1}{4}}{3\frac{1}{20}} = 36\frac{1}{2} = \text{number}$   
of days when A, B, and C will first be together.

## ARITHMETICAL RECREATIONS.

1. The third of 6 = 2, and the fourth of 20 = 5.

Then if 2 becomes 3, what should 5 become? Evidently

 $7\frac{1}{2}$ . *Ans.*

or

$$\begin{array}{l} 6 : 20 \\ \frac{1}{3} : \frac{1}{4} \end{array} \} :: 3 : x = \frac{3 \times 20 \times \frac{1}{4}}{6 \times \frac{1}{3}} = 7\frac{1}{2}.$$

2. The half of 5 =
- $2\frac{1}{2}$
- ; then if 7 becomes
- $2\frac{1}{2}$
- , what will 11 become?

$$\frac{2\frac{1}{2} \times 11}{7} = \frac{55}{14}. \text{ Lastly, what part of 9 is } \frac{55}{14}?$$

$$\frac{\frac{55}{14}}{\frac{9}{4}} = \frac{55}{126}. \text{ Ans.}$$

or

$$\begin{array}{l} 9 : 5 \\ 7 : 11 \end{array} \} :: \frac{1}{4} : x = \frac{\frac{1}{4} \times 5 \times 11}{9 \times 7} = \frac{\frac{55}{2}}{63} = \frac{55}{126}. \text{ Ans.}$$

- 3.
- $99\frac{3}{9}$
- .

- 4.
- $\frac{1}{3}$
- of 2d. =
- $\frac{2}{3}$
- d. Then
- $\frac{2}{3}$
- d. is what part of 3d.?
- Ans.*
- $\frac{2}{9}$
- .

- 5.
- $1\frac{1}{2}$
- d. for a herring and a half is at the rate of 1d. per herring; hence 11 herrings will cost 11d.

6. 12 apples = 21 pears = 7 cents.

If 12 apples cost 7 cents, what will 100 apples cost?

$$12 : 100 :: 7 : \frac{100 \times 7}{12} = 58\frac{1}{3} \text{ cents.}$$

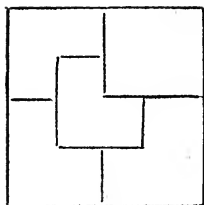
7. If 5 is
- $\frac{2}{3}$
- of a certain number,
- $\frac{1}{3}$
- will be
- $\frac{1}{3}$
- of 5, which is
- $\frac{5}{3}$
- .

If  $\frac{5}{3}$  is  $\frac{1}{3}$  of a certain number, the whole number will be

$$\frac{5}{3} \times 3 = 5. \text{ Ans.}$$

8. The hurdles are arranged so as to form a rectangular enclosure having 49 hurdles on each side and one on each end. Two additional hurdles will give two hurdles to each end, and will thus double the size of the enclosure.

9. The mode of dividing the plot may be learned from the following figure:—



10.  $33\frac{3}{4}$

11. XIII; rub out the lower half, and there remains the expression VIII = 8.

12. 1st Step: Fill the 3-gallon cask and empty it into the 5-gallon cask.

2nd Step: Again fill the 3-gallon cask out of the 8-gallon cask.

3rd Step: Fill up the 5-gallon cask out of the 3-gallon cask. This will leave one gallon in the latter.

4th Step: Empty the 5-gallon cask into the 8-gallon cask.

5th Step: Pour the one gallon out of the 3-gallon cask into the 5-gallon cask.

6th Step: Fill the 3-gallon cask out of the 8-gallon cask, and empty it into the 5-gallon cask.

The following diagrams show this more clearly:

1st Step.



2nd Step.



3rd Step.



4th Step.



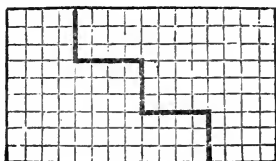
5th Step.



6th Step.



13. The heavy line in the accompanying figure shows how the board is to be cut.



|   |   |   |
|---|---|---|
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

15. Weigh out 7 lbs. as often as possible and there will remain 2 lbs.; add two four pounds and one seven pounds to this, and the sum will be 17 lbs., the share of one.

Weigh 7 lbs. as often as possible out of the remaining 34 lbs. and there will remain 6 lbs., to which add 7 lbs. and 4 lbs., and the sum will be 17 lbs., the share of the second. The remaining 17 lbs. will be the share of the third.

16. The hurdles are, in the first case, placed 12 on a side and one on each end, and then they inclose a space represented by 12 squares whose area is, by the question, 40 square yards. If two hurdles be taken away there will remain 24, and if these be placed in the form of a square, each side containing 6 hurdles, they will enclose a space represented by 36 squares of the same size as the former. Hence they now inclose three times as much space as before, *i. e.* three times 40 square yards, or 120 square yards.

17. He takes the goose to the remote bank and leaves it there, returning, he next carries over the fox, which he leaves, but takes the goose back with him. He now leaves the goose on the first bank, and carries over the oats which he allows to remain on the remote bank with the fox, and returns for the goose.

18. The following diagrams exhibit the solution of this problem:

| I. | II. | III. | IV. |
|----|-----|------|-----|
| 3  | 4   | 2    | 1   |
| 3  | 1   | 5    | 7   |
| 3  | 4   | 2    | 1   |
| 3  | 1   | 5    | 7   |
| 3  | 4   | 2    | 1   |
| 24 | 20  | 28   | 20  |

| V. | VI. |
|----|-----|
| 0  | 5   |
| 9  | 0   |
| 0  | 4   |
| 9  | 0   |
| 0  | 5   |
| 36 | 18  |

19. XII; rub out the lower half, and VII remains.

20.

|    |    |    |    |    |
|----|----|----|----|----|
| 17 | 24 | 1  | 8  | 15 |
| 23 | 5  | 7  | 14 | 16 |
| 4  | 6  | 13 | 20 | 22 |
| 10 | 12 | 19 | 21 | 3  |
| 11 | 18 | 25 | 2  | 9  |

#### RULE FOR FILLING MAGIC SQUARES OF ODD NUMBER OF CELLS.

Begin in centre cell of top horizontal row by placing 1 in it; ascend diagonally to the right, and where this carries us beyond the square, transport the next number to the cell at the remote end of the vertical or horizontal band to which it belongs. When in ascending we come to a cell already filled, we place the number in the cell next below the cell last filled. The following is a square of 7 cells in a side filled after this method :

(Continued on next page.)

|    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|
| 30 | 39 | 48 | 1  | 10 | 19 | 28 |
| 38 | 47 | 7  | 9  | 18 | 27 | 29 |
| 46 | 6  | 8  | 17 | 26 | 35 | 37 |
| 5  | 14 | 16 | 25 | 34 | 36 | 45 |
| 13 | 15 | 24 | 33 | 42 | 44 | 4  |
| 21 | 23 | 32 | 41 | 43 | 3  | 12 |
| 22 | 31 | 40 | 49 | 2  | 11 | 20 |

21. Half-a-dozen dozen =  $6 \times 12 = 72$ .  
 Six dozen dozen =  $6 \times 12 \times 12 = 864$ .  
 $864 - 72 = 792$ . *Ans.*

22. The following shows the mode of performing this.  
 It will be observed that the two side counters are merely moved one counter higher when the other two are taken away.

23. This problem admits of the following two solutions:

#### 1ST SOLUTION.

| Persons. | Full bottles. | Hf.-full bottles. | Empty bottles. |
|----------|---------------|-------------------|----------------|
| 1st      | 2             | 3                 | 2              |
| 2nd      | 2             | 3                 | 2              |
| 3rd      | 3             | 1                 | 3              |
|          | <u>7</u>      | <u>7</u>          | <u>7</u>       |

Each person has  $3\frac{1}{2}$  bottles of wine and 7 bottles.

#### 2ND SOLUTION.

|     |          |          |          |
|-----|----------|----------|----------|
| 1st | 3        | 1        | 3        |
| 2nd | 3        | 1        | 3        |
| 3rd | 1        | 5        | 1        |
|     | <u>7</u> | <u>7</u> | <u>7</u> |

Each person, as before, has 7 bottles and  $3\frac{1}{2}$  bottles of wine.

24. There were in all 8 bottles of wine, of which each drank  $\frac{1}{2}$ , which is 2. The third person, therefore, drank  $\frac{1}{2}$  of a bottle belonging to him who had but 3 bottles, and  $\frac{1}{2}$  of a bottle belonging to him who owned the 5 bottles. Hence the latter should have *seven* times as much of the money as the former, or, in other words, the latter gets 7 shillings, and the former 1 shilling.

25. This problem is merely to find some number between 50 and 100 which is exactly divisible by 2 and by 3, but which divided by 5 leaves a remainder 3.

The only numbers between 50 and 100 that are divisible by both 2 and 3, are 54, 60, 66, 72, 78, 84, 90, and 96, and by inspection the only one of these which gives a remainder 3 when divided by 5 is 78; therefore the basket contained 78 eggs.

26. *Ans.* 1 lb., 3 lbs., 9 lbs., and 27 lbs.

For 1 lb. = 1 lb.; 2 lbs. = 3 lbs. -- 1 lb., i. e. 3 lbs. in one scale and 1 lb. in the other; 3 lbs. = 3 lbs.; 4 lbs. = 3 lbs. + 1 lb.; 5 lbs. = 9 lbs. -- (3 lbs. + 1 lb.); 6 lbs. = 9 lbs. -- 3 lbs.; 7 lbs. = 9 lbs. + 1 lb. -- 3 lbs.; 8 lbs. = 9 lbs. -- 1 lb.; 9 lbs. = 9 lbs.; 10 lbs. = 9 + 1 lb.; 11 lbs. = 9 lbs. + 3 lbs. -- 1 lb.; 12 lbs. = 9 lbs. + 3 lbs.; 13 lbs. = 9 lbs. + 3 lbs. + 1 lb.; 14 lbs. = 27 lbs. -- (9 lbs. + 3 lbs. + 1 lb.); 15 lbs. = 27 lbs. -- (9 lbs. + 3 lbs.); 16 lbs. = 27 lbs. + 1 lb. -- (9 lbs. + 3 lbs.); 17 lbs. = 27 lbs. -- (9 lbs. + 1 lb.); 18 lbs. = 27 lbs. -- 9 lbs.; &c., &c.

27. In order to fill seven out of the eight points, it is merely requisite to remember that the second counter must be carried to the point from which the first *started*, the third to the point from which the second started, &c.

Thus if the first counter is carried from 1 to 4 and there deposited, the second must be taken from 6 to 1 and there deposited; the third from 3 to 6; the fourth from 8 to 3; the fifth from 5 to 8; the sixth from 2 to 5; and the seventh either from 7 to 2 or from 2 to 7.

28. The mouth fills the reservoir in 6 hours, therefore it fills  $\frac{1}{6}$  in 1 hour; the right eye fills it in 38 hours, therefore it fills

$\frac{1}{48}$  in 1 hour; the left eye fills it in 72 hours, therefore it fills  $\frac{1}{72}$  in 1 hour; the foot fills it in 96 hours, therefore it fills  $\frac{1}{96}$  in 1 hour. Hence together they fill  $\frac{1}{6} + \frac{1}{48} + \frac{1}{72} + \frac{1}{96} = \frac{61}{288}$  in 1 hour, and to fill the reservoir they require  $1 \div \frac{61}{288} = \frac{288}{61} = 4$  hours 43 min.  $16\frac{4}{61}$  sec.

29. The person who thinks of the numbers must proceed as follows: He must multiply the 1st by 2 and add 5 to the product; he must next multiply this sum by 5 and add the second number to the product; he must next multiply this result by 10 and add the third number to the product; lastly, he must subtract 250 and name the remainder.

The three digits of the remainder will be the three numbers thought of, and will be in the order in which they were thought of.

The reason is obvious: let  $a = 1st$ ,  $b = 2nd$ , and  $c = 3rd$  number thought of.

$$a \times 2 + 5 = 2a + 5.$$

$$(2a + 5) \times 5 + b = 10a + b + 25.$$

$$(10a + b + 25) \times 10 + c = 100a + 10b + c + 250.$$

$$(100a + 10b + c + 250) - 250 = 100a + 10b + c = a \text{ in hundreds' place, } b \text{ in tens' place, and } c \text{ in units' place.}$$

30. Since each man possesses 63 square rods of land more than his son, we must form three pairs of numbers, such that the difference of their squares shall be 63.

The difference of the squares of two numbers is equal to their sum multiplied by their difference, and hence 63 must be divided into two factors in three distinct ways, thus:

$$63 = 63 \times 1 = 21 \times 3 = 9 \times 7.$$

If sum = 63 and difference = 1, the numbers are 32 and 31.

If sum = 21 and difference = 3, the numbers are 12 and 9.

If sum = 9 and difference = 7, the numbers are 8 and 1.

Hence the squares of Jones, Brown, and Smith, are respectively 32 rods, 12 rods, and 8 rods on the side, and the son's squares are respectively 31, 9, and 1 yards on the side.

Jones' piece was 23 rods longer on each side than Tom's, and since the difference between 32 and 9 is 23, we may conclude that Jones' square was 32 rods to the side, and Tom's 9 rods on a side.



Brown's piece was 11 rods longer on a side than Harry's, and since if the above numbers 12 and 1 have 11 for their difference, we may conclude that Brown's piece was 12 rods on a side, and Harry's piece 1 rod.

Hence Tom was Brown's son, Harry was Smith's son, and Ned was Jones' son

31. The mode of arranging the crew may be remembered by attention to the vowels in the following line.

*Populeam virgam mater regina ferebat.*

The vowels refer to the crew as follows,  $a = 1$ ,  $e = 2$ ,  $i = 3$ ,  $o = 4$ , and  $u = 5$ .

We begin with 4 whites because the first vowel is  $o$ , next  $u = 5$  blacks, next  $e = 2$  whites, next  $a = 1$  black, next  $i = 3$  whites, next  $a = 1$  black, next  $a = 1$  white, next  $e = 2$  blacks, next  $e = 2$  whites, next  $i = 3$  blacks, &c., as follows,  $o$  standing for a white and  $+$  for a black.

oooo+++++oo+ooo+o++++oo+++++o+++++oo+

32. You select the multiplier or the multiplicand, such that the sum of its digits shall be exactly divisible by nine. Hence upon the principle of the proof by casting out the nines, the product has the sum of its digits exactly divisible by nine. By subtracting the sum of the digits of the remainder from the next higher multiple of 9 you determine the digit crossed out.

Thus suppose you select 117, and he takes for multiplicand 21613. Then  $21613 \times 117 = 2528721$ . Now, suppose he crosses out the 7; upon reading you the remaining digits 252821, you find that their sum  $= 20$ , which taken from 27 the next higher multiple of 9 leaves 7 the digit he crossed out.

If he crosses out a 0 or a 9, you cannot determine which, but in all other cases you can tell the exact figure.

33. You write the second, fourth, sixth, &c. lines in such a manner as to make the sum of the first pair, the sum of the second pair, &c. an exact number of 9's. Then having settled the number of pairs, you get the answer by multiplying by that number a row of 9's containing as many digits as there are to be figures in the line.

Thus suppose you agree to write 5 lines each, and that each line is to contain 5 digits, or not more than 5 digits. Then  $99999 \times 5 = 499995$  will be the answer. This is shown as follows :

|                   |       |   |         |   |                     |
|-------------------|-------|---|---------|---|---------------------|
| Suppose he writes | 41113 | } | = 99999 | } |                     |
| You write         | 58886 |   |         |   |                     |
| Suppose he writes | 61451 | } | = 99999 | } |                     |
| You write         | 38548 |   |         |   |                     |
| Suppose he writes | 6500  | } | = 99999 | } | = 99999 $\times$ 5. |
| You write         | 93499 |   |         |   |                     |
| Suppose he writes | 1     | } | = 99999 | } |                     |
| You write         | 99998 |   |         |   |                     |
| Suppose he writes | 99999 | } | = 99999 | } |                     |
| You write         | 00000 |   |         |   |                     |
| <hr/>             |       |   |         |   |                     |
| Sum = 499995      |       |   |         |   |                     |

**THE END.**









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